M THEMATICS 11 INTERMEDIATE PART 1

Bilal Article

Chapter 9. Fundamental of Trigonometry

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Chapter#9	Class 1^{st}	Fundamental of Trigonometry
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Trigonometry:

The word trigonometry has been derived from Greek words: trei (three), Gano (angles) and Metron (measurement). So it means measurement of triangle.

Units of measure of Angles:

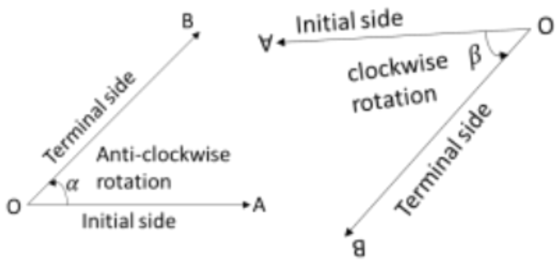
Angle: two rays with a common starting point from an angle.

One of the rays of angle is called initial side and the other as terminal side.

An angle is said to be positive if the rotation is anti-clockwise.

An angle is said to be negative if the rotation is clockwise.

Angle are usually denoted by Greek letters α(alpha) $\beta(beta)\,,\gamma(gamma),\theta(theta)$



Sexagesiml System:

(degree, Minute and second)$(\boldsymbol{D^0},\boldsymbol{M'},\boldsymbol{S''})$

The system in which one complete revolution is divide into 360 parts, each part is called degree. Then one degree is divided into 60 parts, each part is called minute. Now one minute is divide into 60 parts each part is called a second.

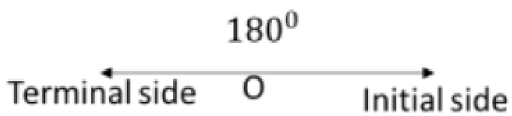
So 1 rotation (anti clock wise)=360^0

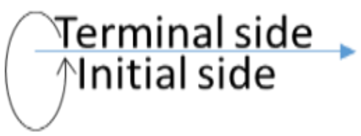
One degree $(1^0) = 60'(60minutes)$

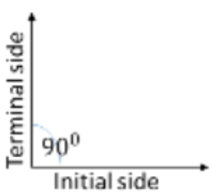
$one\ mintue(1') = 60''(60second)$

$\frac{1}{2}rotation\ (anti-clockwise) = 180^0\ is\ called\ a$ straight angle.

$\frac{1}{4}rotation(anticlockwise) = 90^0\ \ called\ right\ angle$







Note that

$$1^0 = 60' \Rightarrow 1' = \left(\frac{1}{60}\right)^0, 1' = 60'' \Rightarrow 1' = \left(\frac{1}{60}\right)'$$
$$= \left(\frac{1}{60\times 60}\right) = \left(\frac{1}{3600}\right)^0$$

Conversion from $D^0\,M'S''to\ \ a\ decimal\ from\ and\ vice\ versa$:

$$(i)\ \ 16^0 30' = [16 + \frac{30}{60}]^0 = (16 + \frac{1}{2})^0$$
$$= (16 + 0.5)^0 = 16.5^0$$
$$\because 1' = \left(\frac{1}{60}\right)^0 \ \Rightarrow 30' = \left(\frac{30}{60}\right)^0 \quad \because 1'' = 60'$$
$$\because 0.25^o = 0.25 \times 60'$$

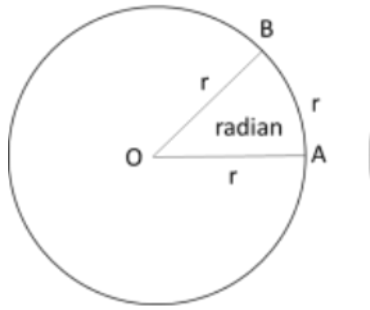
(ii)

$$45.25^o = 45^0 + 0.25 \times 60'$$
$$= 45^0 15'$$

Circular system (Radian)

Radian: an angle subtended at the center of the circle by an arc, whose length is equal to the radius of the circle is called one radian.

$$in\ fig.\ \ AB = \overline{OA} = r$$
$$so\ m\angle AOB = 1\,radian$$



Relation between the length of arc of a circle and the circular measure of its central angle:

Prove that $l = r\theta$

Where $r = radius\,, l = arc\ length$

$\theta\ = circular\ mesure\ of\ central\ angle.$

Proof:

Let r be the radius at point O. let $AB = l\ \ ,\ m\angle AOB = \theta$

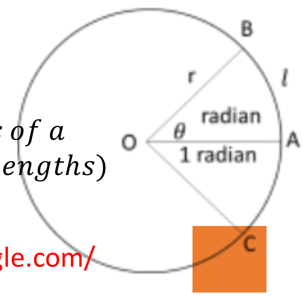
Take an arc $AC = r$

$$by\ def.\ m\angle AOC = 1\,radian$$

By elementary geometry

$$\frac{m\angle AOB}{m\angle AOC} = \frac{mAB}{mAC}$$

$\because measure\ of\ central\ angles\ of\ a$ $(circle\ are\ proportional\ to\ the\ lengths)$ $of\ their\ arcs$



$$\frac{\theta radian}{1\,radian} = \frac{l}{r}$$

$$\theta = \frac{l}{r} \Rightarrow l = r\theta \text{ hence proved}$$

Conservation of radian into Degree and vice versa

$$\because \text{circumference of circle of radius } r = 2\pi r$$

$$\Rightarrow 2\pi r = r\theta$$

$$\Rightarrow \theta = 2\pi \text{ radian}$$

Thus $2\pi = 360^0$

$$\Rightarrow \pi = 180^0 \rightarrow (i)$$

$$\Rightarrow 1\,radian = \frac{180^0}{\pi}$$

$$\Rightarrow 1\,radian = \frac{180^0}{3.1416} \approx 57.296^0$$

Further by (i)

$$\pi = 180^0$$

$$\Rightarrow 1^0 = \frac{\pi}{180} rad$$

$$\approx \frac{3.14}{180}$$

$$\Rightarrow 1^0 \approx 0.0175\,rad$$

Exercise 9.1

Question.1.

Express the following sexagesiml measures of angles in radians:

$(i).30^0$

Solution:

$$30° = 30 \times \frac{\pi}{180} \text{radian}$$

$$30° = \frac{\pi}{6} \text{ radian.}$$

$(ii).45^0$

Solution:

$$45° = 45 \times \frac{\pi}{180} \text{radian}$$

$$45^0 = \frac{\pi}{4} \text{ radian.}$$

$(iii).60^0$

Solution:

$$60° = 60 \times \frac{\pi}{180} \text{radian}$$

$$60° = \frac{\pi}{3} \text{ radian.}$$

$(iv).75^0$

Solution:

$$75° = 75 \times \frac{\pi}{180} \text{radian}$$

$$75° = \frac{5}{12} \text{ radian.}$$

$(v).90^0$

Solution:

$$90° = 90 \times \frac{\pi}{180} \text{radian}$$

$$90° = \frac{\pi}{2} \text{radian.}$$

$(vi).105^0$

Solution:

$$105° = 105 \times \frac{\pi}{180} \text{radian}$$

$$105° = \frac{7\pi}{12} \text{radian.}$$

$(vii).120^0$

Solution:

$$120° = 120 \times \frac{\pi}{180} \text{radian}$$

$$120° = \frac{2\pi}{3} \text{radian.}$$

$(viii).135^0$

Solution:

$$135° = 135 \times \frac{\pi}{180} \text{radian}$$

$$135° = \frac{3\pi}{4} \text{ radian.}$$

$(ix).150^0$

Solution:

$$150° = 150 \times \frac{\pi}{180} \text{radian}$$

$$150° = \frac{5\pi}{6} \text{radian.}$$

$(x).10^0 15'$

Solution:

$$10°15' = 10^0 + \left(\frac{15}{60}\right)^0$$

$$10°15' = 10^0 + \left(\frac{1}{4}\right)^0$$

$$10°15' = \left(10 + \frac{1}{4}\right)^0$$

$$10°15' = \left(\frac{41}{4}\right)^0$$

$$10°15' = \frac{41}{4} \times \frac{\pi}{180} \text{ radian}$$

$$10°15' = \frac{41\pi}{720} \text{ radian.}$$

$(xi).35^0 20'$

Solution:

$$35°20' = 35^0 + (\frac{20}{60})^0$$
$$35°20' = 35^0 + (\frac{1}{3})^0$$
$$35°20' = (35 + \frac{1}{3})^0$$
$$35°20' = (\frac{106}{3})^0$$
$$35°20' = \frac{106}{3} \times \frac{\pi}{180} \text{ radian}$$
$$35°20' = \frac{53\pi}{270} \text{ radian.}$$

$(xii).75^0 6'30''$

Solution:

$$75^0 6'30'' = 75^0 + (\frac{6}{60})^0 + (\frac{36}{3600})^0$$
$$75^0 6'30'' = 35^0 + (\frac{1}{3})^0 + (\frac{1}{100})^0$$
$$75^0 6'30'' = (35 + \frac{1}{3} + \frac{1}{100})^0$$
$$75^0 6'30'' = (\frac{10500 + 100 + 3}{3})^0$$
$$75^0 6'30'' = (\frac{10500 + 100 + 3}{3})^0$$
$$75^0 6'30'' = \frac{10603}{3} \times \frac{\pi}{180} \text{ radian}$$
$$75^0 6'30'' = \frac{10603\pi}{540} \text{ radian.}$$

$(xiii).120'40''$

Solution:

$$120'40'' = (\frac{120}{60})^0 + (\frac{40}{3600})^0$$
$$120'40'' = (2)^0 + (\frac{1}{90})^0$$
$$120'40'' = (\frac{2(90) + 1}{90})^0$$
$$120'40'' = (\frac{181}{90})^0$$
$$120'40'' = \frac{181}{90} \times \frac{\pi}{180} \text{ radian}$$
$$120'40'' = \frac{181\pi}{16200} \text{ radian.}$$

$(xiv).154^0 20''$

Solution:

$$154^0 20'' = 154^0 + (\frac{20}{3600})^0$$
$$154^0 20'' = 154^0 + (\frac{1}{180})^0$$
$$154^0 20'' = (\frac{150(180) + 1}{180})^0$$
$$154^0 20'' = (\frac{27001}{180})^0$$
$$154^0 20'' = \frac{27001}{180} \times \frac{\pi}{180} \text{ radian}$$
$$154^0 20'' = \frac{27001\pi}{32400} \text{ radian.}$$

$(xv).0^0$

Solution:

$$0^0 = 0 \times \frac{\pi}{180} \text{ radian}$$
$$0^0 = 0 \text{ radian.}$$

$(xvi).3''$

Solution:

$$3'' = (\frac{3}{3600})^0$$
$$3'' = (\frac{1}{2700})^0$$
$$3'' = \frac{1}{2700} \times \frac{\pi}{180} \text{ radian}$$
$$3'' = \frac{\pi}{486000} \text{ radian.}$$

Question.2.

Convert the following radian measures of angles into the measures of sexagesimal system

(i). $\frac{\pi}{8}$

Solution:

$$\frac{\pi}{8} rad = (\frac{\pi}{8} \times \frac{180}{\pi})^0$$
$$\frac{\pi}{8} rad = 22.5^0$$

(ii). $\frac{\pi}{6}$

Solution:

$$\frac{\pi}{6} rad = (\frac{\pi}{6} \times \frac{180}{\pi})^0$$
$$\frac{\pi}{6} rad = 30^0$$

(iii). $\frac{\pi}{4}$

Solution:

$$\frac{\pi}{4}rad = (\frac{\pi}{4}\times\frac{180}{\pi})^0$$
$$\frac{\pi}{4}rad = 45^0$$

(iv). $\frac{\pi}{3}$

Solution:

$$\frac{\pi}{3}rad = (\frac{\pi}{3}\times\frac{180}{\pi})^0$$
$$\frac{\pi}{3}rad = 60^0$$

(v). $\frac{\pi}{2}$

Solution:

$$\frac{\pi}{2}rad = (\frac{\pi}{2}\times\frac{180}{\pi})^0$$
$$\frac{\pi}{2}rad = 90^0$$

(vi). $\frac{2\pi}{3}$

Solution:

$$\frac{2\pi}{3}rad = (\frac{2\pi}{3}\times\frac{180}{\pi})^0$$
$$\frac{2\pi}{3}rad = 120^0$$

(vii). $\frac{3\pi}{4}$

Solution:

$$\frac{3\pi}{4}rad = (\frac{3\pi}{4}\times\frac{180}{\pi})^0$$
$$\frac{3\pi}{4}rad = 135^0$$

(viii). $\frac{5\pi}{6}$

Solution:

$$\frac{5\pi}{6}rad = (\frac{5\pi}{6}\times\frac{180}{\pi})^0$$
$$\frac{5\pi}{6}rad = 150^0$$

(ix). $\frac{7\pi}{12}$

Solution:

$$\frac{7\pi}{12}rad = (\frac{7\pi}{12}\times\frac{180}{\pi})^0$$
$$\frac{7\pi}{12}rad = 105^0$$

(x). $\frac{9\pi}{5}$

Solution:

$$\frac{9\pi}{5}rad = (\frac{9\pi}{5}\times\frac{180}{\pi})^0$$
$$\frac{9\pi}{5}rad = 324^0$$

(xi). $\frac{11\pi}{27}$

Solution:

$$\frac{11\pi}{27}rad = (\frac{11\pi}{27}\times\frac{180}{\pi})^0$$
$$\frac{11\pi}{27}rad = 73.33^0$$

(xii). $\frac{13\pi}{16}$

Solution:

$$\frac{13\pi}{16}rad = (\frac{13\pi}{16}\times\frac{180}{\pi})^0$$
$$\frac{13\pi}{16}rad = 146.25^0$$

(xiii). $\frac{17\pi}{24}$

Solution:

$$\frac{17\pi}{24}rad = (\frac{17\pi}{24}\times\frac{180}{\pi})^0$$
$$\frac{17\pi}{24}rad = 127.5^0$$

(xiv). $\frac{25\pi}{36}$

Solution:

$$\frac{25\pi}{36}rad = (\frac{25\pi}{36}\times\frac{180}{\pi})^0$$
$$\frac{25\pi}{36}rad = 125^0$$

(xv). $\frac{19\pi}{32}$

Solution:

$$\frac{19\pi}{32}rad = (\frac{19\pi}{32}\times\frac{180}{\pi})^0$$
$$\frac{19\pi}{32}rad = 106.875^0$$

Question.3.

What is the circular measure of the angle between the hands of a watch at 4 O'clock?

Solution:

Since total angle in watch= $2\pi\ rad$.

Angle made by hands in 1 hour= $\frac{2\pi}{12} = \frac{\pi}{6}\ rad$.

Thus Angle made by hands in 4 hour = $4 \times \frac{\pi}{6}\ rad$

$$= \frac{2\pi}{3}rad.$$

Question.4.
Find θ, when:
(i). $l = 1.5\ cm\ , r = 2.5\ cm$
Solution:
$l = 1.5cm\ \ , r = 2.5cm$

Since $l = r\ \theta$

$$\theta = \frac{l}{r}$$

$$\theta = \frac{1.5}{2.5} = 0.6\ rad$$

(ii). $l = 3.2\ m\ \ , r = 2\ m$
Solution:
$l = 3.2m\ \ , r = 2\ m$
Since $l = r\ \theta$

$$\theta = \frac{l}{r}$$

$$\theta = \frac{3.2}{2} = 1.6\ rad$$

Question.5.
Find l, when:
(i). $\theta = \pi\ rad\ \ , r = 2.5\ cm$

Solution:
$\theta = \pi\ rad\ \ ,\ \ , r = 2.5cm$
Since

$$l = r\theta$$

$$\Rightarrow l = 6\pi$$

$$l = 6(3.14159)$$

$$l = 18.85cm$$

(ii). $\theta = 65^0 20'\ , r = 18\ mm$
Solution:
$\theta = 65°20`, \mathrm{r} = 18\mathrm{mm}$

$$\theta = 65°20` = 65.33°$$

$$\theta = 65.33 \times \frac{\pi}{180} radian$$

$$\theta = 65.33 \times \frac{3.14159}{180} radian$$

$$\theta = 1.1403\ rad.$$

Since $l = r\theta$

$$l = 18 \times 1.1403 = 20.525\ mm$$

Question.6.
Find r , when:
(i). $l = 5cm\ \ , \theta = 45^0$
Solution:

$l = 5cm\ , \theta = 45 \times \frac{\pi}{180}\ rad = \frac{\pi}{4}\ rad$

Since , $l = r\theta$

$$5 = r \times \frac{\pi}{4}$$

$$r = 2 \times \frac{4}{\pi}$$

$$r = 2.5465\ cm$$

(i). $l = 56\ cm\ , \theta = 45^0$
Solution:

$$l = 56\ cm\ , 45 \times \frac{\pi}{180}\ rad = \frac{\pi}{4} rad$$

Since , $l = r\theta$

$$56 = r \times \frac{\pi}{4}$$

$$r = 56 \times \frac{4}{\pi}$$

$$r = 71.3035\ cm$$

Question.7.
What is the length of the arc intercepted on a circle of radius 4 cms by the arms of a central angle of 45^0?
Solution:

$l =?\ \ , \theta = 45 \times \frac{\pi}{180} rad = \frac{\pi}{4} rad\ \ , r = 14\ cm$

Since , $l = r\theta$

$$l = 14 \times \frac{\pi}{4}$$

$$r = 14 \times \frac{3.14159}{4}$$

$$r = 10.0056\ cm$$

(i). $l = 56\ cm\ , \theta = 45^0$
Solution:

$$l = 56\ cm\ , 45 \times \frac{\pi}{180} rad = \frac{1}{2} rad$$

Since , $l = r\theta$

$$56 = r \times \frac{1}{2}$$

$$r = 2 \times 56$$

$$r = 112cm$$

Question.8.
Find the radius of the circle, in which the arms of a central angle of measure 1 radian
cut of an arc of length 35 cm.
Solution:

$$r =?\ \ \ , \theta = 1\ rad\ \ \ , l = 35\ cm$$

Since

$$l = r\theta$$

$$35 = r \times 1$$

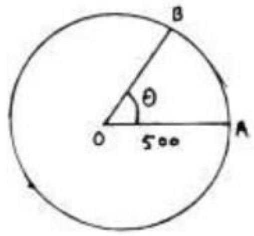
$$r = 35\ cm.$$

Question.10.

A railway train is running on a circular track of radius 500 meters at the rate of 30 km per hour. Through what angle will it turn in 10 sec.?

Solution:

$r = 500\ m$



$$\textbf{Distance} = l = \frac{25}{3} \times 10 = \frac{250}{3}\ m$$

$$\textbf{Speed} = \frac{30km}{h}$$

$$= 30 \times \frac{1000}{60 \times 60}\ m/s = \frac{25}{3}\ m/s$$

$$\textbf{Now,}\quad l = r\theta \Rightarrow \frac{250}{3} = 500 \times \theta$$

$$\Rightarrow \theta = \frac{250}{3 \times 500} = \frac{250}{1500}$$

$$\Rightarrow \theta = \frac{1}{6}\ rad.$$

Question.10:

A horse is tethered to a peg by a rope of 9 meters length and it can move in a circle with the peg as center. If the horse moves along the circumference of the circle, keeping the rope tight, how far will it have gone when the rope has turned through an angle of 70°?

Solution.

$$r = \mathbf{9\ m}\ ,\ \boldsymbol{\theta} = \mathbf{70^\circ} = 70 \times \frac{\pi}{180} = \frac{7\pi}{18}\quad .\ l = ?$$

$$\text{Now,}\quad l = r\theta \Rightarrow l = 9 \times \frac{7\pi}{18}$$

$$l = \frac{7}{2}(3.14159) = 10.9956\text{m} \approx 11\text{m}$$

Thus the horse w ill cover 11m distance.

Question.11:

The pendulum of a clock is 20 cm long and it swings through an angle of 20° each second. How far does the tip of the pendulum move in 1 second?

Solution.

$$r = 20cm$$

$$\theta = 20^\circ = 20 \times \frac{\pi}{180} = \frac{\pi}{9}\ rad \qquad l = ?$$

$$\textbf{Now,}\quad l = r\theta \Rightarrow l = 20 \times \frac{\pi}{9}$$

$$l = 20 \times \frac{3.14159}{9} = 6.98\ \text{cm}$$

Thus pendulum will move **6.98cm**

Question.12:

Assuming the average distance of the earth from the sun to be 148×10^6 km and the angle subtended by the sun at the eye of a person on the earth of measure 9.3×10^{-3} radian. Find the diameter of the sun.

Solution.

$$\textbf{Here,}\quad r = 148 \times 10^6\ km$$

$$\theta = 9.3 \times 10^{-3}\ rad.$$

$$\textbf{Since}\quad l = r\theta$$

$$\Rightarrow l = (148 \times 10^6)(9.3 \times 10^{-3})$$

$$= 1376400\ km = 1.3764\ 10^6\ km$$

Thus the diameter of the sun = $\mathbf{1.3764\ 10^6}\ \boldsymbol{km}$

Question.13:

A circular wire of radius 6 cm is cut straightened and then bent so as to lie along the circumference of a hoop of radius 24 cm. Find the measure of the angle which it subtends at the center of the hoop.

Solution.

$$Length\ of\ wire = circumference\ of\ circle = 2\pi r$$

$$= 2\pi(6) = 12\pi$$

$$\textbf{i.e.}\ l = 12\pi\ cm \quad , \quad r = 24cm$$

Now ,

$$\theta = \frac{l}{r} \Rightarrow \theta = \frac{12\pi}{24} = \frac{\boldsymbol{\pi}}{\mathbf{2}}\ \textbf{rad}$$

Question.14:

Show that the area of a sector of a circular region of radius r is $\frac{1}{2}\theta r^2$, where θ is the circular measure of the central angle of the sector.

Solution.

$$\frac{Area\ of\ sector}{Area\ of\ circle} = \frac{Central\ angle\ of\ sector}{Angle\ of\ circle}$$

$$\frac{Area\ of\ sector}{\pi r^2} = \frac{\theta}{2\pi}$$

$$\textbf{Area of sector} = \frac{\boldsymbol{\theta}}{\mathbf{2}\boldsymbol{\pi}} \times \boldsymbol{\pi r^2}$$

$$\textbf{Area of sector} = \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{\theta}\ \boldsymbol{r^2}$$

Hence proved.

Question.15:

Two cities *A* and *B* lie on the equator such that their longitudes are 45°E and 25°W respectively. Find the distance between the two cities, taking radius of the earth as 6400 kms.

Solution.

$$r = 6400 \ km$$

$$\theta = 45° + 25° = 70° = 70 \times \frac{\pi}{180} = \frac{7}{18}\pi \, rad$$

Now, $l = r\theta$

$$\Rightarrow l = (6400)\left(\frac{7}{18}\pi\right)$$

$$= (6400)(0.3889 \times 3.14159) = 7819.075 \text{ km}$$

Thus the distance between cities 7819.075 km

Question.16:

The moon subtends an angle of 0.5° at the eye of an observer on earth. The distance of the moon from the earth is $\mathbf{3.844 \times 10^5}$ km approx. What is the length of the diameter of the moon?

Solution.

$$\boldsymbol{r = 3.844 \times 10^5 \ km \quad, \theta = 0.5^0}$$

$$= 0.5 \times \frac{\pi}{180} \, rad$$

$$= 0.008727 \, rad$$

Now, $l = r\theta$

$$\Rightarrow l = (3.844 \times 10^5)(0.008727) = \mathbf{3354.505 \ km}$$

Thus the diameter of the moon $= 3354.505$ km

Question.17:

The angle subtended by the earth at the eye of a spaceman, landed on the moon, is 1° 54′. The radius of the earth is 6400 km. Find the approximate distance between the moon and the earth.

Solution.

$$\theta = 1°54` = 1.9° = 1.9 \times \frac{\pi}{180} = 0.03316 \ rad$$

$$l = 2(6400) = 12800 \ km \quad , \quad r = ?$$

Now,

$$l = r\theta \Rightarrow 12800 = r(0.03316)$$

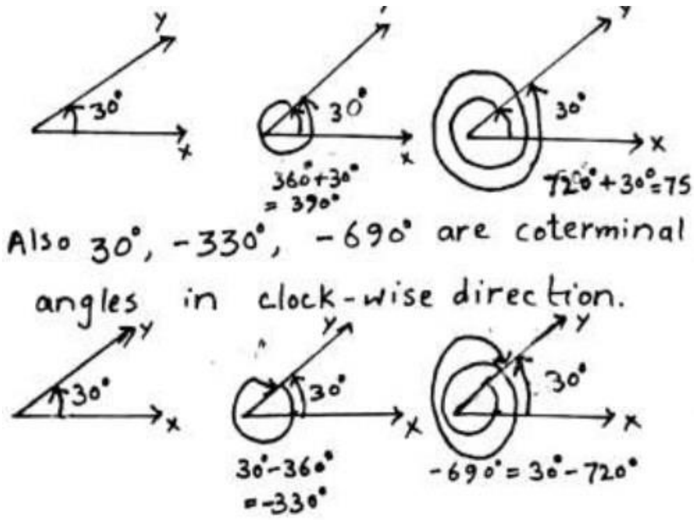
$$\Rightarrow r = \frac{12800}{0.03316} = \mathbf{386007.24 \ km}$$

Thus, the distance between earth and moon $=$ 386007.24 ***km***.

General angle:

Coterminal angles:

The angles having same initial and terminal sides are called coterminal angle. *e.g* $30^0, 390^0, 750^0$ *are coterminal angles* in anti-clock wise direction.



Note:

In general, if angle θ *is indegrees, then* $\theta + 360k, k \in Z$ is an angle coterminal with θ

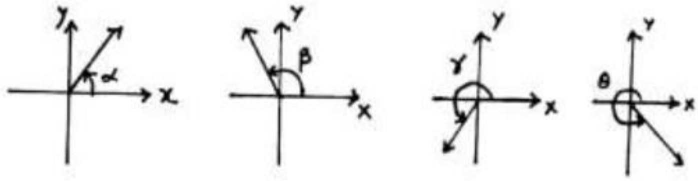
If angle θ is in radians , then $\theta + 2k\pi, k \in Z$ is an angle coterminal with θ

$\Rightarrow$ *general angle is* $\theta + 2k\pi, k \in Z$

trigonmetric functions or trimetric ratios of coterminal angles are some.

Angle in the standard position:

An angle is said to be in standard position if its vertex lies at the origin of a rectangular coordinates system and its initial side along the positive $x - axis$.

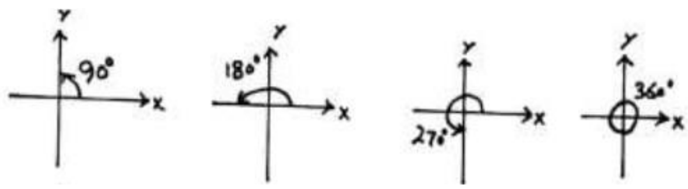


Angles α, β, γ *and* θ *are in standard position.*

Quadrantal angles:

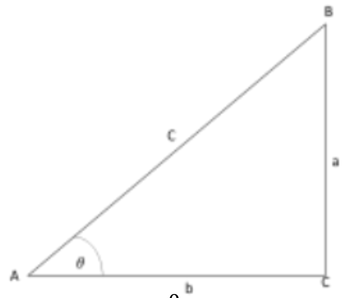
If the terminal side of an angle falls on $x - axis$ *or* $y - axis$

It is called a Quadrantal angle. i.e. $90^0, 180^0, 270^0$ *and* 360^0 are Quadrantal angles.



Trigonometric functions:

Consider a right triangles ABC *with* $\angle C = 90^0$ *and sides* a, b, c let $m\angle A = \theta radian$



The side AB *opposite to* 90^0 *is called hypotenuse*(*hyp*. The side BC opposite to θ is called perpendicular (prp) and the side AC related to angle θ is called adjacent or base.

Trig metric functions of angle θ are defines as below.

$$sin\theta = \frac{a}{c};\quad cosec\theta = \frac{c}{a}$$

$$cos\theta = \frac{b}{a}\quad sec\theta = \frac{c}{b}$$

$$tan\theta = \frac{a}{b};\quad cot\theta = \frac{b}{a}$$

We observe that

$$csc\theta = \frac{1}{sin\theta}\quad or\ sin\theta = \frac{1}{cosec\theta}$$

$$sec\theta = \frac{1}{cos\theta}\quad or\quad cos\theta = \frac{1}{sec\theta}$$

$$tan\theta = \frac{1}{cot\theta}\quad or\ cot\theta = \frac{1}{tan\theta}$$

Also $tan\theta = \frac{sin\theta}{cos\theta}$ or $cot\theta = \frac{cos\theta}{sin\theta}$

Fundamental identities:

For any real numbers θ

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Proof:

For any right triangle ABC Pythagoras theorem is

$$(Base)^2 + (perp)^2 = (Hyp)^2$$

$$\Rightarrow b^2 + a^2 = c^2 \rightarrow (i)$$

Dividing $(i) by\ c^2$ *we get*

$$\Rightarrow \frac{b^2}{c^2} + \frac{a^2}{c^2} = \frac{c^2}{c^2}$$

$$\Rightarrow \left(\frac{b}{a}\right)^2 + \left(\frac{a}{c}\right)^2 = 1$$

$$\Rightarrow (cos\theta)^2 + (sin\theta)^2 = 1$$

$$\Rightarrow \sin^2\theta + \cos^2\theta = 1$$

Dividing (i) by b^2 *we get*

$$\frac{b^2}{b^2} + \frac{a^2}{b^2} = \frac{c^2}{b^2}$$

$$\Rightarrow 1 + \left(\frac{a}{b}\right)^2 = \left(\frac{c}{a}\right)^2$$

$$\Rightarrow 1 + (\tan\theta)^2 = (\sec^2\theta)$$

$$\Rightarrow 1 + \tan^2\theta = \sec^2\theta$$

Dividing (i) by a^2 *we get*

$$\frac{b^2}{a^2} + \frac{a^2}{a^2} = \frac{c^2}{a^2}$$

$$\Rightarrow \left(\frac{b}{a}\right)^2 + 1 = \left(\frac{c}{a}\right)^2$$

$$\Rightarrow (cot\theta)^2 + 1 = \left(\frac{c}{a}\right)^2$$

$$\Rightarrow (cot\theta)^2 + 1 = (\csc^2\theta)$$

$$\Rightarrow q + \cot^2\theta = \csc^2\theta$$

Important note:

$$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = a - \cos^2\theta$$

$$\Rightarrow \boldsymbol{sin\theta = \pm\sqrt{1 - cos^2\theta}}$$

Or $\boldsymbol{cos^2 = 1 - sin^2\theta}$ $\quad\boldsymbol{\Rightarrow cos\theta = \pm\sqrt{1 - sin^2\theta}}$

$$\boldsymbol{1 + tan^2\theta = sec^2\theta \quad \Rightarrow 1 = sec^2 - tan^2\theta}$$

$$\boldsymbol{\Rightarrow tan^2\theta = sec^2\theta - 1 \Rightarrow tan\theta = \pm\sqrt{sce^2\theta - 1}}$$

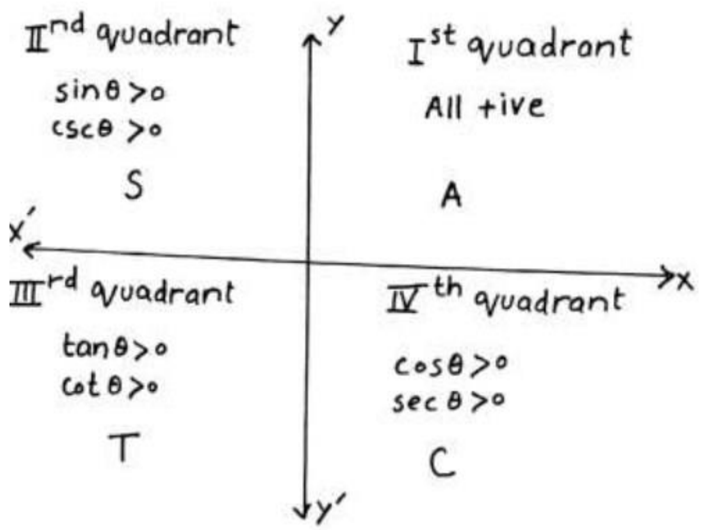
$$\boldsymbol{or\quad sec\theta = \pm\sqrt{a + tan^2\theta}}$$

$$\boldsymbol{1 + cot^2\theta = cse^2\theta \Rightarrow 1 = csc^2\theta - cot^2\theta}$$

$$\boldsymbol{\Rightarrow cot^2\theta = csc^2 - 1 \Rightarrow cot\theta = \pm\sqrt{csc^2 - 1}}$$

$$\boldsymbol{or\quad csc\theta = \pm\sqrt{1 - cot^2\theta}}$$

Signs of the trimetric functions:



CAST

C for $\boldsymbol{cos\ \theta}$ ***and its recprocal*** $\boldsymbol{sec\theta}$

A for all trigonometric functions

S for $\boldsymbol{sin\theta}$ ***and its recprocal*** $\boldsymbol{csce\theta}$

T for $\boldsymbol{tan\theta}$ ***and its recprocal*** $\boldsymbol{cot\theta}$

It is clear from the above figure that $\boldsymbol{sin(-\theta) = -sin\theta}$

$$\boldsymbol{csc(-\theta) - csc\theta}$$

$$\boldsymbol{cos(-\theta) = cos\theta}$$

$$\boldsymbol{sec(-\theta) = sec\theta}$$

$$\boldsymbol{tan(-\theta) = -tan\theta}$$

$$\boldsymbol{cot(-\theta) - cot\theta}$$

Exercise 9.2

Question. 1. Find the signs of the following:

(i). $\boldsymbol{Sin160^0}$

Solution.

As terminal side of the angle 160° is in 2nd quad. So sin160° is positive.

(ii). $\boldsymbol{Cos90^0}$

Solution.

As terminal side of the angle 190^0 is in 3rd quad. So $Cos90^0$ is negative.

(iii). $\boldsymbol{Tan115^0}$

Solution.

As terminal side of the angle 115^0 is in 2nd quad. So $Tan115^0$ is negative.

(iv). $\boldsymbol{Sec245^0}$

Solution.

As terminal side of the angle 245° is in 3rd quad. So $Sec245^0$ is negative.

(v). $\boldsymbol{Cot80^0}$

Solution.

As terminal side of the angle 80° is in 1st quad. So $Cot80^0$ is positive.

(vi). $\boldsymbol{Cosec297^0}$

Solution.

As terminal side of the angle 297° is in 4th quad. So $Cosec297^0$ is negative.

Question.2. Fill in the blanks

(i). $\boldsymbol{Sin(-310^0) = \cdots SIn(310^o))}$

Solution.

$$Sin(-310^0) = -Sin310^0.$$

(ii). $\boldsymbol{Cos(-75^0) = \cdots Cos(75^0)}$

Solution.

$$Cos(-75^0) = +Cos(75^0)$$

(iii). $\boldsymbol{tan(-182^0) = \cdots \tan(182^0)}$

Solution.

$$tan(-182^0) = -\tan 182^0$$

(iv). $\boldsymbol{Cot(-173^0) = \cdots Cot173^0}$

Solution.

$$Cot(-173^0) = +Cot173^0$$

(v). $\boldsymbol{Sec(-216^0) = \underset{\cdots}{} Sec216^0}$

Solution. $Sec(-216^0) = -Sec216^0$

(vi). $\boldsymbol{Cosec(-15^0) = \underset{\cdots}{} Cosec15^0}$

Solution.

$$Cosec(-15^0) = \cdots Cosec15^0.$$

Question.3.In which quadrant are the terminal arms of the angle lie when

(i). $\boldsymbol{Sin\theta < 0 \ \ and \ Cos\theta > 0}$.

Solution.

Since $Sin\theta < 0$, $so\ \theta\ lies\ in\ the\ 3rd\ or\ 4th\ quadrant.$
$Also\ Cos\theta > 0$, $So\ \theta\ lies\ in\ the\ 1st\ or\ 4th\ quadrant$.
Hence θ lies in the $4th\ quadrant$.

(ii). $\boldsymbol{Cot\theta > 0 \ \ and \ Cosec\theta > 0}$.

Solution.

Since $Cot\theta > 0$, $so\ \theta\ lies\ in\ the\ 1st\ or\ 3rd\ quadrant.$
$Also\ Cosec\theta > 0$, $So\ \theta\ lies\ in\ the\ 1st\ or\ 2nd\ quadrant$.
Hence θ lies in the $1st\ quadrant$.

(iii). $\boldsymbol{tan\theta < 0 \ \ and \ Cos\theta > 0}$.

Solution.

Since $tan\theta < 0$, $so\ \theta\ lies\ in\ the\ 1st\ or\ 3rd\ quadrant.$
$Also\ Cos\theta > 0$, $So\ \theta\ lies\ in\ the\ 1st\ or\ 4th\ quadrant$.
Hence θ lies in the $1st\ quadrant$.

(iv). $\boldsymbol{Sec\theta < 0 \ \ and \ Sin\theta < 0}$.

Solution.

Since $Sec\theta < 0$, $so\ \theta\ lies\ in\ the\ 2nd\ or\ 3rd\ quadrant.$
$Also\ Sin\theta < 0$, $So\ \theta\ lies\ in\ the\ 3rd\ or\ 4th\ quadrant$.
Hence θ lies in the $3rd\ quadrant$.

(v). $\boldsymbol{Cot\theta > 0 \ \ and \ Sin\theta < 0}$.

Solution.

Since $Cot\theta < 0$, $so\ \theta\ lies\ in\ the\ 2nd\ or\ 4th\ quadrant.$
$Also\ Sin\theta < 0$, $So\ \theta\ lies\ in\ the\ 3rd\ or\ 4th\ quadrant$.
Hence θ lies in the $4th\ quadrant$.

(vi). $\boldsymbol{Cos\theta < 0 \ \ and \ Tan\theta < 0}$.

Solution.

Since $Cos\theta < 0$, $so\ \theta\ lies\ in\ the\ 2nd\ or\ 3rd\ quadrant.$

$Also\ Tan\theta <$
0, $So\ \theta$ lies in the 2nd or 4th quadrant.
Hence θ lies in the 2nd quadrant.

Question.4. Find the values of the remaining trigonometric functions:

(i). $Sin\theta = \frac{12}{13}$ and the terminal arm of the angle is in the quadrant I.

Solution.

Since,

$$Sin\theta = \frac{11}{13} > 0$$

And terminal arm of the angle lies in I quadrant. SO al the functions will be positive.

Since $Sin\theta = \frac{11}{13} = \frac{p}{h}$

$$SO\ \ p = 11 \quad and\ h = 13$$

We know that by Pythagoras theorem, we have

$$h^2 = p^2 + b^2$$
$$(13)^2 = (11)^2 + (b)^2$$
$$169 = 121 + b^2$$
$$b^2 = 169 - 121 = 48$$
$$b = \sqrt{48} = 4\sqrt{3}$$

Now

$$Sin\theta = \frac{p}{h} = \frac{11}{13}$$
$$Cosec\theta = \frac{1}{Sin\theta} = \frac{13}{11}$$
$$Cos\theta = \frac{b}{h} = \frac{4\sqrt{3}}{13}$$
$$Sec\theta = \frac{1}{Cos\theta} = \frac{13}{4\sqrt{3}}.$$
$$Tan\theta = \frac{p}{b} = \frac{11}{4\sqrt{3}}$$
$$Cot\theta = \frac{1}{tan\theta} = \frac{4\sqrt{3}}{11}.$$

Which is required.

(ii). $Cos\theta = \frac{9}{41}$ *and* the terminal arm of the angle is in the quadrant IV.

Solution.

Since,

$$Cos\theta = \frac{9}{41} > 0$$

And terminal arm of the angle lies in IV quadrant. So all function will be negative except $Cos\theta$ *and* $Sec\theta$.

Since $Cos\theta = \frac{9}{41} = \frac{b}{h}$

$$SO\ \ b = 9 \quad and\ h = 41$$

We know that by Pythagoras theorem, we have

$$h^2 = p^2 + b^2$$
$$(41)^2 = p^2 + (9)^2$$
$$1681 = p^2 + 81$$

$$p^2 = 1681 - 81 = 1600$$
$$p = 40$$

Now

$$Sin\theta = -\frac{p}{h} = -\frac{40}{41}$$
$$Cosec\theta = \frac{1}{Sin\theta} = -\frac{41}{40}$$
$$Cos\theta = \frac{b}{h} = \frac{9}{41}$$
$$Sec\theta = \frac{1}{Cos\theta} = \frac{41}{9}.$$
$$Tan\theta = \frac{p}{b} = \frac{40}{9}$$

$$Cot\theta = \frac{1}{tan\theta} = \frac{9}{40}$$

Which is required

(iii). $Cos\theta = -\frac{\sqrt{3}}{2}$ and the terminal arm of the angle is in the quadrant III.

Solution.

Since,

$$Cos\theta = -\frac{\sqrt{3}}{2} < 0$$

And terminal arm of the angle lies in III quadrant. So all function will be negative except $tan\theta$ *and* $cot\theta$.

$$\textbf{Since } Cos\theta = -\frac{\sqrt{3}}{2} = \frac{b}{h}$$

$$SO\ \ b = \sqrt{3} \quad and\ h = 2$$

We know that by Pythagoras theorem, we have

$$h^2 = p^2 + b^2$$
$$(2)^2 = p^2 + (\sqrt{3})^2$$
$$4 = p^2 + 3$$

$$p^2 = 4 - 3 = 1$$
$$p = 1$$

Now

$$Sin\theta = -\frac{p}{h} = -\frac{1}{2}$$

$$Cosec\theta = \frac{1}{Sin\theta} = -2$$
$$Cos\theta = -\frac{b}{h} = -\frac{\sqrt{3}}{2}$$
$$Sec\theta = \frac{1}{Cos\theta} = -\frac{2}{\sqrt{3}}.$$
$$Tan\theta = \frac{p}{b} = \frac{1}{2}$$
$$Cot\theta = \frac{1}{tan\theta} = 2$$

Which is required.

(iv). $tan\theta = -\frac{1}{3}$ and the terminal arm of the angle is in the quadrant II.

Solution.

Since,

$$tan\theta = -\frac{1}{3} < 0$$

And terminal arm of the angle lies in II quadrant. So all function will be negative except $Sin\theta$ *and* $Cosec\theta$.

$$\textbf{Since } tan\theta = -\frac{1}{3} = \frac{p}{b}$$
$$SO\ \ p = 1 \quad and\ b = 3$$

We know that by Pythagoras theorem, we have

$$h^2 = p^2 + b^2$$
$$h^2 = (1)^2 + (3)^2$$
$$h^2 = 1 + 9$$
$$h^2 = 10$$
$$h = \sqrt{10}$$

Now

$$Sin\theta = \frac{p}{h} = \frac{1}{\sqrt{10}}$$
$$Cosec\theta = \frac{1}{Sin\theta} = \sqrt{10}$$
$$Cos\theta = -\frac{b}{h} = -\frac{3}{\sqrt{10}}$$
$$Sec\theta = \frac{1}{Cos\theta} = -\frac{\sqrt{10}}{3}.$$
$$Tan\theta = -\frac{p}{b} = -\frac{1}{3}$$
$$Cot\theta = \frac{1}{tan\theta} = -3$$

Which is required.

(v). $Sin\theta = -\frac{1}{\sqrt{2}}$ and the terminal arm of the angle is not in the quadrant III.

Solution.

Since,

$$Sin\theta = -\frac{1}{\sqrt{2}} < 0$$

And terminal arm of the angle does not lie in III quadrant. So terminal arm lies in the IV quadrant. Hence $Cos\theta$ *and* $Sec\theta$ *positive all other functions are*

$$\textbf{Since } Sin\theta = -\frac{1}{\sqrt{2}} = -\frac{p}{h}$$
$$SO\ \ p = 1 \quad and\ h = \sqrt{2}$$

We know that by Pythagoras theorem, we have

$$h^2 = p^2 + b^2$$
$$(\sqrt{2})^2 = (1)^2 + (b)^2$$
$$2 = 1 + b^2$$
$$b^2 = 2 - 1 = 1$$
$$b = 1$$

Now

$$Sin\theta = -\frac{p}{h} = -\frac{1}{\sqrt{2}}$$
$$Cosec\theta = \frac{1}{Sin\theta} = -\sqrt{2}$$
$$Cos\theta = \frac{b}{h} = \frac{1}{\sqrt{2}}$$
$$Sec\theta = \frac{1}{Cos\theta} = \sqrt{2}.$$
$$Tan\theta = -\frac{p}{b} = -\frac{1}{\sqrt{2}}$$
$$Cot\theta = \frac{1}{tan\theta} = -\sqrt{2}$$

Which is required.

Question.5.

$Cot\theta = \frac{15}{8}$ *and the* terminal arm of the angle is not in quadrant I , find $Cos\theta$ and $Cosec\theta$.

Solution.

Since,

$$Cot\theta = \frac{15}{8} > 0$$

And terminal arm of the angle does not lie in I quadrant. So terminal arm lies in the III quadrant. Hence $Tan\theta$ *and* $COt\theta$ *positive all other functions are*

$$\textbf{Since } Cot\theta = \frac{15}{8} = \frac{b}{p}$$
$$SO\ \ p = 8 \quad and\ b = 15$$

We know that by Pythagoras theorem, we have

$$h^2 = p^2 + b^2$$
$$(h)^2 = (8)^2 + (15)^2$$
$$h^2 = 64 + 225$$
$$h^2 = 289$$
$$h = 17$$

Now

$$Cosec\theta = -\frac{h}{p} = -\frac{17}{8}$$
$$Cos\theta = -\frac{b}{h} = -\frac{15}{17}$$

Which is required.

Question.6.

If $Cosec\theta = \frac{m^2+1}{2m}$ and $m > 0$ $(0 < \theta < \frac{\pi}{2})$, *find the values of the remaining trigonometric functions.*

Solution.

Since,

$$Cosec\theta = \frac{m^2+1}{2m} > 0$$

And terminal arm of the angle lies in I quadrant. So all function will be positive.

$$\textbf{Since } Cosec\theta = \frac{m^2+1}{2m} = \frac{h}{b}$$
$$SO\ \ b = 2m \quad and\ h = m^1 + 1$$

We know that by Pythagoras theorem, we have

$$h^2 = p^2 + b^2$$
$$(m^2+1)^2 = p^2 + (2m)^2$$
$$m^4 + 2m^2 + 1 = p^2 + 4m^2$$
$$p^2 = m^4 + 2m^2 + 1 - 4m^2$$
$$p^2 = m^4 - 2m^2 + 1$$
$$p^2 = (m^2-1)^2$$
$$p = m^2 - 1$$

Now

$$Sin\theta = \frac{p}{h} = \frac{m^2-1}{m^2+1}$$
$$Cosec\theta = \frac{1}{Sin\theta} = \frac{m^2+1}{m^2-1}$$
$$Cos\theta = \frac{b}{h} = \frac{2m}{m^2+1}$$
$$Sec\theta = \frac{1}{Cos\theta} = \frac{m^2+1}{2m}.$$
$$Tan\theta = \frac{p}{b} = \frac{m^2-1}{2m}$$
$$Cot\theta = \frac{1}{tan\theta} = \frac{2m}{m^2-1}$$

Which is required.

Question.7.

If $tan\theta = \frac{1}{\sqrt{7}}$ **and the terminal arm of the angle is not in the quadrant III, find the va***lue*

$$\frac{COsec^2\theta - Sec^2\theta}{COsec^2\theta + Sec^2\theta}.$$

Solution.

Since,

$$tan\theta = \frac{1}{\sqrt{7}} > 0$$

And terminal arm of the angle lies in III quadrant. So terminal arm of the angle lies in the I quadrant. Hence all function will be positive.

$$\textbf{Since } tan\theta = \frac{1}{\sqrt{7}} = \frac{p}{b}$$
$$SO\ \ p = 1 \quad and\ b = \sqrt{7}$$

We know that by Pythagoras theorem, we have

$$h^2 = p^2 + b^2$$
$$h^2 = (1)^2 + (\sqrt{7})^2$$
$$h^2 = 1 + 7$$
$$h^2 = 8$$
$$h = \sqrt{8} = 2\sqrt{2}$$

Now

$$Cosec\theta = \frac{h}{p} = \frac{2\sqrt{2}}{1} = 2\sqrt{2}$$
$$=> Cosec^2\theta = (2\sqrt{2})^2 = 4(2) = 8$$
$$Sec\theta = \frac{h}{b} = \frac{2\sqrt{2}}{\sqrt{7}}.$$
$$Sec^2\theta = \frac{8}{7}$$

Now

$$\frac{COsec^2\theta - Sec^2\theta}{COsec^2\theta + Sec^2\theta} = \frac{8-\frac{8}{7}}{8+\frac{8}{7}}$$
$$\frac{COsec^2\theta - Sec^2\theta}{COsec^2\theta + Sec^2\theta} = \frac{\frac{56-8}{7}}{\frac{56+8}{7}}$$
$$\frac{COsec^2\theta - Sec^2\theta}{COsec^2\theta + Sec^2\theta} = \frac{48}{64}$$

$$\frac{COsec^2\theta - Sec^2\theta}{COsec^2\theta + Sec^2\theta} = \frac{3}{4}.$$

Which is required.

Question.8.

If $Cot\theta = \frac{5}{2}$ and the terminal arm of the angle is in quadrant I , find $\frac{3Sin\theta+4Cos\theta}{Cos\theta-Sin\theta}$.

Solution.

Since,

$$Cot\theta = \frac{5}{2} > 0$$

And terminal arm of the angle lies in I quadrant.
all functions are positive.

$$Since\ Cot\theta = \frac{5}{2} = \frac{b}{p}$$

$$SO\ \ p = 2 \quad and\ b = 5$$

We know that by Pythagoras theorem, we have

$$h^2 = p^2 + b^2$$

$$(h)^2 = (2)^2 + (5)^2$$

$$h^2 = 4 + 25$$

$$h^2 = 29$$

$$h = \sqrt{29}$$

Now

$$Sin\theta = \frac{p}{h} = \frac{2}{\sqrt{29}}$$

$$Cos\theta = \frac{b}{h} = \frac{5}{\sqrt{29}}$$

Now

$$\frac{3Sin\theta + 4Cos\theta}{Cos\theta - Sin\theta} = \frac{3\left(\frac{2}{\sqrt{29}}\right) + 4\left(\frac{5}{\sqrt{29}}\right)}{\left(\frac{5}{\sqrt{29}}\right) - \left(\frac{2}{\sqrt{29}}\right)}$$

$$\frac{3Sin\theta + 4Cos\theta}{Cos\theta - Sin\theta} = \frac{\left(\frac{6}{\sqrt{29}}\right) + \left(\frac{20}{\sqrt{29}}\right)}{\left(\frac{5}{\sqrt{29}}\right) - \left(\frac{2}{\sqrt{29}}\right)}$$

$$\frac{3Sin\theta + 4Cos\theta}{Cos\theta - Sin\theta} = \frac{\left(\frac{6+20}{\sqrt{29}}\right)}{\left(\frac{5-2}{\sqrt{29}}\right)}$$

$$\frac{3Sin\theta + 4Cos\theta}{Cos\theta - Sin\theta} = \frac{26}{3}$$

Which is required.

The valves of trigonometric functions of acute angles $45^0, 30^0$ and 60^0

(a) Trigonometric functions of 45^0

We take $a = b = 1$

$$c^2 = a^2 + b^2$$

$$\Rightarrow c^2 = a^2 + b^2$$

$$\Rightarrow c^2 = (1)^2 + (1)^2 = 1 + 1$$

$$c^2 = 2 \Rightarrow c = \sqrt{2}$$

Now sides $sin45^0 = \frac{a}{c} = \frac{1}{\sqrt{2}};\ \ cosec45^0 = \frac{c}{a} = \frac{\sqrt{2}}{1}$

$$cos45^0 = \frac{b}{c} = \frac{1}{\sqrt{2}};\quad sec45^0 = \frac{c}{b} = \frac{\sqrt{2}}{1}$$

$Tan45^0 = \frac{a}{b} = \frac{1}{1} = 1;\quad cot45^0 = \frac{b}{a} = \frac{1}{1} = 1$

(b) **Trigonometric functions of 30^0**

By elementary geometry in a right triangles the measure of side opposite to 30^0 *is half of th hypotenouse.*
So take

$$c = 2\ \ then\ a = 1\ \ by\ pathgoras\ therorem$$

$$c^2 = a^2 + b^2 \Rightarrow b^2 = c^2 - a^2$$

$$\Rightarrow b^2 = (2)^2 - (1)^2 = 4 - 1 = 3$$

$$\Rightarrow b^2 = \sqrt{3}Now$$

$$sin30^0 = \frac{a}{c} = \frac{1}{2};\quad csc30^0 = \frac{a}{c} = \frac{2}{1} = 2$$

$$cos30^0 = \frac{b}{c} = \frac{\sqrt{3}}{2};\quad sec30^0 = \frac{c}{b} = \frac{2}{\sqrt{3}}$$

$$tan30^0 = \frac{a}{b} = \frac{1}{\sqrt{3}};\quad cot30^0 = \frac{b}{a} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

© **trigonometric functions of 60^0**

By elementary geometry in a right triangles the measure of side opposite to 30^0 *is half of th hypotenouse.*
So take

$$c = 2\ \ then\ b = 1\ \ by\ pathgoras\ therorem$$

$$c^2 = a^2 + b^2 \Rightarrow a^2 = c^2 - b^2$$

$$\Rightarrow a^2 = (2)^2 - (1)^2 = 4 - 1 = 3$$

$$\Rightarrow a = \sqrt{3}Now$$

$$sin60^0 = \frac{a}{c} = \frac{\sqrt{3}}{2};\quad csc60^0 = \frac{c}{a} = \frac{2}{\sqrt{3}} = 2$$

$$cos60^0 = \frac{b}{c} = \frac{1}{2};\quad sec60^0 = \frac{c}{b} = \frac{2}{1} = 2$$

$$tan60^0 = \frac{a}{b} = \sqrt{3};\quad cot60^0 = \frac{b}{a} = \frac{1}{\sqrt{3}}$$

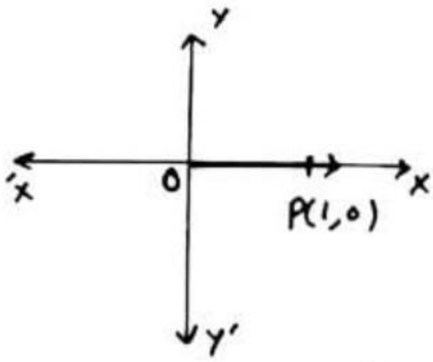
The valves of the trigonometric functions of angles $o^0, 90^0, 180^0, 270^0, 360^0$

(a)

$$\boldsymbol{when\ \theta = 0^0}$$

$$\boldsymbol{the\ point\ (1,0) lies\ on\ the\ terminal\ side\ of\ 0^0}$$

$$\boldsymbol{\Rightarrow x = 1\ \ and\ y = 0}$$



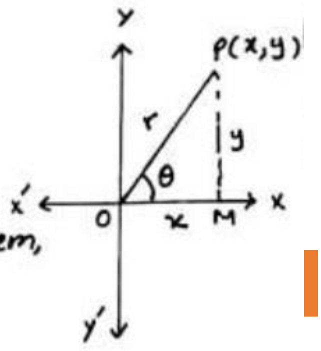
Note:-

In ΔMOP

OM= x, MP=y, OP=r

By Pathagora's theorem,

$r^2 = x^2 + y^2$



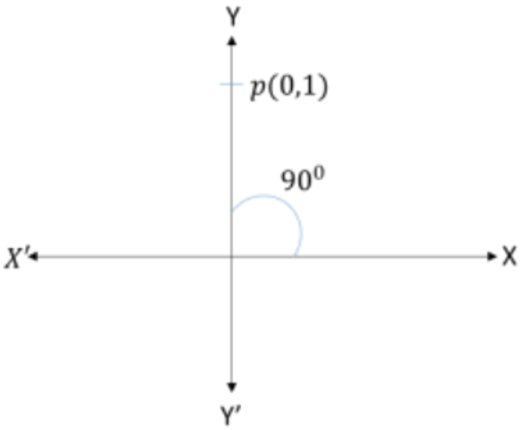
$\because\ r = \sqrt{x^2 + y^2} = \sqrt{(1)^2 + (0)^2} = \sqrt{1+0} = 1$

$\because\ sin0^0 = \frac{y}{r} = \frac{0}{1} = 0;\quad csc0^0 = \frac{r}{y} = \frac{1}{0} = \infty$

$\cos 0^0 = \frac{x}{r} = \frac{1}{1} = 1;\quad sec0^0 = \frac{r}{x} = \frac{1}{1} = 1$

$tan0^0 = \frac{y}{x} = \frac{0}{1} = 0;\quad cot0^0 = \frac{x}{y} = \frac{1}{0} = \infty$

(b) when $\theta = 90^0$



The point $(0,1)$ lies on the terminal side of angle 90^0

$\Rightarrow x = 0, y = 1$

$so\ r = \sqrt{x^2 + y^2} = \sqrt{(0)^2 + (1)^2}$

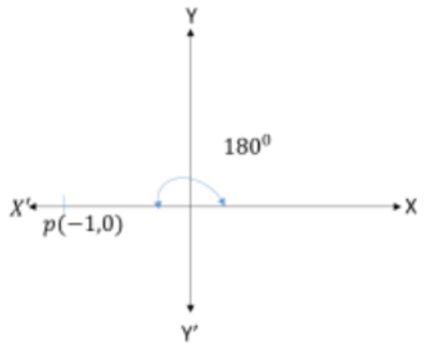
$\Rightarrow r = 11$

$\because\ sin90^0 = \frac{y}{r} = \frac{1}{1} = 1;\quad csc90^0 = \frac{r}{y} = \frac{1}{1} = 1$

$\cos 90^0 = \frac{x}{r} = \frac{0}{1} = 0;\quad sec90^0 = \frac{r}{x} = \frac{1}{0} = \infty$

$tan90^0 = \frac{y}{x} = \frac{1}{0} = \infty;\quad cot90^0 = \frac{x}{y} = \frac{0}{1} = 0$

© when $\theta = 180^0$



The point $(-1,0)$ lies on the terminal side of angle 180^0

$\Rightarrow x = -1, y = 0$

$so\ r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + (0)^2}$

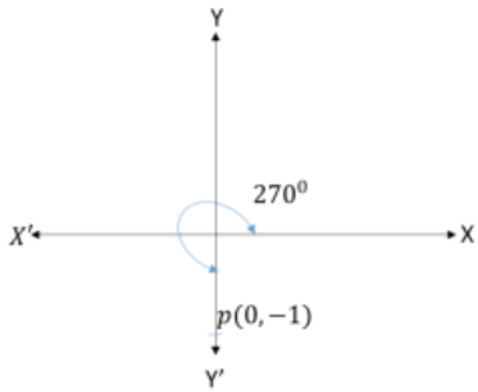
$\Rightarrow r = \sqrt{1+0} = 1$

$\because\ sin180^0 = \frac{y}{r} = \frac{0}{1} = 0;\quad csc180^0 = \frac{r}{y} = \frac{1}{0} = -\infty$

$\cos 180^0 = \frac{x}{r} = \frac{-1}{1} = -1;\quad sec180^0 = \frac{r}{x} = \frac{1}{-1} = -1$

$tan180^0 = \frac{y}{x} = \frac{0}{-1} = 0;\quad cot180^0 = \frac{x}{y} = \frac{1}{0} = \infty$

(d) when $\theta = 270^0$



The point $(0,-1)$ lies on the terminal side of angle 180^0

$\Rightarrow x = 0, y = -1$

$so\ r = \sqrt{x^2 + y^2} = \sqrt{(0)^2 + (-1)^2}$

$\Rightarrow r = \sqrt{0+1} = 1$

$\because\ sin270^0 = \frac{y}{r} = \frac{-1}{1} = -1;\quad csc270^0 = \frac{r}{y} = \frac{1}{-1}$

$= -1$

$\cos 270^0 = \frac{x}{r} = \frac{0}{1} = 0;\quad sec270^0 = \frac{r}{x} = \frac{1}{0} = \infty$

$tan270^0 = \frac{y}{x} = \frac{-1}{0} = \infty;\quad cot270^0 = \frac{x}{y} = \frac{0}{-1} = 0$

Exercise 9.3

Verify the following:

(i). $\boldsymbol{sin60°cos30° - cos60°sin30° = sin30°}$

Solution

$\boldsymbol{L.H.S} = \sin60°\cos30° - \cos60°\sin30°$

$\boldsymbol{L.H.S} = \frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2} - \frac{1}{2}.\frac{1}{2}$

$\boldsymbol{L.H.S} = \frac{3}{4} - \frac{1}{4} = \frac{3-1}{4}$

$\boldsymbol{L.H.S} = \frac{2}{4} = \frac{1}{2}$

$\boldsymbol{L.H.S} = \sin30° = \boldsymbol{L.H.S}$

Hence Proved.

(ii). $\boldsymbol{sin^2\frac{\pi}{6} + sin^2\frac{\pi}{3} + tan^2\frac{\pi}{4} = 2}$

Solution.

$$L.H.S = sin^2\frac{\pi}{6} + sin^2\frac{\pi}{3} + tan^2\frac{\pi}{4}$$

$$L.H.S = (\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2 + (1)^2$$

$$L.H.S = \frac{1}{4} + \frac{3}{4} + 1$$

$$L.H.S = \frac{1+3+4}{4}$$

$$L.H.S = \frac{1}{4} = 2 = R.H.S$$

Hence Proved.

(iii). $2sin45° + \frac{1}{2}cosec45° = \frac{3}{\sqrt{2}}$

Solution.

$$L.H.S = 2sin45° + \frac{1}{2}cosec45° = \frac{3}{\sqrt{2}}$$

$$L.H.S = 2(\frac{1}{\sqrt{2}}) + \frac{1}{2}(\frac{1}{\frac{1}{\sqrt{2}}}) = \frac{2}{\sqrt{2}} + \frac{\sqrt{2}}{2}$$

$$= \frac{(\sqrt{2})^2}{2} + \frac{\sqrt{2}}{(\sqrt{2})^2}$$

$$L.H.S = \sqrt{2} + \frac{1}{\sqrt{2}} = \frac{2+1}{\sqrt{2}} = \frac{3}{\sqrt{2}} = R.H.S$$

$$L.H.S = R.H.S$$

Hence Proved.

(iv). $2sin45° + \frac{1}{2}cosec45° = \frac{3}{\sqrt{2}}$

Solution.

$$L.H.S = sin^2\frac{\pi}{6} : sin^2\frac{\pi}{4} : sin^2\frac{\pi}{3} : sin^2\frac{\pi}{2}$$

$$L.H.S = (\frac{1}{2})^2 : (\frac{1}{\sqrt{2}})^2 : (\frac{\sqrt{3}}{2})^2 : (1)^2$$

$$L.H.S = \frac{1}{4} : \frac{1}{2} : \frac{3}{4} : 1$$

Multiplying by 4

$$L.H.S = 1 : 2 : 3 : 4 = R.H.S$$

$$L.H.S = R.H.S$$

Hence Proved.

Question.2.

Evaluate the following:

(i). $\frac{tan\frac{\pi}{3} - tan\frac{\pi}{6}}{1+tan\frac{\pi}{3}\, tan\frac{\pi}{6}}$

Solution.

$$\frac{tan\frac{\pi}{3} - tan\frac{\pi}{6}}{1+tan\frac{\pi}{3}\, tan\frac{\pi}{6}} = \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1+\sqrt{3}.\frac{1}{\sqrt{3}}}$$

$$\frac{tan\frac{\pi}{3} - tan\frac{\pi}{6}}{1+tan\frac{\pi}{3}\, tan\frac{\pi}{6}} = \frac{\frac{3-1}{\sqrt{3}}}{1+1}$$

$$\frac{tan\frac{\pi}{3} - tan\frac{\pi}{6}}{1+tan\frac{\pi}{3}\, tan\frac{\pi}{6}} = \frac{\frac{2}{\sqrt{3}}}{2}$$

$$\frac{tan\frac{\pi}{3} - tan\frac{\pi}{6}}{1+tan\frac{\pi}{3}\, tan\frac{\pi}{6}} = \frac{2}{2\sqrt{3}}$$

$$\frac{tan\frac{\pi}{3} - tan\frac{\pi}{6}}{1+tan\frac{\pi}{3}\, tan\frac{\pi}{6}} = \frac{1}{\sqrt{3}}$$

Which is required.

(ii) . $\frac{1-tan^2\frac{\pi}{3}}{1+tan^2\frac{\pi}{3}}$

Solution.

$$\frac{1 - tan^2\frac{\pi}{3}}{1 + tan^2\frac{\pi}{3}} = \frac{1-(\sqrt{3})^2}{1+(\sqrt{3})^2}$$

$$\frac{1 - tan^2\frac{\pi}{3}}{1 + tan^2\frac{\pi}{3}} = \frac{1-3}{1+3}$$

$$\frac{1 - tan^2\frac{\pi}{3}}{1 + tan^2\frac{\pi}{3}} = \frac{-2}{4}$$

$$\frac{1 - tan^2\frac{\pi}{3}}{1 + tan^2\frac{\pi}{3}} = -\frac{1}{2}$$

Which is required.

Question.3.

Verify the following when $\theta = 30°, 45°$

(i) $sin2\theta = 2sin\theta cos\theta$

Solution.

When $= 30°$

$$L.H.S = \sin 2\theta = \sin 2(30)$$

$$= \sin 60 = \frac{3}{\sqrt{2}} \rightarrow (i)$$

$$R.H.S = 2\sin\theta\cos\theta = 2\sin 30°\cos 30°$$

$$R.H.S = 2(\frac{1}{\sqrt{2}})(\frac{\sqrt{3}}{2}) = \frac{3}{\sqrt{2}} \rightarrow (ii)$$

From (i) and (ii) , **L.H.S = R.H.S**

Hence Proved.

When $= 45°$

$$L.H.S = \sin 2\theta = \sin 2(45)$$

$$= \sin 90 = 1 \rightarrow (iii)$$

$$R.H.S = 2\sin\theta\cos\theta = 2\sin 45°\cos 45°$$

$$R.H.S = 2(\frac{1}{\sqrt{2}})(\frac{1}{\sqrt{2}}) = \frac{2}{2} = 1 \rightarrow (iv)$$

From (i) and (ii) ,**L.H.S = R.H.S**

Hence proved.

(ii). $cos2\theta = cos^2\theta - sin^2\theta$

Solution.

When $= 30°$

$$L.H.S = cos2\theta = \cos 2(30) = \cos 60 = \frac{1}{2} \rightarrow (i)$$

$$R.H.S = \cos^2\theta - \sin^2\theta = \cos^2 30° - \sin^2 30$$

$$R.H.S = (\frac{\sqrt{3}}{2})^2 - (\frac{1}{2})^2 = \frac{3}{4} - \frac{1}{4} = \frac{1}{2} \rightarrow (ii)$$

From (i) and (ii) , **L.H.S = R.H.S**

Hence Proved.

When $= 45°$

$L.H.S = \boldsymbol{cos2\theta} = \cos 2(45) = \cos 90 = 0 \rightarrow (iii)$

$R.H.S = \cos^2\theta - \sin^2\theta = \cos^2 45° - \sin^2 45$

$R.H.S = \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2 = 0 \rightarrow (iv)$

From (i) and (ii) ,**L.H.S = R.H.S**

Hence proved.

(iii). cos2θ $= 2\cos^2\theta - 1$

Solution.

When $= 30°$

$L.H.S = \boldsymbol{cos2\theta} = \cos 2(30) = \cos 60 = \frac{1}{2} \rightarrow (i)$

$R.H.S = 2\cos^2\theta - 1 = 2\cos^2 30° - 1$

$R.H.S = 2\left(\frac{\sqrt{3}}{2}\right)^2 - 1$

$= 2\left(\frac{3}{4}\right) - 1 = \frac{3}{2} - 1 = \frac{3-2}{2} = \frac{1}{2} \rightarrow (ii)$

From (i) and (ii) , **L.H.S = R.H.S**

Hence Proved.

When $= 45°$

$L.H.S = \boldsymbol{cos2\theta} = \cos 2(45) = \cos 90 = 0 \rightarrow (iii)$

$R.H.S = 2\cos^2\theta - 1$

$= 2\cos^2 45° - 1$

$R.H.S = 2\left(\frac{1}{\sqrt{2}}\right)^2 - 1$

$= 2\left(\frac{1}{2}\right) - 1 = 1 - 1 = 0 \rightarrow (iv)$

From (i) and (ii) ,**L.H.S = R.H.S**

Hence proved.

(iv).***cos2θ*** $= 1 - 2\sin^2\theta$

Solution.

When $= 30°$

$L.H.S = \boldsymbol{cos2\theta} = \cos 2(30) = \cos 60 = \frac{1}{2} \rightarrow (i)$

$R.H.S = 1 - 2\sin^2\theta = 1 - 2\,\mathrm{Sin}^2 30^0$

$R.H.S = 1 - 2\left(\frac{1}{2}\right)^2$

$= 1 - 2\left(\frac{1}{4}\right) = 1 - \frac{1}{2} = \frac{2-1}{2} = \frac{1}{2} \rightarrow (ii)$

From (i) and (ii) , **L.H.S = R.H.S**

Hence Proved.

When $= 45°$

$L.H.S = \boldsymbol{cos2\theta} = \cos 2(45) = \cos 90 = 0 \rightarrow (iii)$

$R.H.S = 1 - 2\sin^2\theta = 1 - 2\sin^2 45^0$

$R.H.S = 1 - 2\left(\frac{1}{\sqrt{2}}\right)^2$

$= 1 - 2\left(\frac{1}{2}\right) = 1 - 1 = 0 \rightarrow (iv)$

From (i) and (ii) ,**L.H.S = R.H.S**

Hence proved.

(v).$\boldsymbol{tan2\theta} = \frac{2\tan\theta}{1-\tan^2\theta}$

Solution.

When $= 30°$

$L.H.S = \tan 2\theta = \tan 2(30) = \tan 60 = \sqrt{3} \rightarrow (i)$

$R.H.S = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\tan 30}{1-\tan^2 30} = \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$

$R.H.S = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{2}{\sqrt{3}} \cdot \frac{3}{2}$

$R.H.S = \frac{2}{\sqrt{3}} \cdot \frac{(\sqrt{3})^2}{2} = \sqrt{3} \rightarrow (ii)$

From (i) and (ii) , **L.H.S = R.H.S**

Hence Proved.

When $= 45°$

$L.H.S = \tan 2\theta = \tan 2(45) = \tan 90 = \infty \rightarrow (iii)$

$R.H.S = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\tan 45}{1-\tan^2 45}$

$R.H.S = \frac{2(1)}{1-(1)^2}$

$R.H.S == \frac{2}{1-1}$

$R.H.S == \frac{2}{0}$

$R.H.S = \infty \rightarrow (iv)$

From (i) and (ii) ,**L.H.S = R.H.S**

Hence proved.

Question.4.

Find x,if

$\tan^2 45° - \cos^2 60° = x \sin 45° \cos 45° \tan 60°.$

Solution.

$\tan^2 45° - \cos^2 60° = x \sin 45° \cos 45° \tan 60°$

$(1)^2 - \left(\frac{1}{2}\right)^2 = x\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)(\sqrt{3})$

$1 - \frac{1}{4} = x\left(\frac{\sqrt{3}}{2}\right)$

$\frac{4-1}{4} = x\left(\frac{\sqrt{3}}{2}\right)$

$\frac{3}{4} \times \frac{2}{\sqrt{3}} = x$

$\frac{\sqrt{3}.\sqrt{3}}{4} \times \frac{2}{\sqrt{3}} = x$

$\frac{\sqrt{3}}{2} = x$

Which is required.

Question No.5 find the valves of the trigonometric functions of the following Quadrantal angles

(i) $\boldsymbol{-\pi}$

solution: *we know that general angles*

$\theta = \theta + 2k\pi \ , k \in Z$

$-\pi = -2\pi + \pi = (-1)2\pi + \pi, k = -1$

Thus valves of trigonometric functions at $-\pi$ *and* π *are same.*

$\because \ \sin(-\pi) = \sin\pi = 0$

$\csc(-\pi) = \csc\pi = \infty$

$$cos(-\pi) = cos\pi = -1$$
$$sec(-\pi) = sec\pi = -1$$
$$tan(-\pi) = tan\pi = 0$$
$$cot(-\pi) = cot\pi = \infty$$

***(i)* -3π**

solution: *we know that general angles*

$-3\pi = -4\pi + \pi = -2(2\pi) + \pi$

Thus valves of trigonometric functions at -3π and π are same.

$$\because\ sin(-3\pi) = sin\pi = 0$$
$$csc(-3\pi) = csc\pi = \infty$$
$$cos(-3\pi) = cos\pi = -1$$
$$sec(-3\pi) = sec\pi = -1$$
$$tan(-3\pi) = tan\pi = 0$$
$$cot(-3\pi) = cot\pi = \infty$$

***(iii)* $\frac{5}{2}\pi$**

Solution:

$$\frac{5}{2}\pi = \frac{4\pi + \pi}{2} = \frac{4\pi}{2} + \frac{\pi}{2} = 2\pi + \frac{\pi}{2}$$
$$= (1)2\pi + \frac{\pi}{2}, \quad k = 1$$

Thus valves of trigonometric functions at $\frac{5\pi}{2}$ and $\frac{\pi}{2}$ are same. So

$$sin\left(\frac{5\pi}{2}\right) = sin\frac{\pi}{2} = 1\ ;\ csc\left(\frac{5\pi}{2}\right) = csc\left(\frac{\pi}{2}\right) = 1$$
$$cos\left(\frac{5\pi}{2}\right) = cos\frac{\pi}{2} = 0;\quad sec\left(\frac{5\pi}{2}\right) = sec\left(\frac{\pi}{2}\right) = 0$$
$$tan\left(\frac{5\pi}{2}\right) = tan\frac{\pi}{2} = \infty;\quad cot\frac{5\pi}{2} = cot\frac{\pi}{2} = 0$$

(iv) $-\frac{9}{2}\pi$

Solution:

$$-\frac{9}{2}\pi = \frac{-12\pi + 3\pi}{2} = \frac{-12\pi}{2} + \frac{3\pi}{2} = -6\pi + \frac{3\pi}{2}$$
$$= (-3)2\pi + \frac{3\pi}{2}, \quad k = -3$$
$$(\because\ \theta = k(2\pi) + \theta, \quad k \in Z)$$

Thus valves of trigonometric functions at $\frac{-9\pi}{2}$ and $\frac{3\pi}{2}$ are same. So

$$sin\left(\frac{-9\pi}{2}\right) = sin\frac{3\pi}{2} = -1$$
$$;\ sc\left(-\frac{9\pi}{2}\right) = csc\left(\frac{3\pi}{2}\right) = -1$$

$$cos\left(\frac{-9\pi}{2}\right) = cos\frac{3\pi}{2} = 0;$$
$$sec\left(\frac{-9\pi}{2}\right) = sec\left(\frac{3\pi}{2}\right) = 0$$
$$tan\left(-\frac{9\pi}{2}\right) = tan\frac{3\pi}{2} = \infty;$$
$$cot\left(-\frac{9\pi}{2}\right) = cot\frac{3\pi}{2} = 0$$

(v) -15π

Solution:

$-15\pi = -16\pi + \pi = (-8)(2\pi) + \pi, \quad k = -8$

Thus valves of trigonometric valves at -15π and π are same, so

$$\because\ sin(-15\pi) = sin\pi = 0$$
$$csc(-15\pi) = csc\pi = \infty$$
$$cos(-15\pi) = cos\pi = -1$$
$$sec(-15\pi) = sec\pi = -1$$
$$tan(-15\pi) = tan\pi = 0$$
$$cot(-15\pi) = cot\pi = \infty$$

(vi) 1530^0

Solution:

$$\because\ \theta = k(2\pi) + \theta, \quad k \in Z$$
$$1530^0 = 1440^0 + 90^0$$
$$= 4(360^0) + 90^0 \quad , \ k = 4$$

Thus valves of trigonometric functions at 1530^0 and 90^0 Are same. So

$$\because\ sin(1530^0) = sin90^0 = 1$$
$$csc(1530^0) = csc90^0 = 1$$
$$cos(1530^0) = cos90^0 = 0$$
$$sec(1530^0) = sec90^0 = \infty$$
$$tan(1530^0) = tan90^0 = \infty$$
$$cot(1530^0) = cot90^0 = 0$$

(vii) -2430^0

Solution:

$$-2430^0 = -2560^0 + 90^0$$
$$= -7(360^0) + 90^0, \quad k = -7$$

Thus valves of trigonometric functions at -2430^0 and 90^0 Are same. So

$$\because\ sin(-2430^0) = sin90^0 = 1$$
$$csc(-2430^0) = csc90^0 = 1$$
$$cos(-2430^0) = cos90^0 = 0$$
$$sec(-2430^0) = sec90^0 = \infty$$
$$tan(-2430^0) = tan90^0 = \infty$$
$$cot(-2430^0) = cot90^0 = 0$$

(viii) $\frac{235\pi}{2}$

Solution:

$$\frac{235\pi}{2} = \frac{232\pi + 3\pi}{2} = \frac{232\pi}{2} + \frac{3\pi}{2}$$
$$= 116\pi + \frac{3\pi}{2} = 58(2\pi) + \frac{3\pi}{2}, \quad k = 58$$

thus trigonometric functions of $\frac{235\pi}{2}$ and $\frac{3\pi}{2}$ are same. So

$$\because\ sin\left(\frac{235\pi}{2}\right) = sin\frac{3\pi}{2} = -1$$
$$csc\left(\frac{235\pi}{2}\right) = csc\frac{3\pi}{2} = -1$$
$$cos\left(\frac{235\pi}{2}\right) = cos\frac{3\pi}{2} = 0$$
$$sec\left(\frac{235\pi}{2}\right) = sec\frac{3\pi}{2} = \infty$$

$$\tan\left(\frac{235\pi}{2}\right) = \tan\frac{3\pi}{2} = \infty$$
$$\cot\left(\frac{235\pi}{2}\right) = \cot\frac{3\pi}{2} = 0$$

(ix) $\frac{407\pi}{2}$

Solution:

$$\frac{407\pi}{2} == \frac{404\pi + 3\pi}{2}$$
$$= \frac{404\pi}{2} + \frac{3\pi}{2} + 202\pi + \frac{3\pi}{2} = 101(2\pi) + \frac{3\pi}{2}$$
$$k = 101$$

Thus trigonometric functions of $\frac{407\pi}{2}$ ***and*** $\frac{3\pi}{2}$ ***are same so,***

$$\because \sin\left(\frac{407\pi}{2}\right) = \sin\frac{3\pi}{2} = -1$$
$$\csc\left(\frac{407\pi}{2}\right) = \csc\frac{3\pi}{2} = -1$$
$$\cos\left(\frac{407\pi}{2}\right) = \cos\frac{3\pi}{2} = 0$$
$$\sec\left(\frac{407\pi}{2}\right) = \sec\frac{3\pi}{2} = \infty$$
$$\tan\left(\frac{407\pi}{2}\right) = \tan\frac{3\pi}{2} = \infty$$
$$\cot\left(\frac{407\pi}{2}\right) = \cot\frac{3\pi}{2} = 0$$

Question No.6 find the valves of the trigonometric functions of the following angles.

(i) 390^0

Solution:

We know that

General angle $\theta = k(2\pi) + \theta, \quad k \in Z$

$$390^0 = 360^0 + 30^0 = (1)360^0 + 30^0, k = 1$$

Thus trigonometric functions of 390^0 and 30^0 *are same*

So

$$\sin 390^0 = \sin 30^0 = \frac{1}{2}$$
$$\csc 390^0 = \csc 30^0 = 2$$
$$\cos 390^0 = \cos 30^0 = \frac{\sqrt{3}}{2}$$
$$\sec 390^0 = \sec 30^0 = \frac{2}{\sqrt{3}}$$
$$\tan 390^0 = \tan 30^0 = \frac{1}{\sqrt{3}}$$
$$\cot 390^0 = \cot 30^0 = \sqrt{3}$$

(ii) -330^0

Solution:

$$-330^0 = -360^0 + 30^0$$
$$= (-1)360^0 + 30^0, k = -1$$

And 30^0 *are same so.*

$$\sin(-330^0) = \sin 30^0 = \frac{1}{2}$$
$$\csc(-330^0) = \csc 30^0 = 2$$
$$\cos(-330^0) = \cos 30^0 = \frac{\sqrt{3}}{2}$$
$$\sec(-330^0) = \sec 30^0 = \frac{2}{\sqrt{3}}$$
$$\tan(-330^0) = \tan 30^0 = \frac{1}{\sqrt{3}}$$
$$\cot(-330^0) = \cot 30^0 = \sqrt{3}$$

(iii) 765^0

Solution:

$$765^0 = 720^0 + 45^0 = 2(360^0) + 45^0, \ k = 2$$

Trigonometric functions of 765^0 ***and*** 45^0 ***are*** **same so,**

$$\sin(765^0) = \sin 45^0 = \frac{1}{\sqrt{2}}$$
$$\csc(765^0) = \csc 45^0 = \sqrt{2}$$
$$\cos(765^0) = \cos 45^0 = \frac{1}{\sqrt{2}}$$
$$\sec(765^0) = \sec 45^0 = \sqrt{2}$$
$$\tan(765^0) = \tan 45^0 = 1$$
$$\cot(765^0) = \cot 45^0 = 1$$

(iv) -675^0

Solution:

$$-675^0 = 720^0 + 45^0$$
$$= (-2)(360^0) + 45^0, \qquad k = -2$$

Trigonometric functions of -675^0 *and* 45^0 *are* same so,

$$\sin(-675^0) = \sin 45^0 = \frac{1}{\sqrt{2}}$$
$$\csc(-675^0) = \csc 45^0 = \sqrt{2}$$
$$\cos(-675^0) = \cos 45^0 = \frac{1}{\sqrt{2}}$$
$$\sec(-675^0) = \sec 45^0 = \sqrt{2}$$
$$\tan(-675^0) = \tan 45^0 = 1$$
$$\cot(-675^0) = \cot 45^0 = 1$$

(v) $-\frac{17}{3}\pi$

Solution:

$$-\frac{17}{3}\pi = \frac{-18\pi + \pi}{3} = \frac{-18\pi}{3} + \frac{\pi}{3}$$
$$= -6\pi + \frac{\pi}{3} = -3(2\pi) + \frac{\pi}{3}$$

Trigonometric functions of $-\frac{17\pi}{3}$ *and* $\frac{\pi}{3}$ *are same so*

$$\sin\left(-\frac{17}{3}\pi\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$
$$\csc\left(-\frac{17}{3}\pi\right) = \csc\frac{\pi}{3} = \frac{2}{\sqrt{3}}$$

$$\cos\left(-\frac{17}{3}\pi\right) = \cos\frac{\pi}{3} = \frac{1}{2}$$
$$\sec\left(-\frac{17}{3}\pi\right) = \sec\frac{\pi}{3} = 2$$
$$\tan\left(-\frac{17}{3}\pi\right) = \tan\frac{\pi}{3} = \sqrt{3}$$
$$\cot\left(-\frac{17}{3}\pi\right) = \cot\frac{\pi}{3} = \frac{1}{\sqrt{3}}$$

(vi) $\frac{13}{3}\pi$

solution:

$$\frac{13}{3}\pi = \frac{12\pi+\pi}{3} = \frac{12\pi}{3} + \frac{\pi}{3}$$
$$= 4\pi + \frac{\pi}{3}, \quad k = 4$$

Trigonometric functions of $\frac{13\pi}{3}$ *and* $\frac{\pi}{3}$ *are same so*

$$\sin\left(\frac{13}{3}\pi\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$
$$\csc\left(\frac{13}{3}\pi\right) = \csc\frac{\pi}{3} = \frac{2}{\sqrt{3}}$$
$$\cos\left(\frac{13}{3}\pi\right) = \cos\frac{\pi}{3} = \frac{1}{2}$$
$$\sec\left(\frac{13}{3}\pi\right) = \sec\frac{\pi}{3} = 2$$
$$\tan\left(\frac{13}{3}\pi\right) = \tan\frac{\pi}{3} = \sqrt{3}$$
$$\cot\left(\frac{13}{3}\pi\right) = \cot\frac{\pi}{3} = \frac{1}{\sqrt{3}}$$

(vii) $\frac{25}{6}\pi$

Solution:

$$\frac{25}{6}\pi = \frac{24\pi+\pi}{6} = \frac{24\pi}{6} + \frac{\pi}{6}$$
$$= 4\pi + \frac{\pi}{6}, = 2(2\pi) + \frac{\pi}{6}\ ; \ k = 2$$

Trigonometric functions of $\frac{25\pi}{6}$ *and* $\frac{\pi}{6}$ *are same so*

$$\sin\left(\frac{25\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$$
$$\csc\left(\frac{25\pi}{6}\right) = \csc\frac{\pi}{6} = 2$$
$$\cos\left(\frac{25\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$
$$\sec\left(\frac{25\pi}{6}\right) = \sec\frac{\pi}{6} = \frac{2}{\sqrt{3}}$$
$$\tan\left(\frac{25\pi}{6}\right) = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$
$$\cot\left(\frac{25\pi}{6}\right) = \cot\frac{\pi}{6} = \sqrt{3}$$

(viii) $-\frac{71\pi}{6}$

Solution:

$$-\frac{71\pi}{6} = \frac{-72\pi+\pi}{6} = \frac{-72\pi}{6} + \frac{\pi}{6}$$
$$= -12\pi + \frac{\pi}{6}, = -6(2\pi) + \frac{\pi}{6}\ ; \ k = -6$$

Trigonometric functions of $\frac{-71\pi}{6}$ *and* $\frac{\pi}{6}$ *are same so*

$$\sin\left(-\frac{71\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$$
$$\csc\left(-\frac{71\pi}{6}\right) = \csc\frac{\pi}{6} = 2$$
$$\cos\left(-\frac{71\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$
$$\sec\left(-\frac{71\pi}{6}\right) = \sec\frac{\pi}{6} = \frac{2}{\sqrt{3}}$$
$$\tan\left(-\frac{71\pi}{6}\right) = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$
$$\cot\left(-\frac{71\pi}{6}\right) = \cot\frac{\pi}{6} = \sqrt{3}$$

(ix) -1035^0

Solution:

$$-1035^0 = 1080^0 + 45^0$$
$$= (-3)(360^0) + 45^0, \quad k = -3$$

Trigonometric functions of -1035^0 *and* 45^0 *are* same so,

$$\sin(-1035^0) = \sin 45^0 = \frac{1}{\sqrt{2}}$$
$$\csc(-1035^0) = \csc 45^0 = \sqrt{2}$$
$$\cos(-1035^0) = \cos 45^0 = \frac{1}{\sqrt{2}}$$
$$\sec(-1035^0) = \sec 45^0 = \sqrt{2}$$
$$\tan(-1035^0) = \tan 45^0 = 1$$
$$\cot(-1035^0) = \cot 45^0 = 1$$

Domains of trigonometric functions and fundamental identities:

(i) $\sin\theta$ for all $\theta \in R$

(ii) $\cos\theta$; for all $\theta \in R$

(iii) $\csc\theta = \frac{1}{\sin\theta}$, for all $\theta \in R$ but $\theta \neq n\pi$

(iv) $\sec\theta = \frac{1}{\cos\theta}$, $\forall\theta \in R$, $\theta \neq \left(\frac{2n+1}{2}\right)x$, $n \in Z$

(V) $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\forall\theta \in R$, $\theta \neq \left(2n + \frac{n\pi}{2}\right)$, $n \in Z$

(vi) $\cot\theta = \frac{\cos\theta}{\sin\theta}$, $\forall\theta \in R, \theta \neq n\pi, n \in Z$

(vii) $\sin^2\theta + \cos^2\theta = 1 \quad \forall\,\theta \in R$

(viii) $1 + \tan^2\theta = \sec^2\theta$

$\forall\,\theta \in R;\ \theta \neq (2n+1)\frac{\pi}{2}\ \ n \in Z$

(ix) $1 + \cot^2\theta = \csc^2\theta$, $\forall\ \theta \in R$ but $\theta \neq n\pi$

Exercise 9.4

Prove the following questions

Question#1

$\tan\theta + \cot\theta = \sec\theta \operatorname{cosec}\theta$

Solution:

$L.H.S = \tan\theta + \cot\theta$

$L.H.S = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$L.H.S = \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$ $\quad \therefore \sin^2\theta + \cos^2\theta = 1$

$L.H.S = \frac{1}{\cos\theta\sin\theta}$

$L.H.S = \sec\theta\operatorname{cosec}\theta = \mathbf{R.H.S}$

Hence Proved

Question#2

$$\boldsymbol{\sec\theta \operatorname{cosec}\theta \sin\theta \cos\theta = 1}$$

Solution:

$L.H.S = \sec\theta \operatorname{cosec}\theta \sin\theta \cos\theta$

$L.H.S = \frac{1}{\cos\theta}\frac{1}{\sin\theta}\sin\theta\cos\theta$

$L.H.S = 1 = \mathbf{R.H.S}$

Hence Proved.

Question#3

$\boldsymbol{\cos\theta + \tan\theta \sin\theta = \sec\theta}$

Solution:

$L.H.S = \cos\theta + \frac{\sin\theta}{\cos\theta}.\sin\theta$

$L.H.S = \cos\theta + \frac{\sin^2\theta}{\cos\theta} = \frac{\cos^2\theta + \sin^2\theta}{\cos\theta}$

$L.H.S = \frac{1}{\cos\theta} = \sec\theta = \mathbf{R.H.S}$

Hence Proved

Question # 4

$\boldsymbol{\operatorname{cosec}\theta + \tan\theta\sec\theta = \operatorname{cosec}\theta\sec^2\theta}$

Solution:

$L.H.S = \mathbf{\operatorname{cosec}\theta + \tan\theta\sec\theta}$

$L.H.S = \frac{1}{\sin\theta} + \frac{\sin\theta}{\cos\theta}.\frac{1}{\cos\theta}$

$L.H.S = \frac{1}{\sin\theta} + \frac{\sin\theta}{\cos^2\theta} = \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos^2\theta}$

$L.H.S = \frac{1}{\sin\theta\cos^2\theta} = \frac{1}{\sin\theta}.\frac{1}{\cos^2\theta}$

$L.H.S = \operatorname{cosec}\theta\sec^2\theta = \mathbf{R.H.S}$

Hence Proved

Question#5

$$\boldsymbol{\sec^2\theta - \operatorname{cosec}^2\theta = \tan^2\theta - \cot^2\theta}$$

Solution:

$L.H.S = \boldsymbol{\sec^2}\theta - \boldsymbol{\operatorname{cosec}^2}\theta$

$L.H.S = (1 + \tan^2\theta) - (1 + \cot^2\theta)$

$\therefore \mathbf{1 + \tan^2\theta = \sec^2\theta}$

$\therefore \mathbf{1 + \cot^2\theta = \operatorname{cosec}^2\theta}$

$L.H.S = 1 + \tan^2\theta - 1 - \cot^2\theta$

$L.H.S = \tan^2\theta - \cot^2\theta = \mathbf{R.H.S}$

Hence Proved.

Question# 6

$$\boldsymbol{\cot^2\theta - \cos^2\theta = \cot^2\theta\cos^2\theta}$$

Solution:

$L.H.S = \cot^2\theta - \cos^2\theta$

$L.H.S = \frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta = \cos^2\theta\left(\frac{1}{\sin^2\theta} - 1\right)$

$L.H.S = \cos^2\theta\left(\frac{1-\sin^2\theta}{\sin^2\theta}\right)$

$L.H.S = \cos^2\theta\left(\frac{\cos^2\theta}{\sin^2\theta}\right)$

Hence Proved.

Question#7

$\boldsymbol{(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1}$

Solution:

$L.H.S = (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)$

$L.H.S = (1 + \tan^2\theta) - \tan^2\theta$

$\therefore 1 + \tan^2\theta = \sec^2\theta$

$L.H.S = 1 + \tan^2\theta - \tan^2\theta$

Hence Proved.

Question#8

$$\boldsymbol{\cos^2\theta - 1 = 1 - 2\sin^2\theta}$$

Solution:

$L.H.S = 2\cos^2\theta - 1$

$L.H.S = 2(1 - \sin^2\theta) - 1$

$\therefore \sin^2\theta + \cos^2\theta = 1$

$L.H.S = 2 - 2\sin^2\theta - 1$

$\therefore \cos^2\theta = 1 - \sin^2\theta$

$L.H.S = 1 - 2\sin^2\theta = \mathbf{R.H.S}$

Hence Proved.

Question# 9

$$\mathbf{\cos^2\theta - \sin^2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}}$$

Solution:

$R.H.S = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$R.H.S = \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}} = \frac{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$

$R.H.S = \frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta}$

$R.H.S = \frac{\cos^2\theta-\sin^2\theta}{1}$

$\therefore \sin^2\theta + \cos^2\theta = 1$

$R.H.S = \mathbf{\cos^2}\theta - \mathbf{\sin^2}\theta = \mathbf{L.H.S}$

Hence Proved.

Question#10

$$\boldsymbol{\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{\cot\theta - 1}{\cot\theta + 1}}$$

Solution:

$R.H.S = \frac{\cot\theta - 1}{\cot\theta + 1}$

$R.H.S = \frac{\frac{\cos\theta}{\sin\theta} - 1}{\frac{\cos\theta}{\sin\theta} + 1} = \frac{\frac{\cos\theta - \sin\theta}{\sin\theta}}{\frac{\cos\theta + \sin\theta}{\sin\theta}}$

$R.H.S = \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \mathbf{L.H.S}$

Hence Proved.

Question#11

$$\boldsymbol{\frac{\sin\theta}{1+\cos\theta} + \cot\theta = \operatorname{cosec}\theta}$$

Solution:

$L.H.S = \frac{\sin\theta}{1+\cos\theta} + \cot\theta$

$L.H.S = \frac{\sin\theta}{1+\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$L.H.S = \frac{\sin^2\theta + \cos\theta + \cos^2\theta}{(1+\cos\theta)\sin\theta}$

$L.H.S = \frac{(\sin^2\theta + \cos^2\theta) + \cos\theta}{(1+\cos\theta)\sin\theta}$

$L.H.S = \frac{1+\cos\theta}{(1+\cos\theta)\sin\theta}$ $\quad \therefore \sin^2\theta + \cos^2\theta = 1$

$L.H.S = \frac{1}{\sin\theta} = \operatorname{cosec}\theta = \mathbf{R.H.S}$

Hence Proved.

Question#12

$$\frac{\cot^2\theta - 1}{1+\cot^2\theta} = 2\cos^2\theta - 1$$

Solution:

$$L.H.S = \frac{\cot^2\theta - 1}{1+\cot^2\theta}$$

$$L.H.S = \frac{\frac{\cos^2\theta}{\sin^2\theta} - 1}{1+\frac{\cos^2\theta}{\sin^2\theta}} = \frac{\frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta}}{\frac{\cos^2\theta + \sin^2\theta}{\sin^2\theta}}$$

$$L.H.S = \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$$

$$L.H.S = \frac{\cos^2\theta - \sin^2\theta}{1}$$

$L.H.S = \cos^2\theta - \sin^2\theta$ $\quad \therefore \sin^2\theta + \cos^2\theta = 1$

$L.H.S = \cos^2\theta - (1-\cos^2\theta)$ $\quad \therefore 1-\sin^2\theta = \cos^2\theta$

$L.H.S = \cos^2\theta - 1 + \cos^2\theta = 2\cos^2\theta - 1 = \mathbf{R.H.S}$

Hence Proved.

Question#13

$$\frac{1+\cos\theta}{1-\cos\theta} = (cosec\theta + \cot\theta)^2$$

Solution:

$$L.H.S = (\mathbf{cosec\theta + \cot\theta})^2$$

$$L.H.S = \left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{1+\cos\theta}{\sin\theta}\right)^2$$

$$L.H.S = \frac{(1+\cos\theta)^2}{\sin^2\theta} = \frac{(1+\cos\theta)^2}{1-\cos^2\theta}$$

$$L.H.S = \frac{(1+\cos\theta)(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)} = \frac{1+\cos\theta}{1-\cos\theta} = \mathbf{L.H.S}$$

Hence Proved.

Question#14

$$(\sec\theta - \tan\theta)^2 = \frac{1-\sin\theta}{1+\sin\theta}$$

Solution:

$$L.H.S = (\sec\theta - \tan\theta)^2$$

$$L.H.S = \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2 = \left(\frac{1-\sin\theta}{\cos\theta}\right)^2$$

$$L.H.S = \frac{(1-\sin\theta)^2}{\cos^2\theta} = \frac{(1-\sin\theta)^2}{1-\sin^2\theta}$$

$$L.H.S = \frac{(1-\sin\theta)(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} = \frac{1-\sin\theta}{1+\sin\theta} = \mathbf{R.H.S}$$

Hence Proved.

Question#15

$$\frac{2\tan\theta}{1+\tan^2\theta} = 2\sin\theta\cos\theta$$

Solution:

$$L.H.S = \frac{2\tan\theta}{1+\tan^2\theta}$$

$L.H.S = \frac{2\tan\theta}{\sec^2\theta} = 2\tan\theta\cos^2\theta$ $\quad \therefore 1+\tan^2\theta = \sec^2\theta$

$L.H.S = 2\frac{\sin\theta}{\cos\theta}.\cos^2\theta = 2\sin\theta\cos\theta = \mathbf{R.H.S}$

Hence Proved.

Question#16

$$\frac{1-\sin\theta}{\cos\theta} = \frac{\cos\theta}{1+\sin\theta}$$

Solution:

$$L.H.S = \frac{\mathbf{1-\sin\theta}}{\mathbf{\cos\theta}}$$

Rationalizing

$$L.H.S = \frac{1-\sin\theta}{\cos\theta}\cdot\frac{1+\sin\theta}{1+\sin\theta}$$

$$L.H.S = \frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)}$$

$$L.H.S = \frac{\cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$\therefore \mathbf{1-\sin^2\theta = \cos^2\theta}$$

$$L.H.S = \frac{\cos\theta}{1+\sin\theta} = \mathbf{R.H.S}$$

Hence Proved.

Question#17

$$(\tan\theta + \cot\theta)^2 = \sec^2\theta cosec^2\theta$$

Solution:

$$L.H.S = (\mathbf{\tan\theta + \cot\theta})^2$$

$$L.H.S = \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)^2$$

$L.H.S = \left(\frac{1}{\cos\theta\sin\theta}\right)^2$ $\quad \therefore \sin^2\theta + \cos^2\theta = 1$

$L.H.S = (\sec\theta + cosec\theta)^2 = \sec^2\theta cosec^2\theta = \mathbf{R.H.S}$

Hence Proved.

Question#18

$$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \tan\theta + \sec\theta$$

Solution:

$$\frac{\tan\theta + \sec\theta - 1}{\tan\theta + \sec\theta + 1} = \tan\theta + \sec\theta$$

$$\tan\theta + \sec\theta - 1 = (\tan\theta + \sec\theta)(\tan\theta + \sec\theta + 1)$$

$$(\tan\theta + \sec\theta)(\tan\theta + \sec\theta + 1) + \tan\theta + \sec\theta = -1$$

$$L.H.S$$

$$= \tan^2\theta - \tan\theta\sec\theta + \tan\theta\sec\theta + \tan\theta - \sec^2\theta + \sec\theta$$

$$L.H.S = \tan^2\theta - \sec^2\theta + \tan\theta + \sec\theta$$

$$\therefore 1 + \tan^2\theta = \sec^2\theta$$

$$\Rightarrow \tan^2\theta - \sec^2\theta = -1$$

$$L.H.S = -1 + \tan\theta + \sec\theta$$

$$L.H.S = \tan\theta + \sec\theta - 1 = \mathbf{L.H.S}$$

Hence Proved.

Question#19

$$\frac{1}{cosec\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{cosec\theta + \cot\theta}$$

Solution:

$$\frac{1}{cosec\theta - \cot\theta} + \frac{1}{cosec\theta + \cot\theta} = \frac{1}{\sin\theta} + \frac{1}{\sin\theta}$$

$$\frac{cosec\theta + \cot\theta + cosec\theta - \cot\theta}{(cosec\theta - \cot\theta)(cosec\theta + \cot\theta)} = \frac{1+1}{\sin\theta}$$

$$\frac{2cosec\theta}{cosec^2\theta - \cot^2\theta} = \frac{2}{\sin\theta}$$

$$\therefore \mathbf{cosec^2\theta = 1 + \cot^2\theta}$$

$$\Rightarrow \mathbf{cosec^2\theta - \cot^2\theta = 1}$$

$$\frac{2cosec\theta}{cosec^2\theta - \cot^2\theta} = \frac{2}{\sin\theta}$$

$$\frac{2}{\sin\theta} = \frac{2}{\sin\theta}$$

Hence Proved.

Question#20

$$\boldsymbol{sin^3\theta + cos^3\theta = (sin\theta - cos\theta)(1 + sin\theta cos\theta)}$$

Solution:

$$L.H.S = \sin^3\theta + \cos^3\theta$$

$$L.H.S = \sin^3\theta + \cos^3\theta$$

$L.H.S = (\sin\theta)^3 - (cos\theta)^3$

$L.H.S = (\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)$

$L.H.S = (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)$ $\therefore \mathbf{\sin^2\theta + \cos^2\theta = 1}$

$L.H.S = \mathbf{R.H.S}$

Hence Proved.

Question#21

$$\boldsymbol{sin^6\theta - cos^6\theta = (sin^2\theta - cos^2\theta)(1 - sin^2\theta\, cos^2\theta)}$$

Solution:

$$L.H.S = \sin^6\theta - \cos^6\theta$$

$L.H.S = (\sin^2\theta)^{\mathbf{3}} - (\cos^2\theta)^{\mathbf{3}}$

$$L.H.S = (\sin^2\theta - \cos^2\theta)((\sin^2\theta)^{\mathbf{2}} + (\cos^2\theta)^{\mathbf{2}} + \sin^2\theta\cos^2\theta)$$

$$L.H.S = (\sin^2\theta - \cos^2\theta)((\sin^2\theta)^{\mathbf{2}} + (\cos^2\theta)^{\mathbf{2}} + 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta)$$

$L.H.S = (\sin^2\theta - \cos^2\theta)((\sin^2\theta + \cos^2\theta)^{\mathbf{2}} - \sin^2\theta\cos^2\theta)$

$L.H.S = (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta) = \mathbf{R.H.S}$

Hence Proved.

Question#22

$$\boldsymbol{sin^6\theta + cos^6\theta = 1 - 3sin^2\theta\, cos^2\theta}$$

Solution:

$$L.H.S = \sin^6\theta + \cos^6\theta$$

$L.H.S = (\sin^2\theta)^3 + (\cos^2\theta)^3$

$$L.H.S = (\sin^2\theta + \cos^2\theta)((\sin^2\theta)^2 + (\cos^2\theta)^2 - \sin^2\theta\cos^2\theta)$$

$$L.H.S = (1)((\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta - 3\sin^2\theta\cos^2\theta\ )$$

$L.H.S = ((\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta)$

$L.H.S = ((1)^2 - 3\sin^2\theta\cos^2\theta)$ $\therefore \mathbf{\sin^2\theta + \cos^2\theta = 1}$

$L.H.S = 1 - 3\sin^2\theta\cos^2\theta = \mathbf{R.H.S}$

Hence Proved.

Question#23

$$\boldsymbol{\frac{1}{1+sin\theta} + \frac{1}{1-sin\theta} = 2sec^2\theta}$$

Solution:

$$L.H.S = \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$$

$L.H.S = \frac{1+\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)} = \frac{2}{1-\sin^2\theta}$

$L.H.S = \frac{2}{\cos^2\theta}$ $\therefore \mathbf{\sin^2\theta + \cos^2\theta = 1}$

$L.H.S = 2\sec^2\theta = \mathbf{R.H.S}$

Hence Proved.

Question#24

$$\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} + \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{2}{1 - 2\sin^2\theta}$$

Solution:

$$L.H.S = \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} + \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$

$$L.H.S = \frac{(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2}{\cos^2\theta - \sin^2\theta}$$

$$L.H.S = \frac{\cos^2\theta + \sin^2\theta + 2\cos^2\theta\sin^2\theta + \cos^2\theta + \sin^2\theta - 2\cos^2\theta\sin^2\theta}{\cos^2\theta - \sin^2\theta}$$

$$L.H.S = \frac{2\cos^2\theta + 2\sin^2\theta}{\cos^2\theta - \sin^2\theta} = \frac{2(\cos^2\theta + \sin^2\theta)}{(1 - \sin^2\theta) - \sin^2\theta}$$

$$\therefore 1 - \sin^2\theta = \cos^2\theta$$

$$L.H.S = \frac{2(1)}{1 - \sin^2\theta - \sin^2\theta}$$

$$\therefore \sin^2\theta + \cos^2\theta = 1$$

$$L.H.S = \frac{2}{1 - 2\sin^2\theta} = \mathbf{R.H.S}$$

Hence Proved.