

Bilal Article

# Chapter 5.

## LINER INEQUALITIES AND LINER PROGRAMMING

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# Contents

Exercise 5.1 .....	1
Exercise 5.2 .....	11
Exercise .5.3 .....	17



**Linear Inequalities in one variable:**

The inequalities of the form  $ax + b < 0$ ,  $ax + b > c$

$ax + b \geq c$  and  $ax + by \leq c$  are called *linear inequalities*

In one variable.

**Linear inequalities in two variables:**

The inequalities of the form  $ax + by + c$ ,  $ax + b > c$  and  $ax + by \geq c$  and  $ax + by \leq 0$  are called *linear inequalities in two variables* *ca* *nd* *y* where  $a, b$  and  $c$  are constants.

**Corresponding Equation/ associated Equation:**

- (i) The corresponding equation to any inequality is an equation formed by replacing the inequality symbol with an equal sign. For example

*Corresponding equation of  $x + 2y < 6$  is*

$$x + 2y = 6$$

- (ii) Corresponding equations of  $x \geq 0$  and  $2x + y \geq 2$  are  $x = 0$  and  $2x + y = 2$

Respectively.

**Graphing of a linear inequalities in two variables:**

- The corresponding equation is useful for graphing inequalities, because this equation forms the boundary line to the graph of given inequality.
- A vertical line (line || to  $y$  axis) divides the  $xy$  plane into two regions called *half plane* (*left half plane*  $x < 0$  and *right half plane*  $x \geq 0$ )
- A non-vertical line (line || to  $x$  - axis) divides  $xy$  plane in two regions called "half planes". (*Upper half plane*  $x \geq 0$  and *lower half plane*  $x \leq 0$ )
- If the inequality is strict ( $<$  or  $>$ ) then we draw dashed or broken boundary line.
- If the inequality is non-strict ( $\leq$  or  $\geq$  then we draw a solid boundary line.

**Procedure for graphing a linear inequality in two variables:**

- Graph the corresponding equation of given inequality.
- Select any test point (not on the graph of corresponding equation of inequality) The region  $(0,0)$  is most convenient point to choose as a test pt.

- Put the coordinates of the test pt. in the inequality.
- If the test point satisfied the given inequality, then shade the half plane containing the test point.
- If the test point does not satisfied the given inequality then the shade the half plane that does not contain the test point.

**Solution set of linear inequalities:**

The ordered pair  $(a, b)$  which satisfy the linear inequality in two variables  $x$  and  $y$  form the solution.

**Solution Region:**

Solution region of system of inequalities is the common region that satisfies all given inequality in the system.

**Corner point / vertex:**

A point of the solution region where two of its boundary lines intersect is called the corner point or vertex of the solution region.

## Exercise 5.1

- Graph the solution of each of the following linear inequality in  $xy$  - plane.

(i)  $2x + y \leq 6$

**Solution:**

$$2x + y \leq 6 \rightarrow (i)$$

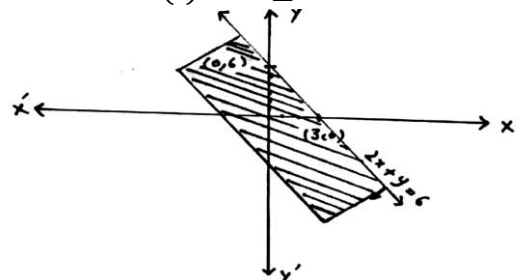
The associated eq. of (i) is  $2x + y = 6 \rightarrow (ii)$

$$(ii) \Rightarrow \text{put } x = 0, y = 6 \text{ so that pt. } (0, 6)$$

Put  $y = 0, x = 3$  so that pt.  $(3, 0)$

Test pt  $(0, 0)$ : We test (i) at  $(0, 0)$

$$(i) \Rightarrow 0 \leq 6 \rightarrow \text{true}$$



(ii)  $3x + 7y \geq 21$

**Solution:**

$$3x + 7y \geq 21 \rightarrow (i)$$

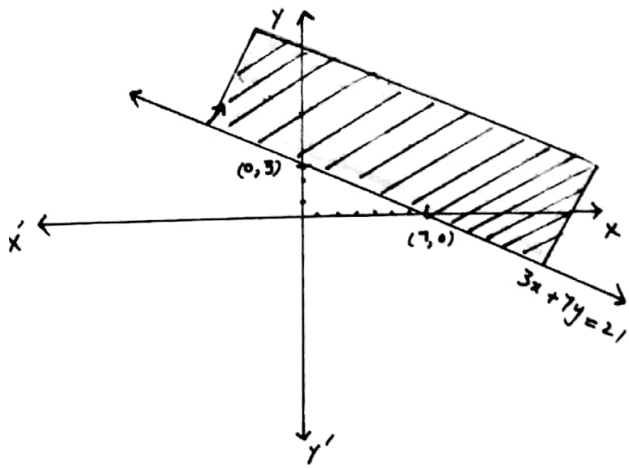
The associated eq. of (i) is  $3x + 7y = 21$

$$(ii) \Rightarrow \text{Put } x = 0, y = 3 \text{ so that pt. } (0, 3)$$

$$\Rightarrow \text{put } y = 0, x = 7 \text{ so pt. } (7, 0)$$

Test pt  $(0, 0)$  we test (i) at  $(0, 0)$

$$(i) \Rightarrow 0 \geq 21 \rightarrow \text{false.}$$



(iii)  $3x - 2y \geq 6$

Solution:

$3x - 2y \geq 6 \rightarrow (i)$

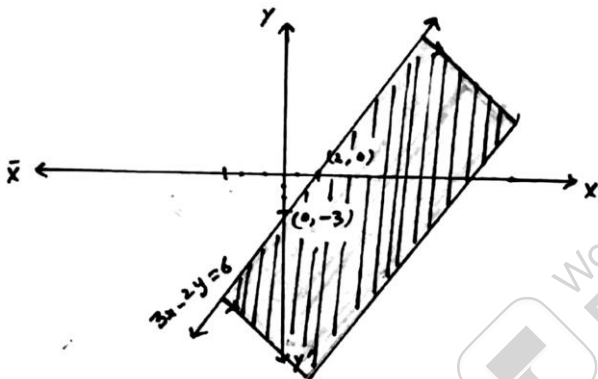
the associated eq. of (i) is  $3x - 2y = 6$

(ii)  $\Rightarrow$  put  $x = 0, y = -3$  so the pt.  $(0, -3)$

put  $y = 0, x = 2$  so the pt.  $(2, 0)$

Test pt  $(0,0)$ : We test (i) at  $(0,0)$  so

$(i) \Rightarrow 0 \geq 6 \rightarrow$  false



(iv)  $5x - 4y \leq 20$

Solution:

$5x - 4y \leq 20 \rightarrow (i)$

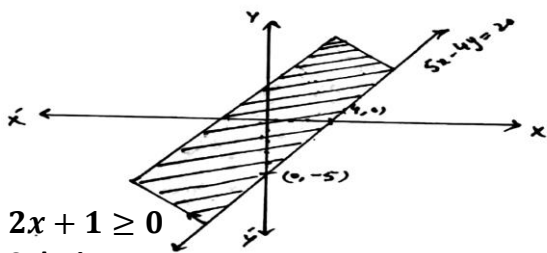
The associated eq. of (i) is  $5x - 4y = 20 \rightarrow (ii)$

(ii)  $\Rightarrow$  Put  $x = 0, y = -5$  so the pt  $(0, -5)$

put  $y = 0, x = 4$  so the pt  $(4, 0)$

Test pt  $(0,0)$ : we test (i) at  $(0,0)$  so

$(i) \Rightarrow 0 \leq 20 \rightarrow$  true



(v)  $2x + 1 \geq 0$

Solution:

$2x + 1 \geq 0 \rightarrow (i)$

The associated eq. of (i) is  $2x + 1 = 0$

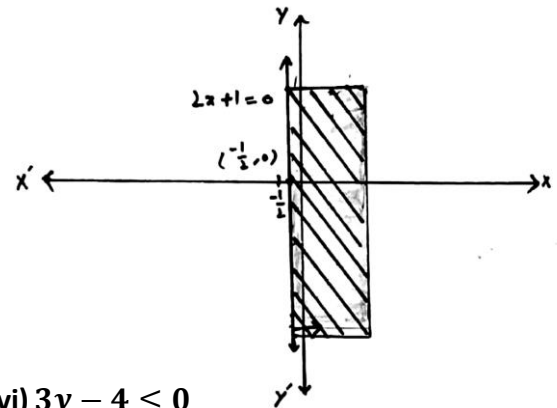
$\Rightarrow 2x = -1 \Rightarrow x$

$= -\frac{1}{2}$  (line || to y-axis passing

Through  $(-\frac{1}{2}, 0)$

Test pt  $(0,0)$ : we test (i) at  $(0,0)$  so

$(i) \Rightarrow 2(0) + 1 \geq 0 \Rightarrow 1 \geq 0 \rightarrow$  true



(vi)  $3y - 4 \leq 0$

Solution:

$3y - 4 \leq 0 \rightarrow (i)$

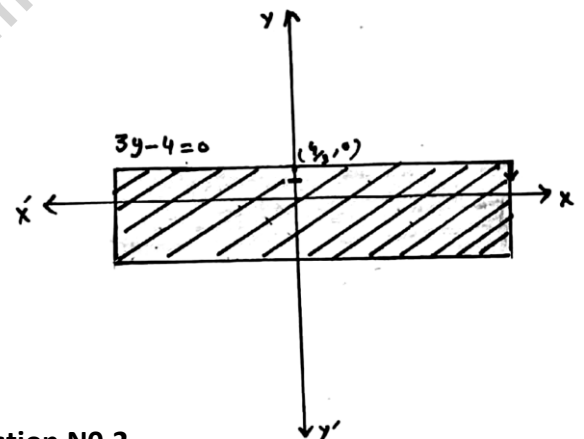
The associated eq. of (i) is  $3y - 4 = 0$

$\Rightarrow 3y = 4 \Rightarrow y = \frac{4}{3}$  (line || to x-axis passing

Through  $(0, \frac{4}{3})$

Test pt  $(0,0)$ : we test (i) at  $(0,0)$  so

$(i) \Rightarrow 3(0) - 4 \leq 0 \Rightarrow -4 \leq 0 \rightarrow$  true



Question NO.2

Indicate the solution set of the following systems of linear inequalities of shading:

(i)  $2x - 3y \leq 6$

$2x + 3y \leq 12$

Solution:

$2x - 3y \leq 6 \rightarrow (i)$

$2x + 3y \leq 12 \rightarrow (ii)$

the associated eqs. of (i) and (ii) are

$l_1; 2x - 3y = 6 \rightarrow (iii) l_2; 2x + 3y = 12 \rightarrow (iv)$

(iii)  $\Rightarrow$  put  $x = 0, y = -2$  so the pt  $(0, -2)$

put  $y = 0, x = 3$  so the pt  $(3, 0)$

(iv)  $\Rightarrow$  put  $x = 0, y = 4$  so the pt  $(0, 4)$

put  $y = 0, x = 6$  so the pt  $(6, 0)$

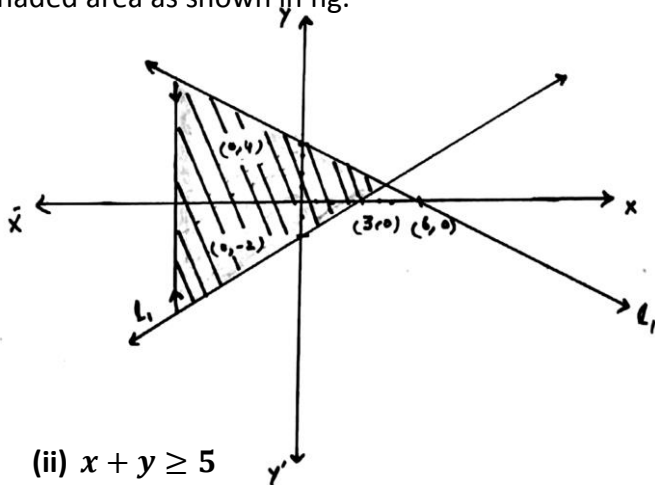
Test pt  $(0,0)$ : we test (i) and (ii) at  $(0,0)$  so

$(i) \Rightarrow 0 \leq 6 \rightarrow$  true

(ii)  $\Rightarrow 0 \leq 12 \rightarrow \text{true}$

Solution region:

The solution of the given is intersection of the graphs of (i) and (ii). so solution region is shaded area as shown in fig.



(ii)  $x + y \geq 5$   
 $-y + x \leq 1$

Solution:

$x + y \rightarrow (i)$

$-y + x \leq 1 \rightarrow (ii)$

the associated eqs. of (i) and (ii) are

$l_1; x + y = 5 \rightarrow (iii); l_2; -y + x = 1 \rightarrow (iv)$

(iii)  $\Rightarrow$  put  $x = 0, y = 5$  so the pt(0,5)

put  $y = 0, x = 5$  so the pt(5,0)

(iv)  $\Rightarrow$  put  $x = 0, y = -1$  so the pt(0,-1)

put  $y = 0, x = 1$  so the pt(1,0)

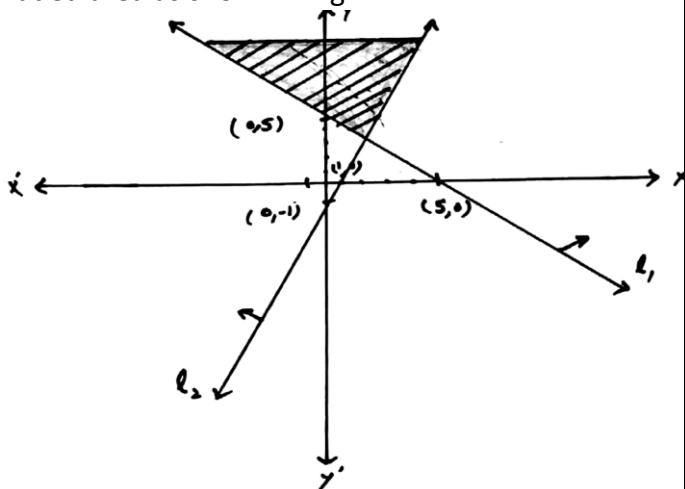
Test pt(0,0): we test (i) and (ii) at (0,0) so

(i)  $\Rightarrow 0 \geq 5 \rightarrow \text{true}$

(ii)  $\Rightarrow 0 \leq 1 \rightarrow \text{true}$

Solution region:

The solution of the given is intersection of the graphs of (i) and (ii). so solution region is shaded area as shown in fig.



(iii)  $3x + 7y \geq 21$   
 $x - y \geq 2$

Solution:

$3x + 7y \geq 21 \rightarrow (i)$

$x - y \geq 2 \rightarrow (ii)$

the associated eqs. of (i) and (ii) are

$l_1; 3x + 7y = 21 \rightarrow (iii); l_2; x - y = 2 \rightarrow (iv)$

(iii)  $\Rightarrow$  put  $x = 0, y = 3$  so the pt(0,3)

put  $y = 0, x = 7$  so the pt(7,0)

(iv)  $\Rightarrow$  put  $x = 0, y = -2$  so the pt(0,-2)

put  $y = 0, x = 2$  so the pt(2,0)

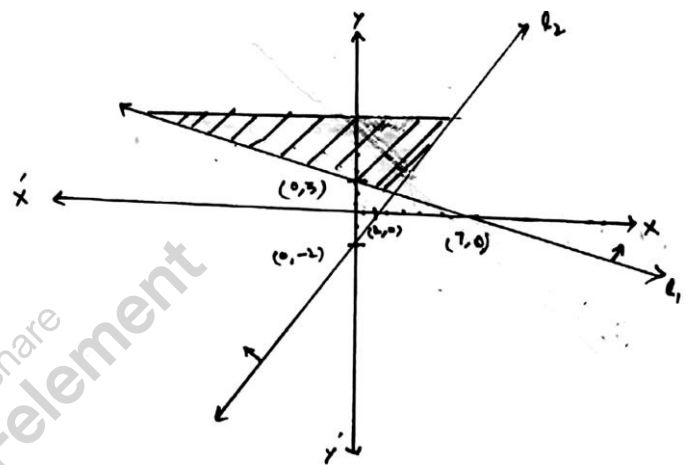
Test pt(0,0): we test (i) and (ii) at (0,0) so

(i)  $\Rightarrow 0 \geq 21 \rightarrow \text{true}$

(ii)  $\Rightarrow 0 \leq 2 \rightarrow \text{true}$

Solution region:

The solution of the given system of the graphs of (i) and (ii). so solution region is shaded area as shown in fig.



(iv)  $4x - 3y \leq 12, x \geq -\frac{3}{2}$

Solution:

$4x - 3y \leq 12 \rightarrow (i)$

$x \geq -\frac{3}{2} \rightarrow (ii)$

the associated eqs. of (i) and (ii) are

$l_1; 4x - 3y = 12 \rightarrow (iii); l_2; x = -\frac{3}{2} \rightarrow (iv)$

(iii)  $\Rightarrow$  put  $x = 0, y = -4$  so the pt(0,-4)

put  $y = 0, x = 3$  so the pt(3,0)

(iv)  $\Rightarrow$  put  $x = -\frac{3}{2}$ , (line || to y-axis through pt( $-\frac{3}{2}, 0$ ))

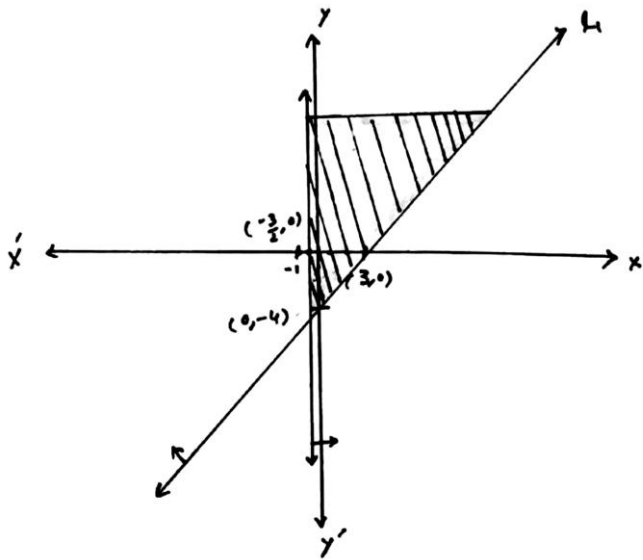
Test pt(0,0): we test (i) and (ii) at (0,0) so

(i)  $\Rightarrow 0 \leq 12 \rightarrow \text{true}$

(ii)  $\Rightarrow 0 \leq -\frac{3}{2} \rightarrow \text{true}$

Solution region:

The solution of the given system is intersection of the graph of (i) and (ii). so solution region is shaded area as shown in fig.



(v)  $3x + 7y \geq 21, y \leq 4$

Solution:

$3x + 7y \geq 21 \rightarrow (i)$

$y \leq 4 \rightarrow (ii)$

the associated eqs. of (i) and (ii) are

$l_1; 3x + 7y = 21 \rightarrow (iii) l_2; y = 4 \rightarrow (iv)$

(iii)  $\Rightarrow$  put  $x = 0, y = 3$  so the pt(0,3)

put  $y = 0, x = 7$  so the pt(7,0)

(iv)  $\Rightarrow$  put  $y = 4$  (line || to  $x$ -axis through pt(0,4)

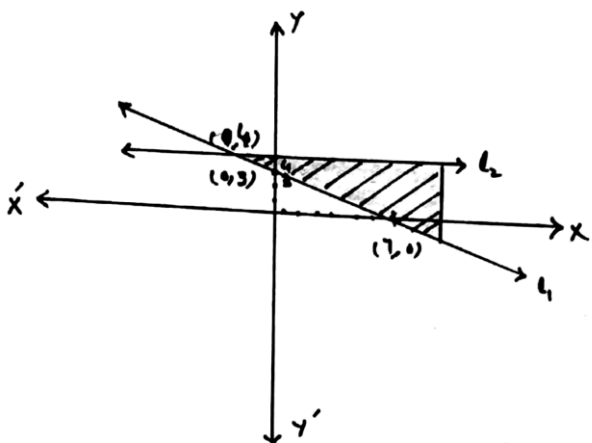
Test pt(0,0): we test (i) and (ii) at (0,0) so

(i)  $\Rightarrow 0 \geq 21 \rightarrow$  false

(ii)  $\Rightarrow 0 \leq 4 \rightarrow$  true

Solution region:

The solution of the given system is intersection of the graph of (i) and (ii). so solution region is shaded area as shown in fig.



Question No.3

Indicate the solution region of the following systems of linear inequalities of shading.

(i)  $2x - 3y \leq 6, 2x + 3y \leq 12; y \geq 0$

Solution:

$2x - 3y \leq 6 \rightarrow (i)$

$2x + 3y \leq 12 \rightarrow (ii)$

the associated eqs. of (i) and (ii) are  
 $l_1; 2x - 3y = 6 \rightarrow (iii) l_2; 2x + 3y \leq 12 \rightarrow (iv)$   
 (iii)  $\Rightarrow$  put  $x = 0, y = -2$  so the pt(0, -2)  
 put  $y = 0, x = 3$  so the pt(3, 0)  
 (iv)  $\Rightarrow$  put  $x = 0, y = 4$  so the pt(0, 4)  
 put  $y = 0, x = 6$  so the pt(6, 0)

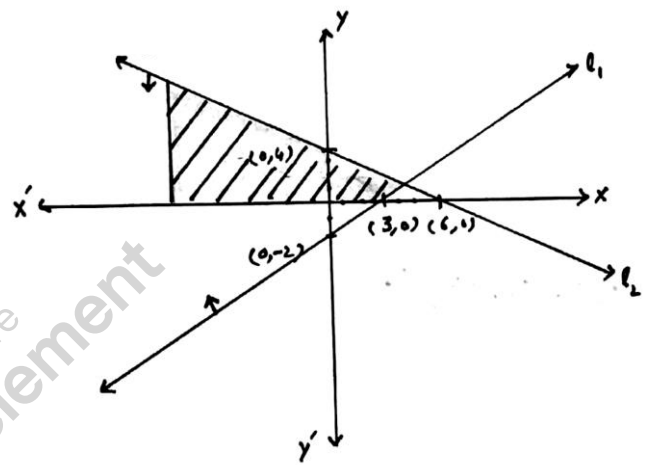
Test pt(0,0): we test (i) and (ii) at (0,0) so

(i)  $\Rightarrow 0 \geq 6 \rightarrow$  true

(ii)  $\Rightarrow 0 \leq 12 \rightarrow$  true

Solution region:

The solution of the given system is intersection of the graphs of (i) and (ii). so solution set is upper half plane including the graph of boundary line  $y = 0$  as shown in fig.



(ii)  $x + y \leq 5, y - 2x \leq 2; x \geq 0$

Solution:

$x + y \leq 5 \rightarrow (i) y - 2x \leq 2 \rightarrow (ii)$

the associated eqs. of (i) and (ii) are

$l_1; x + y \leq 5 \rightarrow (iii) l_2; y - 2x = 2 \rightarrow (iv)$

(iii)  $\Rightarrow$  put  $x = 0, y = 5$  so the pt(0,5)

put  $y = 0, x = 5$  so the pt(5,0)

(iv)  $\Rightarrow$  put  $x = 0, y = 2$  so the pt(0,2)

put  $y = 0, x = -1$  so the pt(-1,0)

Test pt(0,0): we test (i) and (ii) at (0,0) so

(i)  $\Rightarrow 0 \leq 5 \rightarrow$  true

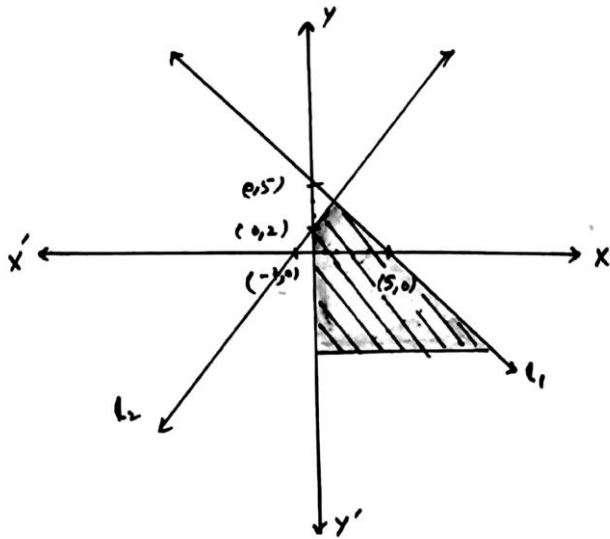
(ii)  $\Rightarrow 0 \leq 2 \rightarrow$  true

Solution region:

The solution of the given system is intersection of the graphs of (i) and (ii).

Also  $x \geq 0$  shows that the graph the solution set of right half plane including the graph of boundary line

$x = 0$  As shown in fig.



(iii)  $x + y \geq 5$ ;  $x - y \geq 1$ ;  $y \geq 0$

**Solution:**

$x + y \geq 5 \rightarrow (i)$   $x - y \geq 1 \rightarrow (ii)$

the associated eqs. of (i) and (ii) are

$l_1; x + y \geq 5 \rightarrow (iii) l_2; x - y = 1 \rightarrow (iv)$

(iii)  $\Rightarrow$  put  $x = 0, y = 5$  so the pt  $(0, 5)$

put  $y = 0, x = 5$  so the pt  $(5, 0)$

(iv)  $\Rightarrow$  put  $x = 0, y = -1$  so the pt  $(0, -1)$

put  $y = 0, x = 1$  so the pt  $(1, 0)$

Test pt  $(0, 0)$ : we test (i) and (ii) at  $(0, 0)$  so

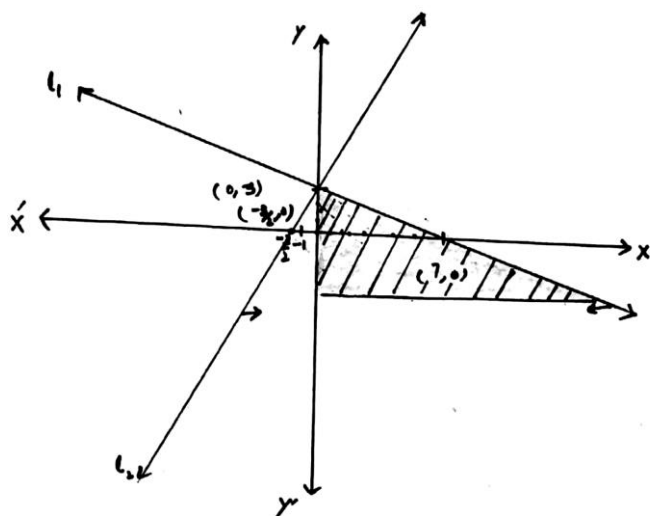
(i)  $\Rightarrow 0 \geq 5 \rightarrow$  false

(ii)  $\Rightarrow 0 \geq 1 \rightarrow$  true

**Solution region:**

The solution of the given system is intersection of the graphs of (i) and (ii).

Also  $y \geq 0$  shows that the solution set is upper half plane including the graph of boundary line  $y = 0$  as shown in fig.



(iv)  $3x + 7y \leq 21$ ,  $x - y \leq 2$ ,  $x \geq 0$

**Solution:**

$3x + 7y \leq 21 \rightarrow (i)$   $x - y \leq 2 \rightarrow (ii)$

the associated eqs. of (i) and (ii) are  $l_1; 3x + 7y \leq 21 \rightarrow (iii) l_2; x - y \leq 2 \rightarrow (iv)$   
 (iii)  $\Rightarrow$  put  $x = 0, y = 3$  so the pt  $(0, 3)$   
 put  $y = 0, x = 7$  so the pt  $(7, 0)$   
 (iv)  $\Rightarrow$  put  $x = 0, y = -2$  so the pt  $(0, -2)$   
 put  $y = 0, x = 2$  so the pt  $(2, 0)$

Test pt  $(0, 0)$ : we test (i) and (ii) at  $(0, 0)$  so

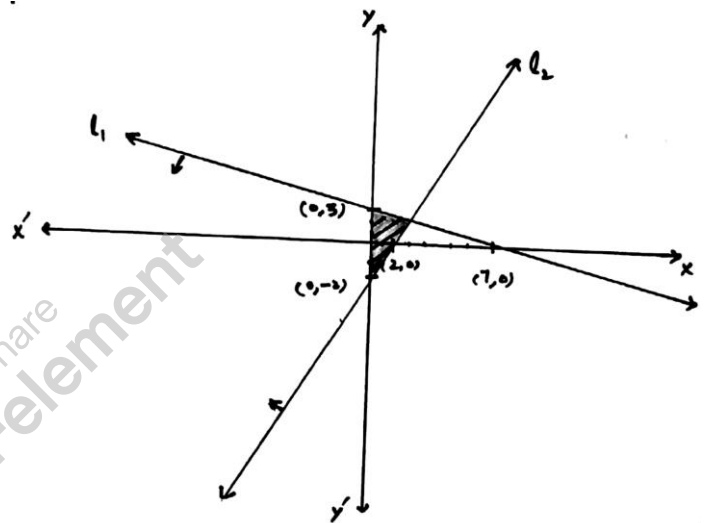
(i)  $\Rightarrow 0 \leq 21 \rightarrow$  true

(ii)  $\Rightarrow 0 \leq 2 \rightarrow$  true

**Solution region:**

The solution of the given system is intersection of the graphs of (i) and (ii).

Also  $x \geq 0$  shows that the solution set is right half plane including the graph of boundary line  $x = 0$  as shown in fig.



(v)  $3x + 7y \leq 21$ ;  $x - y \leq 2$ ;  $y = 0$

**Solution:**

$3x + 7y \leq 21 \rightarrow (i)$   $x - y \leq 2 \rightarrow (ii)$

the associated eqs. of (i) and (ii) are

$l_1; 3x + 7y \leq 21 \rightarrow (iii) l_2; x - y \leq 2 \rightarrow (iv)$

(iii)  $\Rightarrow$  put  $x = 0, y = 3$  so the pt  $(0, 3)$

put  $y = 0, x = 7$  so the pt  $(7, 0)$

(iv)  $\Rightarrow$  put  $x = 0, y = -2$  so the pt  $(0, -2)$

put  $y = 0, x = 2$  so the pt  $(2, 0)$

Test pt  $(0, 0)$ : we test (i) and (ii) at  $(0, 0)$  so

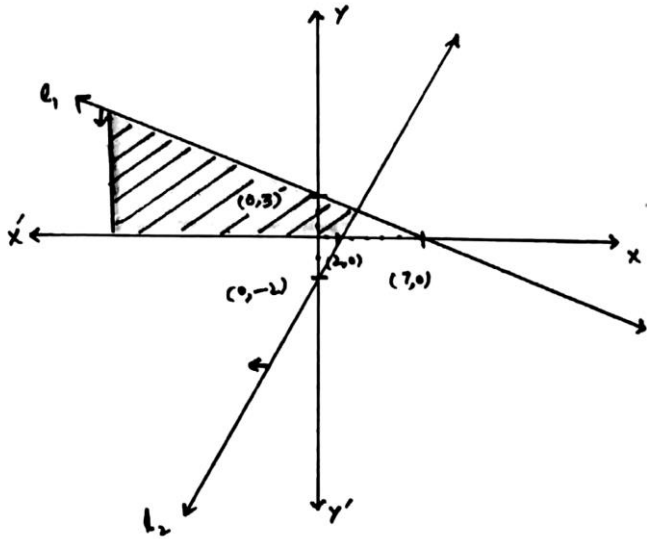
(i)  $\Rightarrow 0 \leq 21 \rightarrow$  true

(ii)  $\Rightarrow 0 \leq 2 \rightarrow$  true

**Solution region:**

The solution of the given system is intersection of the graphs of (i) and (ii).

Also  $y \geq 0$  shows that the solution set is upper half plane including the graph of boundary line  $y = 0$  as shown in fig.



(vi)  $3x + 7y \leq 21$ ,  $2x - y \geq -3$ ,  $x \geq 0$

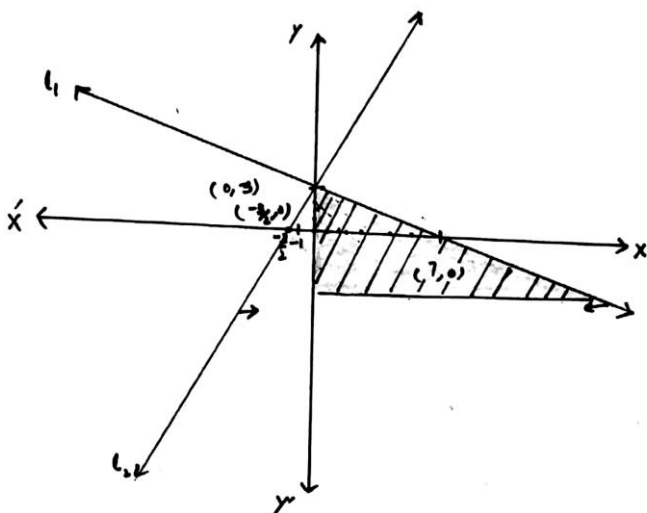
**Solution:**

$3x + 7y \leq 21 \rightarrow$  (i)  $2x - y \geq -3 \rightarrow$  (ii)  
 the associated eqs. of (i) and (ii) are  
 $l_1; 3x + 7y \leq 21 \rightarrow$  (iii)  $l_2; 2x - y \leq -3 \rightarrow$  (iv)  
 (iii)  $\Rightarrow$  put  $x = 0, y = 3$  so the pt  $(0, 3)$   
 put  $y = 0, x = 7$  so the pt  $(7, 0)$   
 (iv)  $\Rightarrow$  put  $x = 0, y = 3$  so the pt  $(0, 3)$   
 put  $y = 0, x = -\frac{3}{2}$  so the pt  $(-\frac{3}{2}, 0)$

Test pt  $(0, 0)$ : we test (i) and (ii) at  $(0, 0)$  so  
 (i)  $\Rightarrow 0 \leq 21 \rightarrow$  true  
 (ii)  $\Rightarrow 0 \geq -3 \rightarrow$  true

**Solution region:**

The solution of the given system is intersection of the graphs of (i) and (ii).  
 Also  $x \geq 0$  shows that the solution set of right half plane including the graph of boundary line  $x = 0$  as shown in fig.



Question No.4

Graph the solution region of the following system of linear inequalities and find the corner points in each case.

(i)  $2x - 3y \leq 6$ ;  $2x + 3y \leq 12$ ;  $x \geq 0$

**Solution:**

$2x - 3y \leq 6 \rightarrow$  (i)  $2x + 3y \leq 12 \rightarrow$  (ii)  
 the associated eqs. of (i) and (ii) are  
 $l_1; 2x - 3y = 6 \rightarrow$  (iii)  $l_2; 2x + 3y \leq 12 \rightarrow$  (iv)  
 (iii)  $\Rightarrow$  put  $x = 0, y = -2$  so the pt  $(0, -2)$   
 put  $y = 0, x = 3$  so the pt  $(3, 0)$   
 (iv)  $\Rightarrow$  put  $x = 0, y = 4$  so the pt  $(0, 4)$   
 put  $y = 0, x = 6$  so the pt  $(6, 0)$

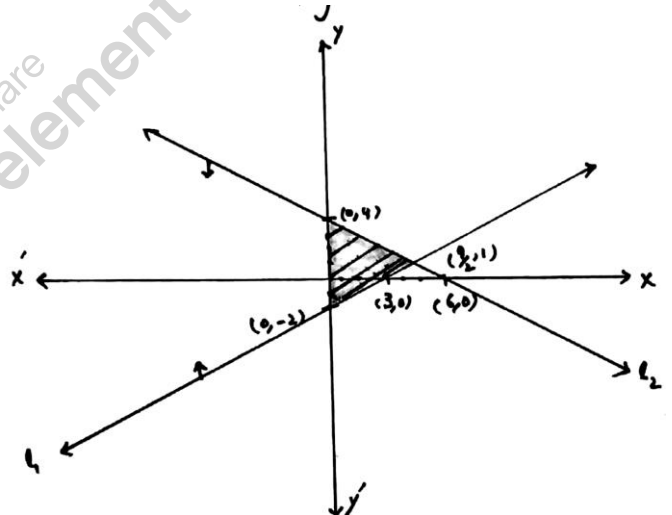
Test pt  $(0, 0)$ : we test (i) and (ii) at  $(0, 0)$  so

(i)  $\Rightarrow 0 \leq 6 \rightarrow$  true

(ii)  $\Rightarrow 0 \leq 12 \rightarrow$  true

**Solution region:**

The solution of the given system is intersection of the graphs of (i) and (ii).  
 Also  $x \geq 0$  shows that the solution set of right half plane including the graph of boundary line  $x = 0$  as shown in fig.



**Corner point:**

$2x - 3y = 6 \rightarrow$  (i)  $2x + 3y = 12 \rightarrow$  (ii)  
 by (i) + (ii)  $\Rightarrow 4x = 18 \Rightarrow x = \frac{9}{2}$  put in (ii)  
 $2(\frac{9}{2}) + 3y = 12 \Rightarrow 3y = 12 - 9 \Rightarrow y = 1$   
 so  $(\frac{9}{2}, 1)$  is pt. of intersection of lines (i) and (ii). hence corner points are  $(0, -2)$ ,  $(0, 4)$ ,  $(\frac{9}{2}, 1)$

(ii)  $x + y \leq 5$ ;  $-2x + y \leq 2$ ,  $y \geq 0$

**Solution:**

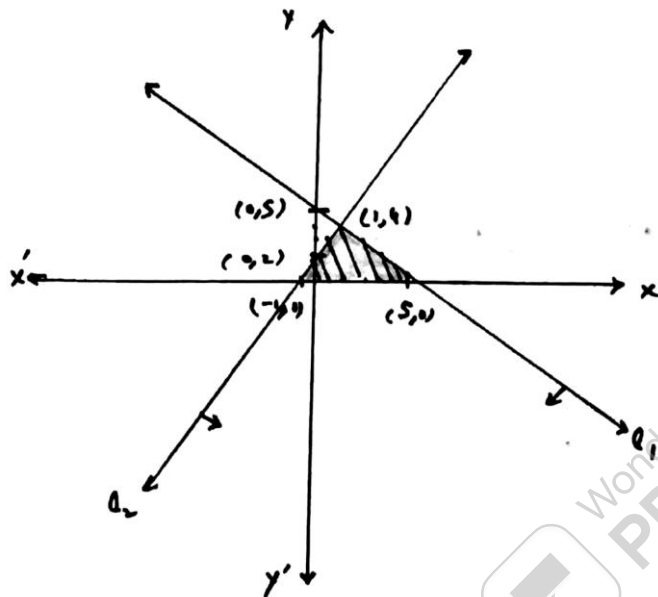
$x + y \leq 5 \rightarrow$  (i) and  $-2x + y \leq 2 \rightarrow$  (ii)  
 the associated eqs. of (i) and (ii) are  
 $l_1; x + y = 5 \rightarrow$  (iii)  $l_2; -2x + y = 2 \rightarrow$  (iv)  
 (iii)  $\Rightarrow$  put  $x = 0, y = 5$  so the pt  $(0, 5)$



put  $y = 0, x = 5$  so the pt(5,0)  
 (iv)  $\Rightarrow$  put  $x = 0, y = 2$  so the pt(0,2)  
 put  $y = 0, x = -1$  so the pt(-1,0)  
 Test pt(0,0): we test (i) and (ii) at (0,0) so  
 (i)  $\Rightarrow 0 \leq 5 \rightarrow$  true  
 (ii)  $\Rightarrow 0 \leq 2 \rightarrow$  true

**Solution region:**

The solution of the given system is intersection of the graphs of (i) and (ii).  
 Also  $y \geq 0$  shows that the solution set is upper half plane including the graph of boundary line  $y = 0$  as shown in fig.



**Corner point:**

$x + y = 5 \rightarrow$  (i)  $-2x + y = 2 \rightarrow$  (ii)  
 by (ii) - (i)  $\Rightarrow 3x = 3 \Rightarrow x = 1$  put in (i)  $1 + y = 5$   
 $\Rightarrow y = 4$  so (1,4) is the pt. of intersection of lines  
 (i) and (ii). hence corner point are (-1,0), (5,0), (1,4)

(iii)  $3x + 7y \leq 21 ; 2x - y \leq -3 , y \geq 0$

**Solution:**

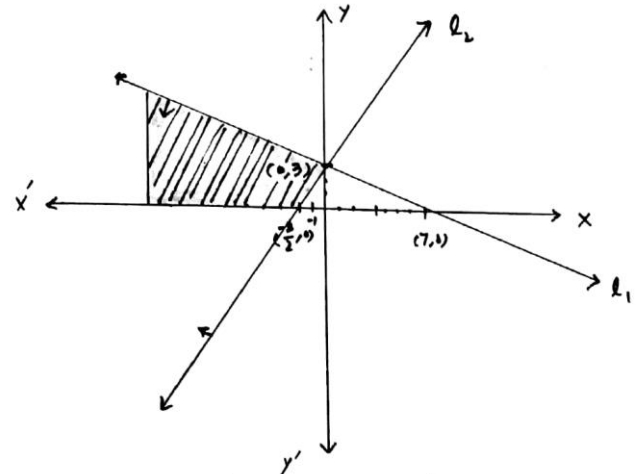
$3x + 7y \leq 21 \rightarrow$  (i)  $2x - y \leq -3 \rightarrow$  (ii)

the associated eqs. of (i) and (ii) are  
 $l_1; 3x + 7y = 21 \rightarrow$  (iii)  $l_2; 2x - y = -3 \rightarrow$  (iv)  
 (iii)  $\Rightarrow$  put  $x = 0, y = 3$  so the pt(0,3)  
 put  $y = 0, x = 7$  so the pt(7,0)  
 (iv)  $\Rightarrow$  put  $x = 0, y = 3$  so the pt(0,3)  
 put  $y = 0, x = -\frac{3}{2}$  so the pt( $-\frac{3}{2}, 0$ )

Test pt(0,0): we test (i) and (ii) at (0,0) so  
 (i)  $\Rightarrow 0 \leq 21 \rightarrow$  true  
 (ii)  $\Rightarrow 0 \leq -3 \rightarrow$  false

**Solution region:**

The solution of the given system is intersection of the graphs of (i) and (ii).  
 Also  $y \geq 0$  shows that the solution set is upper half plane including the graph of boundary line  $y = 0$  as shown in fig.



**Corner point:**

Corner pts. are (0,3) and ( $-\frac{3}{2}, 0$ )

(iv)  $3x + 2y \geq 6 \rightarrow$  (i)  $x + 3y \leq 6 ; y \geq 0$

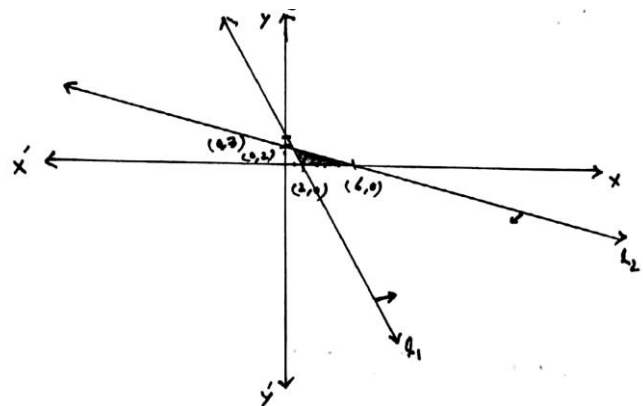
**Solution:**

$3x + 2y \geq 6 \rightarrow$  (i) and  $x + 3y \leq 6 \rightarrow$  (ii)  
 the associated eqs. of (i) and (ii) are  
 $l_1; 3x + y = 6 \rightarrow$  (iii)  $l_2; x + 3y = 6 \rightarrow$  (iv)  
 (iii)  $\Rightarrow$  put  $x = 0, y = 3$  so the pt(0,3)  
 put  $y = 0, x = 2$  so the pt(2,0)  
 (iv)  $\Rightarrow$  put  $x = 0, y = 2$  so the pt(0,2)  
 put  $y = 0, x = 6$  so the pt(6,0)

Test pt(0,0): we test (i) and (ii) at (0,0) so  
 (i)  $\Rightarrow 0 \geq 6 \rightarrow$  false  
 (ii)  $\Rightarrow 0 \leq 6 \rightarrow$  true

**Solution region:**

The solution of the given system is intersection of the graphs of (i) and (ii).  
 Also  $y \geq 0$  shows that the solution set is upper half plane including the graph of boundary line  $y = 0$  as shown in fig.



**Corner point:**

as  $3x + 2y = 6 \rightarrow (i)$

$x + 3y = 6 \rightarrow (ii)$

By  $3(ii) - (i) \Rightarrow \frac{\pm 3x \pm 2y = \pm 6}{7y = 12}$

$\Rightarrow y = \frac{12}{7}$  put in (ii)

$\Rightarrow x = 6 - \frac{36}{7} = \frac{42 - 36}{7} = \frac{6}{7}$

So pt. of intersection of lines (i) and (ii) is

$(\frac{6}{7}, \frac{12}{7})$  hence corner pts are (2,0), (6,0),  $(\frac{6}{7}, \frac{12}{7})$

(v)  $5x + 7y \leq 35$ ;  $-x + 3y \leq 3$ ;  $x \geq 0$

**Solution:**

$5x + 7y \leq 35 \rightarrow (i)$   $-x + 3y \leq 3 \rightarrow (ii)$

the associated eqs. of (i) and (ii) are

$l_1; 5x + 7y = 35 \rightarrow (iii)$   $l_2; -x + 3y = 3 \rightarrow (iv)$

(iii)  $\Rightarrow$  put  $x = 0, y = 5$  so the pt(0,5)

put  $y = 0, x = 7$  so the pt(7,0)

(iv)  $\Rightarrow$  put  $x = 0, y = 1$  so the pt(0,1)

put  $y = 0, x = -3$  so the pt(-3,0)

Test pt(0,0): we test (i) and (ii) at (0,0) so

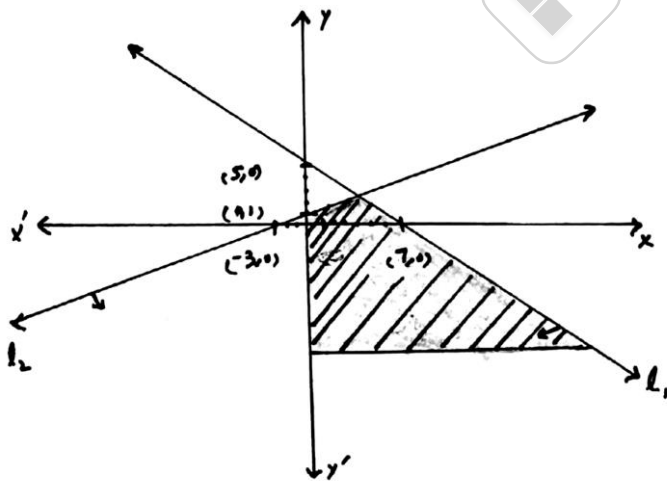
(i)  $\Rightarrow 0 \leq 35 \rightarrow$  true

(ii)  $\Rightarrow 0 \leq 3 \rightarrow$  true

**Solution region:**

The solution of the given system is intersection of the graphs of (i) and (ii).

Also  $x \geq 0$  shows that the solution set of right half plane including the graph of boundary line  $x = 0$  as shown in fig.



**Corner point:** as  $5x + 7y = 35 \rightarrow (i)$

$-x + 3y = 3 \rightarrow (ii)$

By  $5(ii) + (i) \Rightarrow 22y - 50 \Rightarrow y = \frac{50}{22}$  put in (ii)

$-x + 3(\frac{50}{22}) = 3$

$\Rightarrow -x = 3 - \frac{150}{22} = 3 - \frac{75}{11} = \frac{33 - 75}{11} = -\frac{42}{11}$

$\Rightarrow -x = -\frac{42}{11} \Rightarrow x = \frac{42}{11}$

So pt. of intersection of lines (i) and (ii) is

$(\frac{42}{11}, \frac{25}{11})$  so corner points are (0,1), (7,0),  $(\frac{42}{11}, \frac{25}{11})$ .

(vi)  $5x + 7y \leq 35$ ;  $x - 2y \leq 2$ ;  $x \geq 0$

**Solution:**

$5x + 7y \leq 35 \rightarrow (i)$   $x - 2y \leq 2 \rightarrow (ii)$

the associated eqs. of (i) and (ii) are

$l_1; 5x + 7y = 35 \rightarrow (iii)$   $l_2; x - 2y = 2 \rightarrow (iv)$

(iii)  $\Rightarrow$  put  $x = 0, y = 5$  so the pt(0,5)

put  $y = 0, x = 7$  so the pt(7,0)

(iv)  $\Rightarrow$  put  $x = 0, y = -1$  so the pt(0, -1)

put  $y = 0, x = 2$  so the pt(2,0)

Test pt(0,0): we test (i) and (ii) at (0,0) so

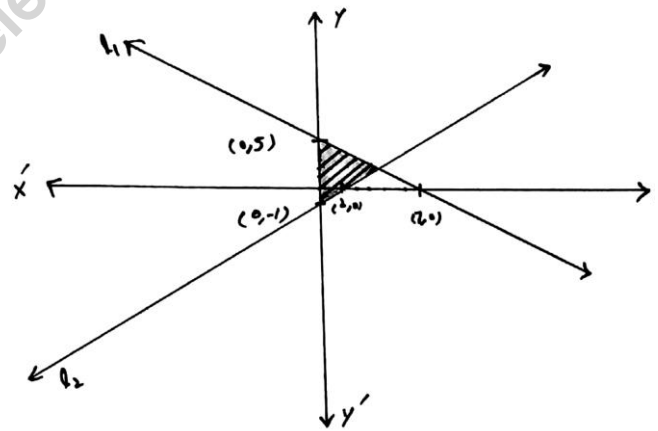
(i)  $\Rightarrow 0 \leq 35 \rightarrow$  true

(ii)  $\Rightarrow 0 \leq 2 \rightarrow$  true

**Solution region:**

The solution of the given system is intersection of the graphs of (i) and (ii).

Also  $x \geq 0$  shows that the solution set of right half plane including the graph of boundary line  $x = 0$  as shown in fig.



**Corner points:**

as  $5x + 7y = 35 \rightarrow (i)$

$x - 2y = 2 \rightarrow (ii)$

By  $5(ii) - (i) \Rightarrow 5x - 10y = 10$

$\pm 5x \pm 7y = \pm 35$

$-17y = -25 \Rightarrow y = \frac{25}{17}$

put in (ii)  $\Rightarrow x - 2(\frac{25}{17}) = 2 \Rightarrow x = 2 + \frac{50}{17}$

$x = \frac{34 + 50}{17} \Rightarrow x = \frac{84}{17}$

So pt. of intersection of lines (i) and (ii) are

$(\frac{84}{17}, \frac{25}{17})$  .so corner pts. are (0, -2), (0,5)

$(\frac{84}{17}, \frac{25}{17})$ .

**Question No.5**

**Graph the solution region of the following system of linear inequalities by shading**

(i)  $3x - 4y \leq 12$ ;  $3x + 2y \geq 3$ ,  $x + 2y \leq 9$

**Solution:**

$3x - 4y \leq 12 \rightarrow (i)$ ;  $3x + 2y \geq 3 \rightarrow (ii)$   
 $x + 2y \leq 9 \rightarrow (iii)$

**The associated eqs. of (i), (ii) and (iii) are**

$l_1; 3x - 4y = 12 \rightarrow (iv)$ ,  $l_2; 3x + 2y = 3 \rightarrow (ii)$

$l_3; x + 2y = 9 \rightarrow (vi)$

(iv)  $\Rightarrow$  put  $x = 0, y = -3$  so that pt.  $(0, -3)$   
 put  $y = 0, x = 4$  so the pt.  $(4, 0)$

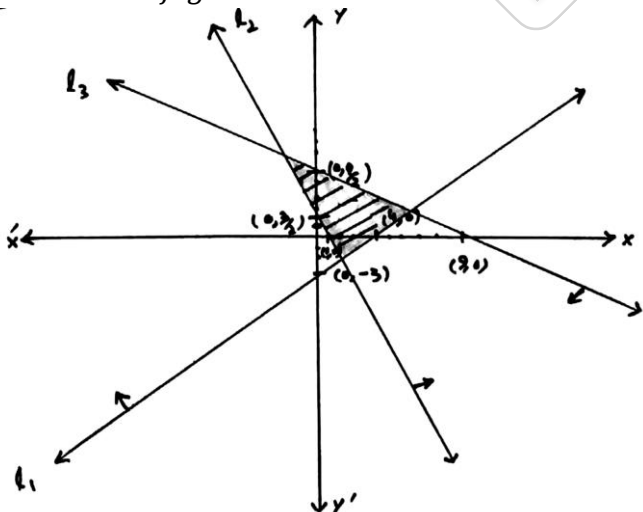
(v)  $\Rightarrow$  put  $x = 0, y = -\frac{3}{2}$  so the pt.  $(0, \frac{3}{2})$   
 put  $y = 0, x = 1$  so the pt.  $(1, 0)$

(vi)  $\Rightarrow$  put  $x = 0, y = \frac{9}{2}$  so the pt.  $(0, \frac{9}{2})$   
 put  $y = 0, x = 9$  so the pt.  $(9, 0)$

Test pt  $(0,0)$ : we test (i), (ii) and (iii) at  $(0,0)$   
 so (i)  $\Rightarrow 0 \leq 12 \rightarrow$  true (ii)  $\Rightarrow 0 \geq 3 \rightarrow$  false  
 (iii)  $\Rightarrow 0 \leq 9 \rightarrow$  true

**Solution region:**

The solution of the given system is intersection of (i), (ii) and (iii) so solution region is shaded area as shown in fig.



(ii)  $3x - 4y \leq 12$ ;  $x + 2y \leq 6$ ;  $x + y \geq 1$

**Solution:**

$3x - 4y \leq 12 \rightarrow (i)$ ;  $x + 2y \geq 6 \rightarrow (ii)$   
 $x + y \geq 1 \rightarrow (iii)$

**The associated eqs. of (i), (ii) and (iii) are**

$l_1; 3x - 4y = 12 \rightarrow (iv)$ ,  $l_2; x + 2y = 6 \rightarrow (ii)$

$l_3; x + y = 1 \rightarrow (vi)$

(iv)  $\Rightarrow$  put  $x = 0, y = -3$  so that pt.  $(0, -3)$

put  $y = 0, x = 4$  so the pt.  $(4, 0)$

(v)  $\Rightarrow$  put  $x = 0, y = 3$  so the pt.  $(0, 3)$

put  $y = 0, x = 6$  so the pt.  $(6, 0)$

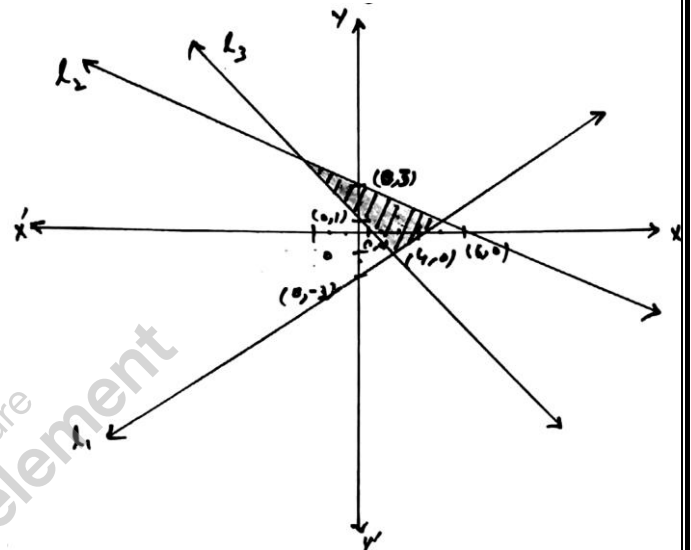
(vi)  $\Rightarrow$  put  $x = 0, y = 1$  so the pt.  $(0, 1)$

put  $y = 0, x = 1$  so the pt.  $(1, 0)$

Test pt  $(0,0)$ : we test (i), (ii) and (iii) at  $(0,0)$   
 so (i)  $\Rightarrow 0 \leq 12 \rightarrow$  true (ii)  $\Rightarrow 0 \leq 6 \rightarrow$  true  
 (iii)  $\Rightarrow 0 \geq 1 \rightarrow$  false

**Solution region:**

The solution of the given system is intersection of (i), (ii) and (iii) so solution region is shaded area as shown in fig.



(iii)  $2x + y \leq 4$ ;  $2x - 3y \geq 12$ ;  $x + 2y \leq 12$

$x + 2y \leq 6$

**Solution:**

$2x + y \leq 4 \rightarrow (i)$ ;  $2x - 3y \geq 12 \rightarrow (ii)$ ;  
 $x + 2y \leq 12 \rightarrow (iii)$

**The associated eqs. of (i), (ii) and (iii) are**

$l_1; 2x + y = 4 \rightarrow (iv)$ ,  $l_2; 2x - 3y = 12 \rightarrow (ii)$

$l_3; 2x - 3y = 12 \rightarrow (vi)$

(iv)  $\Rightarrow$  put  $x = 0, y = 4$  so that pt.  $(0, 4)$

put  $y = 0, x = 2$  so the pt.  $(2, 0)$

(v)  $\Rightarrow$  put  $x = 0, y = -4$  so the pt.  $(0, -4)$

put  $y = 0, x = 6$  so the pt.  $(6, 0)$

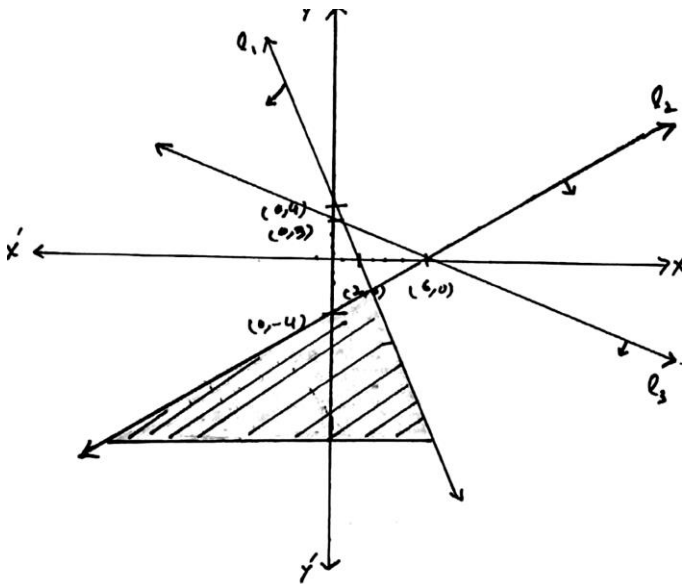
(vi)  $\Rightarrow$  put  $x = 0, y = 3$  so the pt.  $(0, 3)$

put  $y = 0, x = 6$  so the pt.  $(6, 0)$

Test pt  $(0,0)$ : we test (i), (ii) and (iii) at  $(0,0)$   
 so (i)  $\Rightarrow 0 \leq 4 \rightarrow$  true (ii)  $\Rightarrow 0 \geq 12 \rightarrow$  false  
 (iii)  $\Rightarrow 0 \leq 6 \rightarrow$  true

**Solution region:**

The solution of the given system is intersection of (i), (ii) and (iii) so solution region is shaded area as shown in fig.



(iv)  $2x + y \leq 10$ ;  $x + y \leq 7$ ;  $-2x + y \leq 4$

**Solution:**

$2x + y \leq 10 \rightarrow (i)$ ;  $x + y \leq 7 \rightarrow (ii)$ ;  
 $-2x + y \leq 4 \rightarrow (iii)$

The associated eqs. of (i), (ii) and (iii) are

$l_1; 2x + y = 10 \rightarrow (iv)$ ,  $l_2; x + y = 7 \rightarrow (ii)$   
 $l_3; -2x + y = 4 \rightarrow (vi)$

(iv)  $\Rightarrow$  put  $x = 0, y = 10$  so that pt. (0, 10)  
 put  $y = 0, x = 5$  so the pt. (5, 0)

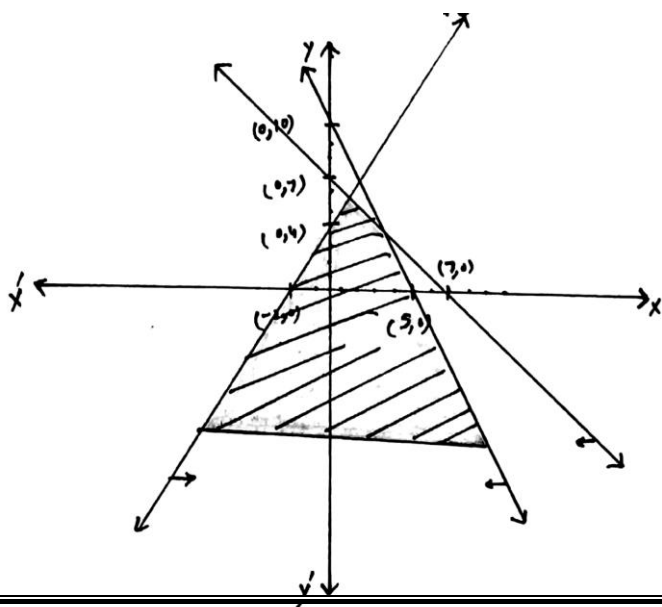
(v)  $\Rightarrow$  put  $x = 0, y = 7$  so the pt. (0, 7)  
 put  $y = 0, x = 7$  so the pt. (7, 0)

(vi)  $\Rightarrow$  put  $x = 0, y = 4$  so the pt. (0, 4)  
 put  $y = 0, x = -2$  so the pt. (-2, 0)

Test pt(0,0): we test (i), (ii) and (iii) at (0,0)  
 so (i)  $\Rightarrow 0 \leq 10 \rightarrow$  true (ii)  $\Rightarrow 0 \leq 7 \rightarrow$  true  
 (iii)  $\Rightarrow 0 \leq 4 \rightarrow$  true

**Solution region:**

The solution of the given system is intersection of (i), (ii) and (iii) so solution region is shaded area as shown in fig.



(v)  $2x + 3y \leq 18$ ;  $2x + y \leq 10$ ;  $-2x + y \leq 2$

**Solution:**

$2x + 3y \leq 18 \rightarrow (i)$   $2x + y \leq 10 \rightarrow (ii)$   
 $-2x + y \leq 2 \rightarrow (iii)$

The associated eqs. of (i), (ii) and (iii) are

$l_1; 2x + 3y = 18 \rightarrow (iv)$ ,  $l_2; 2x + y = 10 \rightarrow (ii)$

$l_3; -2x + y = 2 \rightarrow (vi)$

(iv)  $\Rightarrow$  put  $x = 0, y = 6$  so that pt. (0, 6)  
 put  $y = 0, x = 9$  so the pt. (9, 0)

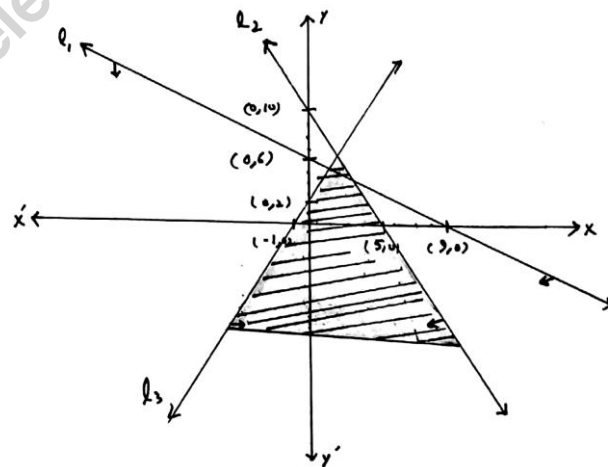
(v)  $\Rightarrow$  put  $x = 0, y = 10$  so the pt. (0, 10)  
 put  $y = 0, x = 5$  so the pt. (5, 0)

(vi)  $\Rightarrow$  put  $x = 0, y = 2$  so the pt. (0, 2)  
 put  $y = 0, x = -1$  so the pt. (-1, 0)

Test pt(0,0): we test (i), (ii) and (iii) at (0,0)  
 so (i)  $\Rightarrow 0 \leq 18 \rightarrow$  true (ii)  $\Rightarrow 0 \leq 10 \rightarrow$  true  
 (iii)  $\Rightarrow 0 \leq 2 \rightarrow$  true

**Solution region:**

The solution of the given system is intersection of (i), (ii) and (iii) so solution region is shaded area as shown in fig.



(vi)  $3x - 2y \geq 3$ ;  $x + 4y \leq 12$ ;  $3x + y \leq 12$

**Solution:**

$3x - 2y \geq 3 \rightarrow (i)$   $x + 4y \leq 12 \rightarrow (ii)$   
 $3x + y \leq 12 \rightarrow (iii)$

The associated eqs. of (i), (ii) and (iii) are

$l_1; 3x - 2y = 3 \rightarrow (iv)$ ,  $l_2; x + 4y = 12 \rightarrow (ii)$   
 $l_3; 3x + y = 12 \rightarrow (vi)$

(iv)  $\Rightarrow$  put  $x = 0, y = -\frac{3}{2}$  so that pt.  $(0, -\frac{3}{2})$   
 put  $y = 0, x = 1$  so the pt. (1, 0)

(v)  $\Rightarrow$  put  $x = 0, y = 3$  so the pt. (0, 3)  
 put  $y = 0, x = 12$  so the pt. (12, 0)

(vi)  $\Rightarrow$  put  $x = 0, y = 12$  so the pt  $(0,12)$

put  $y = 0, x = 4$  so the pt.  $(4,0)$

Test pt  $(0,0)$ : we test (i), (ii) and (iii) at  $(0,0)$

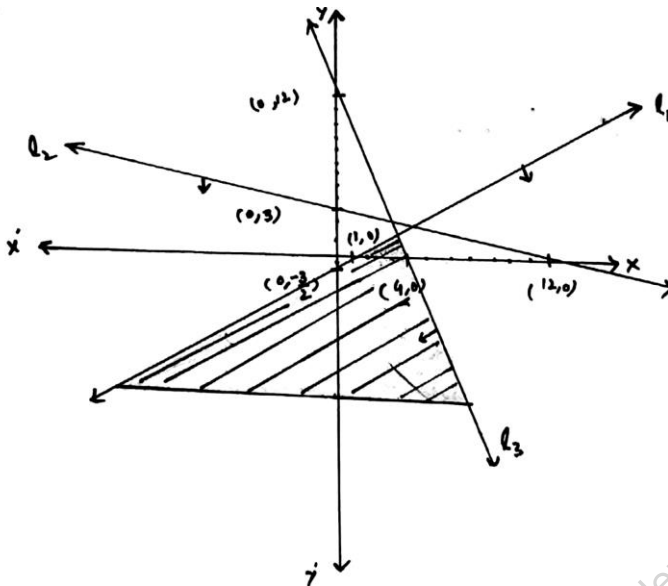
so (i)  $\Rightarrow 0 \geq 3 \rightarrow$  false (ii)  $\Rightarrow 0 \leq 12 \rightarrow$  true

(iii)  $\Rightarrow 0 \leq 12 \rightarrow$  true

### Solution region:

The solution of the given system is intersection of

(i), (ii) and (iii) so solution region is shaded area as shown in fig.



### Problem constraints:

The restrictions applied on the everyday life problems are called problem concentration.

### Non- Negative constraints:

The constraints that are always satisfied are called natural constraints or non- negative constraints.

### Decision variable:

The variable used in non- negative constraints are called decision variable.

### Feasible region:

The solution region which is restricted to the first quadrant is called feasible region. We restricted the solution region by using non-negative constraints  $x \geq 0$  and  $y \geq 0$

### Feasible solution:

Each point of feasible region is called feasible solution of the system.

### Feasible solution Set:

A set consists of all the feasible solution of the system is called feasible solution.

## Exercise 5.2

Graph the feasible region of the following system of linear inequalities and find the corner points in each case

(i)  $2x - 3y \leq 6$  ;  $2x + 3y \leq 12$  ;  $x \geq 0, y \geq 0$

### Solution:

$2x - 3y \leq 6 \rightarrow$  (i)  $2x + 5y \leq 12 \rightarrow$  (ii)

the associated eqs. of (i) and (ii) are

$l_1; 2x - 3y = 6 \rightarrow$  (iii),  $l_2; 2x + 5y = 12 \rightarrow$  (iv)

(iii)  $\Rightarrow$  put  $x = 0, y = -2$  so the pt  $(0, -2)$

put  $x = 0, y = 3$  so the pt  $(0, 3)$

(iv)  $\Rightarrow$  put  $x = 0, y = 4$  so the pt  $(0, 4)$

put  $y = 0, x = 6$  so the pt  $(6, 0)$

Test pt  $(0,0)$ : we test (i) and (ii) at  $(0,0)$  so

(i)  $\Rightarrow 0 \leq 6 \rightarrow$  true (ii)  $\Rightarrow 0 \leq 12 \rightarrow$  true.

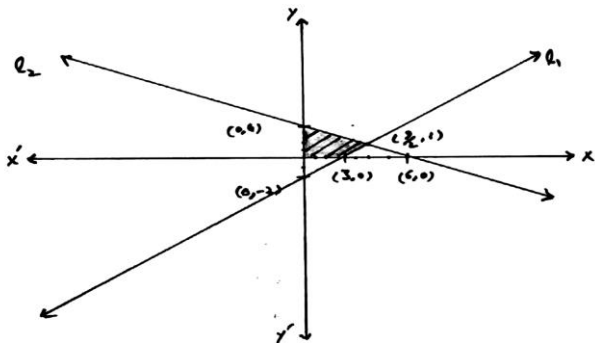
### Feasible region:

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also  $x \geq 0, y \geq 0$

0 indicates that graph of solution

Set in 1st Quadrant as shown in fig.



**Corner point:**

as  $2x - 3y = 6 \rightarrow (i)$

$2x + 3y = 12 \rightarrow (ii)$

By  $(i) + (ii) \Rightarrow 4x = 18 \Rightarrow x = \frac{9}{2}$  put in  $(i)$

$\Rightarrow 2\left(\frac{9}{2}\right) - 3y = 6 \Rightarrow -3y = 6 - 9$   
 $-3y = -3$

$\Rightarrow y = 1$  so  $\left(\frac{9}{2}, 1\right)$  is the pt. of intersection of lines  $(i)$  and  $(ii)$  Thus cornerpoints of feasible region are  $(0,0), (3,0), \left(\frac{9}{2}, 1\right)$  and  $(0,4)$

**(ii)  $x + y \leq 5 ; -2x + y \leq 2 ; x \geq 0, y \geq 0$**

**Solution:**

$x + y \leq 5 \rightarrow (i); -2x + y \leq 2 \rightarrow (ii)$

the associated eqs. of  $(i)$  and  $(ii)$  are

$l_1; x + y = 5 \rightarrow (iii), l_2; -2x + y = 2 \rightarrow (iv)$

$(iii) \Rightarrow$  put  $x = 0, y = 5$  so the pt  $(0,5)$

put  $x = 0, y = 5$  so the pt  $(5,0)$

$(iv) \Rightarrow$  put  $x = 0, y = 2$  so the pt  $(0,2)$

put  $y = 0, x = -1$  so the pt  $(-1,0)$

Test pt  $(0,0)$ : we test  $(i)$  and  $(ii)$  at  $(0,0)$  so

$(i) \Rightarrow 0 \leq 5 \rightarrow$  true  $(ii) \Rightarrow 0 \leq 2 \rightarrow$  true.

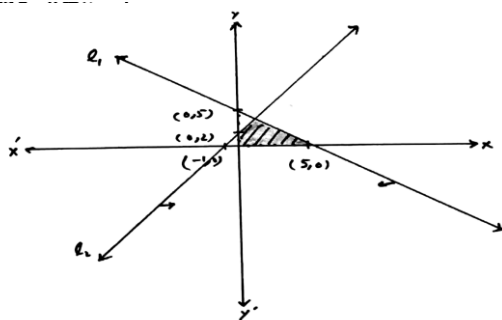
**Feasible region:**

The feasible region of the given system is the intersection of the graphs of  $(i)$  and  $(ii)$ .

Also  $x \geq 0, y \geq 0$

0 indicates that graph of solution

Set in 1st Quadrant as shown in fig.



**Corner point:**

as  $x + y = 5 \rightarrow (i)$

$-2x + y = 2 \rightarrow (ii)$

By  $(i) - (ii) \Rightarrow 3x = 3 \Rightarrow x = 1$  put in  $(i)$

$\Rightarrow 1 + y = 5 \Rightarrow y = 5 - 1 = 4$

so  $(1,4)$  is pt. of intersection of lines  $(i)$  and  $(ii)$

thus corner pts. of feasible region are  $(0,0), (5,0)$

$(1,4)$  and  $(0,2)$

**(iii)  $x + y \leq 5 ; -2x + y \geq 2 ; x \geq 0, y \geq 0$**

**Solution:**

$x + y \leq 5 \rightarrow (i), -2x + y \geq 2 \rightarrow (ii)$

the associated eqs. of  $(i)$  and  $(ii)$  are

$l_1; x + y = 5 \rightarrow (iii), l_2; -2x + y = 2 \rightarrow (iv)$

$(iii) \Rightarrow$  put  $x = 0, y = 5$  so the pt  $(0,5)$

put  $x = 0, y = 5$  so the pt  $(5,0)$

$(iv) \Rightarrow$  put  $x = 0, y = 2$  so the pt  $(0,2)$

put  $y = 0, x = -1$  so the pt  $(-1,0)$

Test pt  $(0,0)$ : we test  $(i)$  and  $(ii)$  at  $(0,0)$  so

$(i) \Rightarrow 0 \leq 5 \rightarrow$  true  $(ii) \Rightarrow 0 \geq 2 \rightarrow$  false.

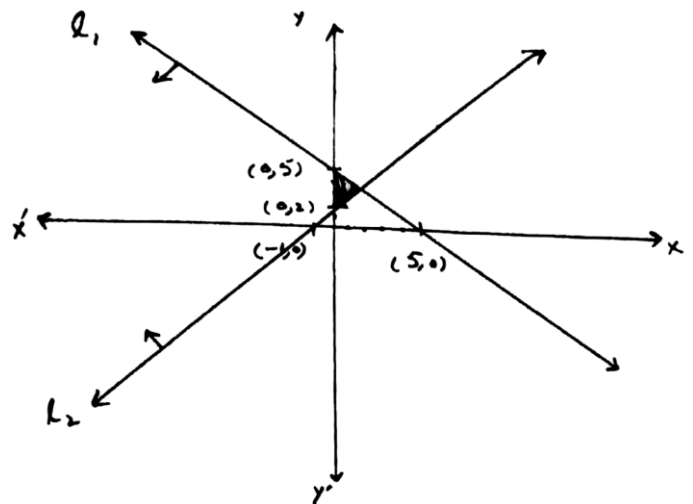
**Feasible region:**

The feasible region of the given system is the intersection of the graphs of  $(i)$  and  $(ii)$ .

Also  $x \geq 0, y \geq 0$

0 indicates that graph of solution

Set in 1st Quadrant as shown in fig.



**Corner point:**

as  $x + y = 5 \rightarrow (i)$

$-2x + y = 2 \rightarrow (ii)$

By  $(i) - (ii) \Rightarrow 3x = 3 \Rightarrow x = 1$  put in  $(i)$

$\Rightarrow 1 + y = 5 \Rightarrow y = 5 - 1 = 4$

so  $(1,4)$  is pt. of intersection of lines  $(i)$  and

$(ii)$  thus corner pts. of feasible region are

$(0,5), (1,4)$  and  $(0,2)$

(iv)  $3x + 7y \leq 21$ ;  $x - y \leq 3$ ;  $x \geq 0, y \geq 0$

**Solution:**

$3x + 7y \leq 21 \rightarrow$  (i);  $x - y \leq 3 \rightarrow$  (ii)  
 the associated eqs. of (i) and (ii) are  $l_1$ ;  $3x + 7y = 21 \rightarrow$  (iii),  $l_2$ ;  $x - y = 3 \rightarrow$  (iv)  
 (iii)  $\Rightarrow$  put  $x = 0, y = 3$  so the pt(0,3)  
 put  $x = 7, y = 0$  so the pt(7,0)  
 (iv)  $\Rightarrow$  put  $x = 0, y = -3$  so the pt(0,-3)  
 put  $y = 0, x = 3$  so the pt(3,0)

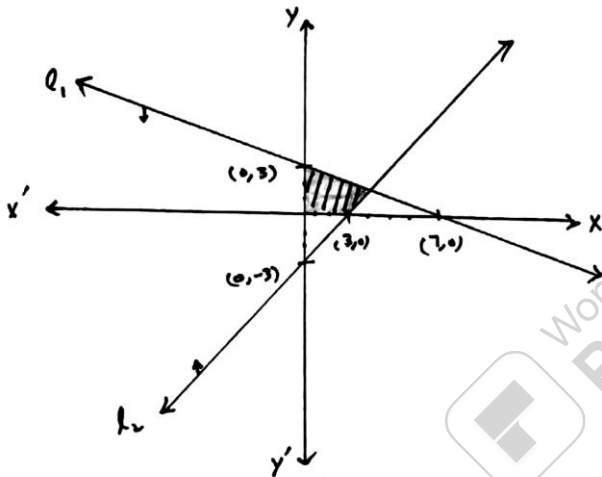
Test pt(0,0): we test (i) and (ii) at (0,0) so  
 (i)  $\Rightarrow 0 \leq 21 \rightarrow$  true (ii)  $\Rightarrow 0 \leq 3 \rightarrow$  false.

**Feasible region:**

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also  $x \geq 0, y \geq 0$

0 indicates that graph of solution Set in 1st Quadrant as shown in fig.



**Corner point:**

as  $3x + 7y = 21 \rightarrow$  (i)  
 $x - y = 3 \rightarrow$  (ii)

By  $7(ii) + (i) \Rightarrow 10x = 42 \Rightarrow x = \frac{21}{5}$  put in (ii)  
 $\Rightarrow \frac{21}{5} - y = 3 \Rightarrow y = \frac{21}{5} - 3 = \frac{6}{5}$

so  $(\frac{21}{5}, \frac{6}{5})$  is pt. of intersection of lines (i) and (ii) thus corner pts. of feasible region are (0,0), (3,0),  $(\frac{21}{5}, \frac{6}{5})$  and (0,3)

(v)  $3x + 2y \geq 6$ ;  $x + y \leq 4$ ;  $x \geq 0, y \geq 0$

**Solution:**

$3x + 2y \geq 6 \rightarrow$  (i) and  $x + y \leq 4 \rightarrow$  (ii)  
 the associated eqs. of (i) and (ii) are  $l_1$ ;  $3x + 2y = 6 \rightarrow$  (iii),  $l_2$ ;  $x + y = 4 \rightarrow$  (iv)  
 (iii)  $\Rightarrow$  put  $x = 0, y = 3$  so the pt(0,3)  
 put  $x = 2, y = 0$  so the pt(2,0)  
 (iv)  $\Rightarrow$  put  $x = 0, y = 4$  so the pt(0,4)

put  $y = 0, x = 4$  so the pt(4,0)

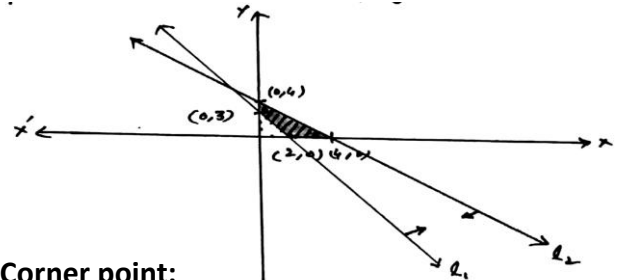
Test pt(0,0): we test (i) and (ii) at (0,0) so  
 (i)  $\Rightarrow 0 \geq 6 \rightarrow$  false (ii)  $\Rightarrow 0 \leq 4 \rightarrow$  true.

**Feasible region:**

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also  $x \geq 0, y \geq 0$

0 indicates that graph of solution Set in 1st Quadrant as shown in fig.



**Corner point:**

thus corner pts. of feasible region are (2,0)(4,0)(0,0) and (0,3)

(vi)  $5x + 7y \leq 35$ ;  $x - 2y \leq 4$ ;  $x \geq 0, y \geq 0$

**Solution:**

$5x + 7y \leq 35 \rightarrow$  (i)  $x - 2y \leq 4 \rightarrow$  (ii)  
 the associated eqs. of (i) and (ii) are  $l_1$ ;  $5x + 7y = 35 \rightarrow$  (iii),  $l_2$ ;  $x - 2y = 4 \rightarrow$  (iv)

(iii)  $\Rightarrow$  put  $x = 0, y = 5$  so the pt(0,5)  
 put  $x = 7, y = 0$  so the pt(7,0)  
 (iv)  $\Rightarrow$  put  $x = 0, y = -2$  so the pt(0,-2)  
 put  $y = 0, x = 4$  so the pt(4,0)

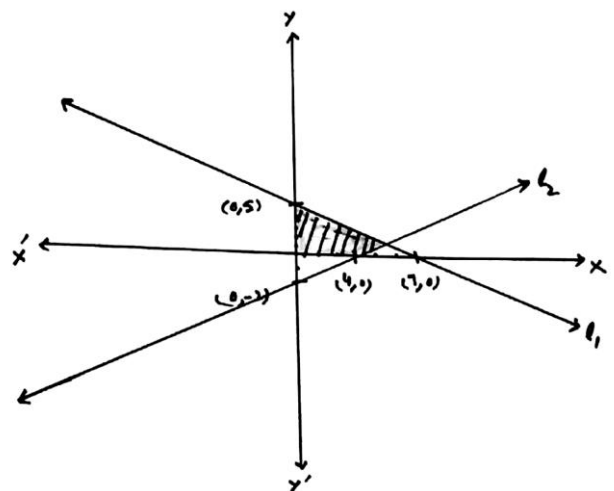
Test pt(0,0): we test (i) and (ii) at (0,0)

**Feasible region:**

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also  $x \geq 0, y \geq 0$

0 indicates that graph of solution Set in 1st Quadrant as shown in fig.



**Corner point:**

As  $5x + 7y = 35 \rightarrow (i)$   
 $x - 2y = 4 \rightarrow (ii)$

By  $5(ii) - (i) \Rightarrow 5x - 10y = 20$   
 $\frac{\pm 5x \pm 7y = \pm 35}{-17y = -15}$

$\Rightarrow y = \frac{15}{17}$  put in (ii)

$\Rightarrow x - 2\left(\frac{15}{17}\right) = 4 \Rightarrow x = 4 + \frac{30}{17}$

So,

$\left(\frac{99}{17}, \frac{15}{17}\right)$  is the pt. of intersection of lines (i) and (ii)

Hence corner pts. are  $(0, 0), (4, 0), \left(\frac{99}{17}, \frac{15}{17}\right)$  and  $(0, 5)$

**Question No.2**

**Graph the feasible region of the following system of linear inequalities and find the corner points in each case.**

(i)  $2x + y \leq 10 ; x + 4y \leq 12 ; x + 2y \leq 10 ;$   
 $x \geq 0, y \geq 0$

Solution:

$2x + y \leq 10 \rightarrow (i); x + 4y \leq 12 \rightarrow (ii)$   
 $; x + 2y \leq 10 \rightarrow (iii)$

The associated eqs. of (i), (ii) and (iii) are

$l_1; 2x + y = 10 \rightarrow (iv) ; l_2; x + 4y = 12 \rightarrow (ii)$   
 $l_3; x + 2y = 10 \rightarrow (vi)$

(iv)  $\Rightarrow$  put  $x = 0, y = 10$  so that pt.  $(0, 10)$   
 put  $y = 0, x = 5$  so the pt.  $(5, 0)$

(v)  $\Rightarrow$  put  $x = 0, y = 3$  so the pt.  $(0, 3)$   
 put  $y = 0, x = 12$  so the pt.  $(12, 0)$

(vi)  $\Rightarrow$  put  $x = 0, y = 5$  so the pt.  $(0, 5)$   
 put  $y = 0, x = 10$  so the pt.  $(10, 0)$

Test pt  $(0, 0)$ : we test (i), (ii) and (iii) at  $(0, 0)$

so (i)  $\Rightarrow 0 \leq 10 \rightarrow$  true (ii)  $\Rightarrow 0 \leq 12 \rightarrow$  True  
 (iii)  $\Rightarrow 0 \leq 10 \rightarrow$  true

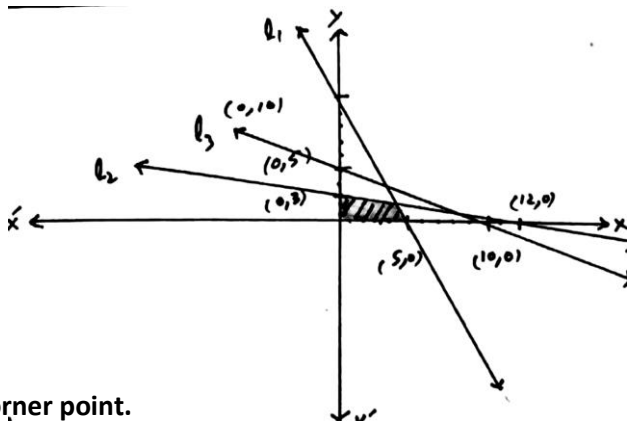
**Feasible region:**

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also  $x \geq 0, y \geq 0$

0 indicates that graph of solution

Set in 1st Quadrant as shown in fig



**Corner point.**

We find pt. of intersection of lines  $l_1, l_2$  so

$l_1; 2x + y = 10 \rightarrow (i)$

$l_2; x + 4y = 12 \rightarrow (ii)$

By  $2(ii) - (i) \Rightarrow 2x + 8y = 24$

$\pm 2x \pm y = -10$

$\frac{7y = 14 \Rightarrow y = 2$  put in (i)

$\Rightarrow x = 4$

So  $(4, 2)$  is pt of intersection of lines (i)

and (ii) thus corner pt. of feasible region

are  $(0, 0), (5, 0), (4, 2)$  and  $(0, 3)$

(ii)  $2x + 3y \leq 18 ; 2x + y \leq 10 ; x + 4y \leq 12$   
 $x \geq 0, y \geq 0$

Solution:

$2x + 3y \leq 18 \rightarrow (i); 2x + y \leq 10 \rightarrow (ii)$   
 $; x + 4y \leq 12 \rightarrow (iii)$

The associated eqs. of (i), (ii) and (iii) are

$l_1; 2x + 3y = 18 \rightarrow (iv) ; l_2; 2x + y = 10 \rightarrow (ii)$   
 $l_3; x + 4y = 12 \rightarrow (vi)$

(iv)  $\Rightarrow$  put  $x = 0, y = 6$  so that pt.  $(0, 6)$   
 put  $y = 0, x = 9$  so the pt.  $(9, 0)$

(v)  $\Rightarrow$  put  $x = 0, y = 10$  so the pt.  $(0, 10)$   
 put  $y = 0, x = 5$  so the pt.  $(5, 0)$

(vi)  $\Rightarrow$  put  $x = 0, y = 3$  so the pt.  $(0, 3)$   
 put  $y = 0, x = 12$  so the pt.  $(12, 0)$

Test pt  $(0, 0)$ : we test (i), (ii) and (iii) at  $(0, 0)$

so (i)  $\Rightarrow 0 \leq 18 \rightarrow$  true (ii)  $\Rightarrow 0 \leq 10 \rightarrow$  True  
 (iii)  $\Rightarrow 0 \leq 12 \rightarrow$  true

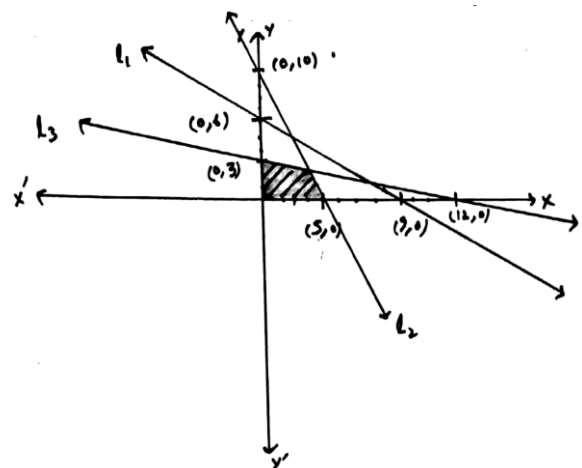
**Feasible region:**

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also  $x \geq 0, y \geq 0$

0 indicates that graph of solution

Set in 1st Quadrant as shown in fig.



**Corner point.**

We find pt. of intersection of lines  $l_1, l_2$  so

$l_1; 2x + 3y = 18 \rightarrow (i)$

$l_2; 2x + y = 10 \rightarrow (ii)$

By  $2(ii) - (i) \Rightarrow 2x + 8y = 24$



$$\frac{\pm 2x \pm y = -10}{7y = 14 \Rightarrow y = 2 \text{ put in (i)} \Rightarrow x = 4}$$

So (4,2) is pt of intersection of lines (i) and (ii) thus corner pt. of feasible region are (0,0), (5,0), (4,2) and (0,3)

(iii)  $2x + 3y \leq 18; x + 4y \leq 12; 3x + y \leq 12$

$x \geq 0, y \geq 0$

**Solution:**

$2x + 3y \leq 18 \rightarrow (i); x + 4y \leq 12 \rightarrow (ii)$   
 $3x + y \leq 12 \rightarrow (iii)$   
 $; x + 2y \leq 0 \rightarrow (iii)$

The associated eqs. of (i), (ii) and (iii) are

$l_1; 2x + 3y = 18 \rightarrow (iv), l_2; x + 4y = 12 \rightarrow (ii)$

$l_3; 3x + y = 12 \rightarrow (vi)$

(iv)  $\Rightarrow$  put  $x = 0, y = 3$  so that pt. (0,3)

put  $y = 0, x = 6$  so the pt. (6,0)

(v)  $\Rightarrow$  put  $x = 0, y = 9$  so the pt. (0,9)

put  $y = 0, x = 12$  so the pt. (12,0)

(vi)  $\Rightarrow$  put  $x = 0, y = 12$  so the pt. (0,12)

put  $y = 0, x = 4$  so the pt. (4,0)

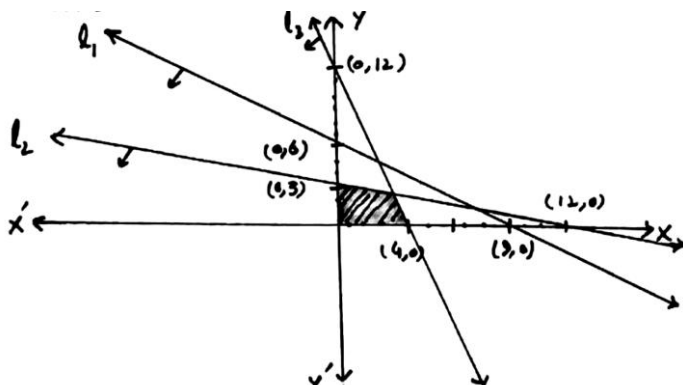
Test pt(0,0): we test (i), (ii) and (iii) at (0,0)  
 so (i)  $\Rightarrow 0 \leq 18 \rightarrow$  true (ii)  $\Rightarrow 0 \leq 12 \rightarrow$  True  
 (iii)  $\Rightarrow 0 \leq 12 \rightarrow$  true

**Feasible region:**

The feasible region of the given system is the intersection of the graphs of (i) and (iii).

Also  $x \geq 0, y \geq 0$

0 indicates that graph of solution Set in 1st Quadrant as shown in fig



**Corner point.**

We find pt. of intersection of lines  $l_1, l_2$  so

$l_1; x + 4y = 12 \rightarrow (i)$

$l_2; 3x + y = 12 \rightarrow (ii)$

By  $3(ii) - (i) \Rightarrow 3x + 12y = 36$

$$\frac{\pm 3x \pm y = -12}{11y = 24 \Rightarrow y = \frac{24}{11} \text{ put in (i)}$$

$$\Rightarrow x + 4\left(\frac{24}{11}\right) = 12 \Rightarrow x = 12 - \frac{96}{11}$$

$$x = \frac{132 - 96}{11} = \frac{36}{11}$$

So  $(\frac{36}{11}, \frac{24}{11})$  is pt of intersection of lines (i)

and (ii) thus corner pt. of feasible region are (0,0), (4,0),  $(\frac{36}{11}, \frac{24}{11})$  and (0,3)

(iv)  $x + 2y \leq 14; 3x + 4y \leq 36; 2x + y \leq 10$

$x \geq 0; y \geq 0$

**Solution:**

$x + 2y \leq 14 \rightarrow (i); 3x + 4y \leq 36 \rightarrow (ii)$   
 $; 2x + y \leq 10 \rightarrow (iii)$

The associated eqs. of (i), (ii) and (iii) are

$l_1; x + 2y = 14 \rightarrow (iv), l_2; 3x + 4y = 36 \rightarrow (ii)$

$l_3; 2x + y = 10 \rightarrow (vi)$

(iv)  $\Rightarrow$  put  $x = 0, y = 7$  so that pt. (0,7)

put  $y = 0, x = 14$  so the pt. (14,0)

(v)  $\Rightarrow$  put  $x = 0, y = 9$  so the pt. (0,9)

put  $y = 0, x = 12$  so the pt. (12,0)

(vi)  $\Rightarrow$  put  $x = 0, y = 10$  so the pt. (0,10)

put  $y = 0, x = 5$  so the pt. (5,0)

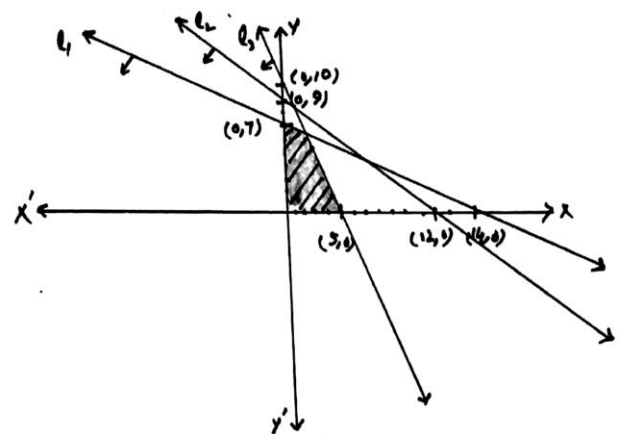
Test pt(0,0): we test (i), (ii) and (iii) at (0,0)  
 so (i)  $\Rightarrow 0 \leq 14 \rightarrow$  true (ii)  $\Rightarrow 0 \leq 36 \rightarrow$  True  
 (iii)  $\Rightarrow 0 \leq 10 \rightarrow$  true

**Feasible region:**

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also  $x \geq 0, y \geq 0$

0 indicates that graph of solution Set in 1st Quadrant as shown in fig.



**Corner point.**

We find pt. of intersection of lines  $l_1, l_2$  so

$l_1; x + 2y = 14 \rightarrow (i)$

$$l_2; 2x + y = 10 \rightarrow (ii)$$

$$\text{By } 2(i) - (ii) \Rightarrow 2x + 4y = 28$$

$$\quad \quad \quad \pm 2x \pm y = -10$$

$$\hline 3y = 18 \Rightarrow y = 6 \text{ put in (i)}$$

$$\Rightarrow x + 12 = 14 \Rightarrow x = 2$$

So (2,6) is pt of intersection of lines (i) and (ii) thus corner pt. of feasible region are (0,0), (5,0), (2,6) and (0,7)

$$(v) \quad x + 3y \leq 15; \quad 2x + y \leq 12; \quad 4x + 3y \leq 24$$

$$x \geq 0, y \geq 0$$

Solution:

$$x + 3y \leq 15 \rightarrow (i) \quad 2x + y \leq 12 \rightarrow (ii)$$

$$4x + 3y \leq 24 \rightarrow (iii)$$

The associated eqs. of (i), (ii) and (iii) are

$$l_1; x + 3y = 15 \rightarrow (iv) \quad , l_2; 2x + y = 12 \rightarrow (ii)$$

$$l_3; 4x + 3y = 24 \rightarrow (vi)$$

$$(iv) \Rightarrow \text{put } x = 0, y = 5 \text{ so that pt. } (0,5)$$

$$\text{put } y = 0, x = 15 \text{ so the pt. } (15,0)$$

$$(v) \Rightarrow \text{put } x = 0, y = 12 \text{ so the pt. } (0,12)$$

$$\text{put } y = 0, x = 6 \text{ so the pt. } (6,0)$$

$$(vi) \Rightarrow \text{put } x = 0, y = 8 \text{ so the pt. } (0,8)$$

$$\text{put } y = 0, x = 6 \text{ so the pt. } (6,0)$$

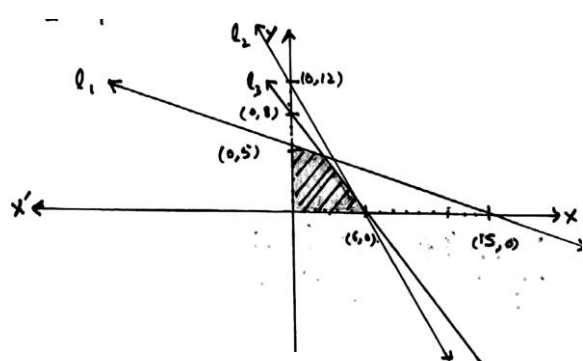
Test pt(0,0): we test (i), (ii) and (iii) at (0,0)  
 so (i)  $\Rightarrow 0 \leq 15 \rightarrow \text{true}$  (ii)  $\Rightarrow 0 \leq 12 \rightarrow \text{True}$   
 (iii)  $\Rightarrow 0 \leq 24 \rightarrow \text{true}$

**Feasible region:**

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also  $x \geq 0, y \geq 0$

0 indicates that graph of solution Set in 1st Quadrant as shown in fig



**Corner point.**

We find pt. of intersection of lines  $l_1, l_2$  so

$$l_1; x + 3y = 15 \rightarrow (i)$$

$$l_2; 4x + 3y = 24 \rightarrow (ii)$$

$$\text{By } (i) - (ii) \Rightarrow -3x = -9 \Rightarrow x = 3 \text{ put in (i)}$$

$$\Rightarrow 3y = 15 - 3 = 12 \Rightarrow y = 4$$

So (3,4) is pt. of intersection of lines (i)

and (ii) thus corner pt. of feasible region are (0,0), (6,0), (3,4) and (0,5)

$$(vi) \quad 2x + y \leq 20; \quad 8x + 15y \leq 20; \quad x + y \leq 11$$

$$x \geq 0, y \geq 0$$

Solution:

$$2x + y \leq 20 \rightarrow (i); \quad 8x + 15y \leq 20$$

$$\rightarrow (ii)$$

$$; x + y \leq 11 \rightarrow (iii)$$

The associated eqs. of (i), (ii) and (iii) are

$$l_1; 2x + y = 20 \rightarrow (iv) \quad , l_2; 8x + 15y = 20$$

$$\rightarrow (ii)$$

$$l_3; x + y = 11 \rightarrow (vi)$$

$$(iv) \Rightarrow \text{put } x = 0, y = 20 \text{ so that pt. } (0,20)$$

$$\text{put } y = 0, x = 10 \text{ so the pt. } (10,0)$$

$$(v) \Rightarrow \text{put } x = 0, y = 8 \text{ so the pt. } (0,8)$$

$$\text{put } y = 0, x = 15 \text{ so the pt. } (15,0)$$

$$(vi) \Rightarrow \text{put } x = 0, y = 11 \text{ so the pt. } (0,11)$$

$$\text{put } y = 0, x = 11 \text{ so the pt. } (11,0)$$

Test pt(0,0): we test (i), (ii) and (iii) at (0,0)  
 so (i)  $\Rightarrow 0 \leq 20 \rightarrow \text{true}$  (ii)  $\Rightarrow 0 \leq 120$   
 $\rightarrow \text{True}$

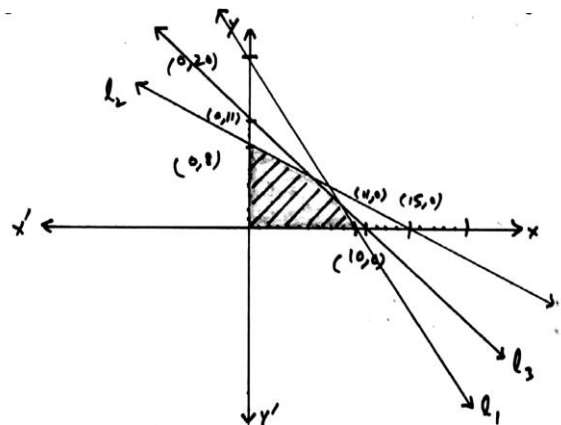
$$(iii) \Rightarrow 0 \leq 11 \rightarrow \text{true}$$

**Feasible region:**

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also  $x \geq 0, y \geq 0$

0 indicates that graph of solution Set in 1st Quadrant as shown in fig.



**Corner point.**

We find pt. of intersection of lines

$l_1, l_3$  also  $l_2$  and  $l_3$

$$l_1; 2x + y = 20 \rightarrow (i)$$

$$l_2; x + y = 11 \rightarrow (ii)$$

$$\text{By } 2(ii) - (i) \Rightarrow 2x + 2y = 22$$

$$\quad \quad \quad \pm 2x \pm y = \pm 20$$

$$\hline y = 2 \text{ put in (ii) } x = 9$$

So

(9,2) is the pt. of intersection of lines (i) and (ii)

Also  $l_2 ; 8x + 15y = 120 \rightarrow (iii)$

$l_3; x + y = 11 \rightarrow (iv)$

By  $(8(iv) - (iii)) \Rightarrow 8x + 8y = 88$   
 $\pm 8x \pm 15y = \pm 120$

$$\frac{-7y = -32 \Rightarrow y = \frac{32}{7} \text{ put in (iv)}}$$

$$\Rightarrow x + \frac{32}{7} = 11 = 11 - \frac{32}{7} = \frac{77 - 32}{7} = \frac{45}{7}$$

So  $(\frac{45}{7}, \frac{32}{7})$  is pt of intersection of lines (i) and (ii) thus corner pt. of feasible region are  $(0,0), (9,2), (\frac{45}{7}, \frac{32}{7})$  and  $(0,8)$

**Linear programming**

**Objective function:**

A function which is to be maximized or minimized is called an objective function:

**Optimal solution:**

The feasible solution which maximizes or minimize the objective function is called optimal solution.

**Procedure for finding optimal:**

**Solution:**

- (i) Graph the solution set of linear inequality constants to determine feasible region.
- (ii) Find the corner points of the feasible region.
- (iii) Evaluate the objective function at each corner point to find the optimal solution:

**Exercise No.5.3**

**Question No.1**

Maximize  $f(x, y) = 2x + 5y$  subject to the constraints  $2y - x \leq 8; x - y \leq 4; x \geq 0, y \geq 0$

**Solution:**

$-x + 2y \leq 8 \rightarrow (i)$   $x - y \leq 4 \rightarrow (ii)$   
 the associated eqs. of (i) and (ii) are  $l_1; -x + 2y = 8 \rightarrow (iii), l_2; x - y = 4 \rightarrow (iv)$   
 $(iii) \Rightarrow$  put  $x = 0, y = 4$  so the pt  $(0,4)$   
 put  $x = 0, y = -8$  so the pt  $(-8,0)$   
 $(iv) \Rightarrow$  put  $x = 0, y = -4$  so the pt  $(0,-4)$   
 put  $y = 0, x = 4$  so the pt  $(4,0)$

Test pt  $(0,0)$ : we test (i) and (ii) at  $(0,0)$  so  
 $(i) \Rightarrow 0 \leq 8 \rightarrow$  true  $(ii) \Rightarrow 0 \leq 4 \rightarrow$  true.

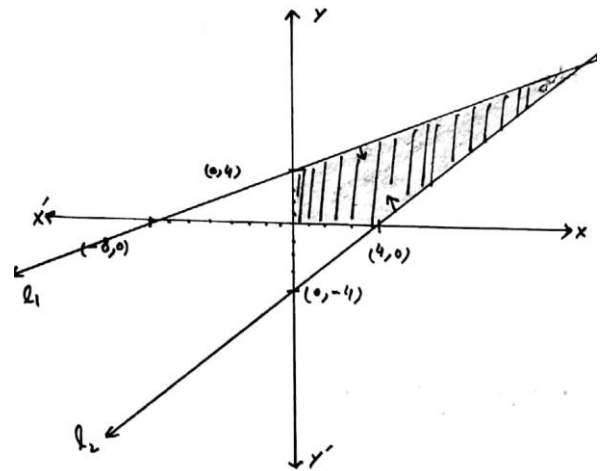
**Feasible region:**

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also  $x \geq 0, y \geq 0$

0 indicates that graph of solution

Set in 1st Quadrant as shown in fig.



**Corner point:**

We find pt. of intersection of lines  $l_1, l_2$  so

$$l_1; -x + 2y = 8 \rightarrow (i)$$

$$x - y = 4 \rightarrow (iii)$$

By  $(i) + (iii) \Rightarrow y = 12$  put in  $(ii) \Rightarrow x = 16$

So  $(16,12)$  is the pt. of intersection of lines (i) and (ii) hence corner pts. of feasible region are  $(0,0), (4,0), (16,2)$  and  $(0,4)$ .

**Optimal solution:**

we find values of  $f(x, y)$   
 $= 2x + 5y$  at corner pts.  
 $f(0,0) = 2(0) + 5(0) = 0, f(4,0) = 2(4) + 5(0) = 8$   
 $f(16,12) = 2(16) + 5(12) = 92, f(0,4) = 2(0) + 5(11) = 20$

Thus  $f(x, y)$  has maximum value 92 at  $(16,12)$

**Question No.2**

Maximize  $f(x, y) = x + 3y$  subject to constraints  $2x + 5y \leq 30, 5x + 4y \leq 20, x \geq 0, y \geq 0$

**Solution:**

$2x + 5y \leq 30 \rightarrow (i)$   $5x + 4y \leq 20 \rightarrow (ii)$   
 the associated eqs. of (i) and (ii) are  $l_1; 2x + 5y = 30 \rightarrow (iii), l_2; 5x + 4y = 20 \rightarrow (iv)$   
 $(iii) \Rightarrow$  put  $x = 0, y = 6$  so the pt  $(0,6)$   
 put  $x = 0, y = 15$  so the pt  $(15,0)$   
 $(iv) \Rightarrow$  put  $x = 0, y = 5$  so the pt  $(0,5)$   
 put  $y = 0, x = 4$  so the pt  $(4,0)$

Test pt  $(0,0)$ : we test (i) and (ii) at  $(0,0)$  so  
 $(i) \Rightarrow 0 \leq 30 \rightarrow$  true  $(ii) \Rightarrow 0 \leq 20 \rightarrow$  true.



**Solution:**

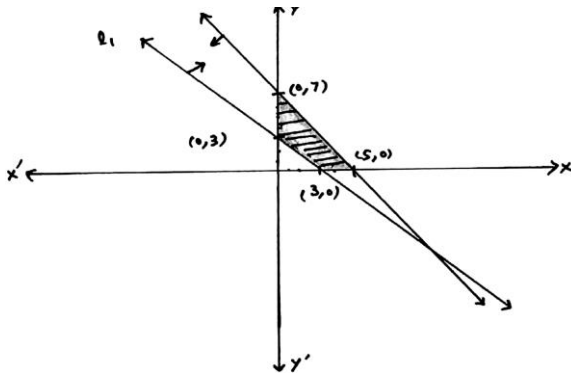
$x + y \geq 3 \rightarrow (i), 7x + 5y \leq 35 \rightarrow (ii)$   
 the associated eqs. of (i) and (ii) are  $l_1; x + y = 3 \rightarrow (iii), l_2; 7x + 5y = 35 \rightarrow (iv)$   
 (iii)  $\Rightarrow$  put  $x = 0, y = 3$  so the pt(0,3)  
 put  $x = 0, y = 3$  so the pt(3,0)  
 (iv)  $\Rightarrow$  put  $x = 0, y = 7$  so the pt (0,7)  
 put  $y = 0, x = 5$  so the pt(5,0)  
 Test pt(0,0): we test (i) and (ii) at (0,0) so  
 (i)  $\Rightarrow 0 \geq 3 \rightarrow$  false (ii)  $\Rightarrow 0 \leq 35 \rightarrow$  true.

**Feasible region:**

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also  $x \geq 0, y \geq 0$

0 indicates that graph of solution Set in 1st Quadrant as shown in fig.



**Corner point:**

Corner point of feasible region are (3,0)(5,0), (0,7) and(0,3)

**Optimal solution:**

we find values of  $Z = 2x + y$  at corner pts.  
 (3,0),  $x = 2(3) + 0 = 6$ , (5,0),  $z = 2(5) + 0 = 10$   
 (0,4),  $z = 2(0) + 7 = 7$ , (0,3),  $z = 2(0) + 3 = 3$   
 So,  $z = 2x + y$  has minimum value 3 at (0,3)?

**Question No.5 Maximize the function defined constraints  $2x + y \leq 8; x + 2y \leq 14; x \geq 0, y \geq 0$**

**Solution:**

$2x + y \leq 8 \rightarrow (i), x + 2y \leq 14 \rightarrow (ii)$   
 the associated eqs. of (i) and (ii) are  $l_1; 2x + y = 8 \rightarrow (iii), l_2; x + 2y = 14 \rightarrow (iv)$   
 (iii)  $\Rightarrow$  put  $x = 0, y = 8$  so the pt(0,8)  
 put  $x = 0, y = 4$  so the pt(4,0)  
 (iv)  $\Rightarrow$  put  $x = 0, y = 14$  so the pt (0,14)  
 put  $y = 0, x = 7$  so the pt(7,0)

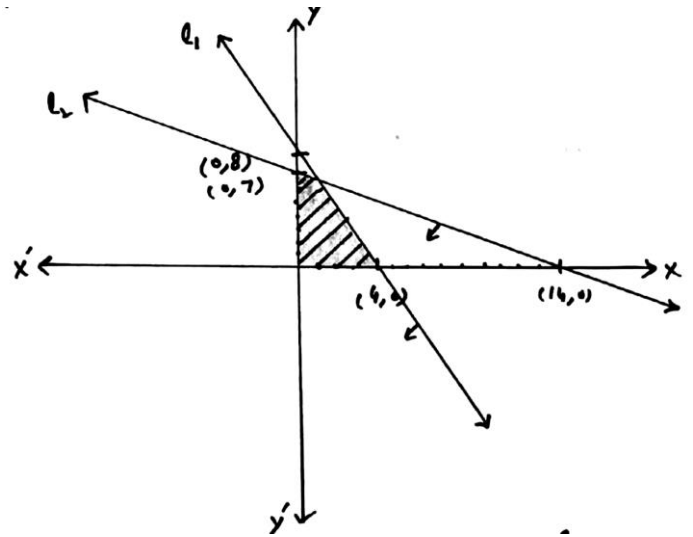
Test pt(0,0): we test (i) and (ii) at (0,0) so  
 (i)  $\Rightarrow 0 \leq 8 \rightarrow$  true (ii)  $\Rightarrow 0 \leq 14 \rightarrow$  true.

**Feasible region:**

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

Also  $x \geq 0, y \geq 0$

0 indicates that graph of solution Set in 1st Quadrant as shown in fig.



**Corner point:**

We find pt. of intersection of lines  $l_1, l_2$  so

$$l_1; 2x + y = 8 \rightarrow (i)$$

$$x + 2y = 14 \rightarrow (ii)$$

$$\text{By } 2(ii) + (i) \Rightarrow 2x + 4y = 28$$

$$\underline{+ 2x + 2y = 14}$$

$$3y = 20$$

$$\underline{20}$$

$$y = \frac{20}{3} \text{ put in (i)}$$

$$2x + \frac{20}{3} = 8 \Rightarrow 2x = 8 - \frac{20}{3} \Rightarrow 2x = \frac{4}{3}$$

$$\Rightarrow x = \frac{4}{6} = \frac{2}{3}$$

so  $(\frac{2}{3}, \frac{20}{3})$  is the pt. of intersection of lines (i) and (ii). Hence coner pts. of feasible region are

$$(0,0), (4,0), (\frac{2}{3}, \frac{20}{3}) \text{ and } (0,8).$$

**Optimal solution:**

We find valves of  $f(x, y) = 2x + 3y$  at corner pts.

$$f(0,0) = 2(0) + 2(0) = 0, f(4,0) = 2(4) + 3(0) = 8$$

$$f(\frac{2}{3}, \frac{20}{3}) = 2(\frac{2}{3}) + 3(\frac{20}{3}) = \frac{64}{3} = 21.33$$

$$f(0,7) = 2(0) + 3(7) = 21$$

So  $f(x, y) = 2x + 3y$

Has maximum value at  $(\frac{2}{3}, \frac{20}{3})$

**Question No.6**

**Minimize  $z = 3x + y$ ; subject to constraints:**

$$3x + 5y \geq 15; x + 6y \geq 9, x \geq 0, y \geq 0$$

**Solution:**

$$3x + 5y \geq 15 \rightarrow (i)$$

$$x + 6y \geq 9 \rightarrow (ii)$$

the associated eqs. of (i) and (ii) are

$$l_1; 3x + 5y = 15 \rightarrow (iii), l_2; x + 6y = 9 \rightarrow (iv)$$

$$(iii) \Rightarrow \text{put } x = 0, y = 3 \text{ so the pt}(0,3)$$

$$\text{put } x = 0, y = 5 \text{ so the pt}(5,0)$$

$$(iv) \Rightarrow \text{put } x = 0, y = \frac{3}{2} \text{ so the pt } (0, \frac{3}{2})$$

$$\text{put } y = 0, x = 9 \text{ so the pt}(9,0)$$

Test pt. (0,0): we test (i) and (ii) at (0,0) so

(i)  $\Rightarrow 0 \geq 15 \rightarrow \text{false}$  (ii)  $\Rightarrow 0 \geq 9 \rightarrow \text{false}$

**Feasible region:**

The feasible region of the given system is the intersection of the graphs of (i) and (ii).

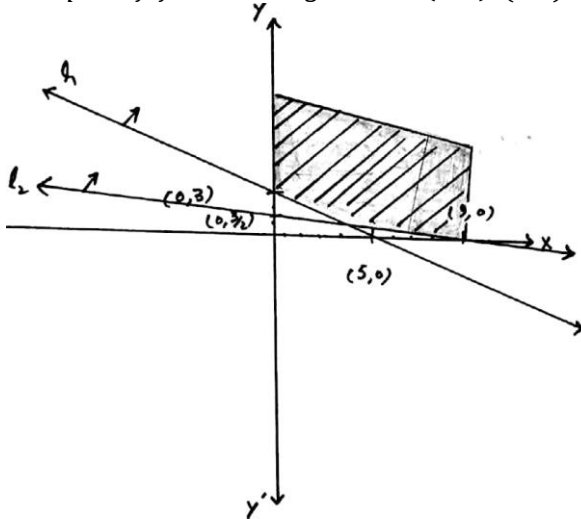
Also  $x \geq 0, y \geq 0$

0 indicates that graph of solution

Set in 1st Quadrant as shown in fig.

**Corner point:**

Corner pts. of feasible region are  $(9, 0), (0, 3)$



**Optimal solution:**

we find values of  $z = 3x + y$  at corner pts.

$$(0, 3), \quad z = 3(0) + 3 = 3$$

$$(9, 0), \quad z = 3(9) + 0 = 27$$

So  $z = 3x + y$  has minimum value 3 at  $(0, 3)$