MATHEMATICS 11 INTERMEDIATE PART 1

Bilal Article

Chapter 11.

TRIGONOMETRIC FUNCTIONS AND THEIR GRAPHS

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Chapter#11	Class 1^{st}	Trigonometric Functions and their Graphs
Contacts:	Definitions →Theory → Exercise	

Function	Domain	Range
$y = sinx$	$-\infty < x < +\infty$	$-1 \le y \le 1$
$y = cosx$	$-\infty < x < +\infty$	$-1 \le y \le 1$
$y = tanx$	$-\infty < x < \infty, x \ne \frac{(2n+1)\pi}{2}, n \in Z$	$-\infty < y < +\infty$
$y = cotx$	$-\infty < x < \infty, x \ne n\pi, n \in Z$	$-\infty < y < +\infty$
$y = secx$	$-\infty < x < \infty, x \ne \frac{(2n+1)\pi}{2}, n \in Z$	$y \ge 1$ or $y \le -1$
$y = cosecx$	$-\infty < x < +\infty, x \ne n\pi, n \in Z$	$y \ge 1$ or $y \le -1$

Period of trigonometric functions:

Period of a trigonometric functions is the smallest +ve number which, when added to the original circular measure of the angle, gives the same valve of the functions.

Alternative definition

"periodic function"

A function $f(x) is$ said to be periodic if for a least positive number $p, f(x+p) = f(x)$ then p is called periodic of $f(x)$.

Theorem-1

Sine is a periodic function and its period is 2π.

Proof:

Suppose p is period of sine function such that

$$\sin(\theta + p) = sin\theta, \ \ \forall\theta \in R \rightarrow (i)$$

$$put\ \theta = 0$$

$$\Longrightarrow sin(o + p) = sin0$$

$$\Rightarrow sinp = 0 \Rightarrow p = \sin^{-1}(0)$$

$$\Rightarrow p = 0, \pm\pi, \pm2\pi, \pm3\pi$$

$$\boldsymbol{if\ p = \pi\ \ then}$$

$$(i) \Rightarrow \sin(\theta + \pi) = sin\theta$$

False because $\sin(\pi + \theta) = sin\theta$

$if\ \ p = 2\pi\ \ then$

$$(i) \Rightarrow \sin(\theta + 2\pi) = sin\theta$$

Which is true because $\sin(2\pi + \theta) = sin\theta$

Hence sine is a periodic functions of periodic function of period 2π

Theorem -2

Cosine is a periodic function is a period $is\ 2\pi$.

Proof:

Suppose p is period of cosine function such that

$$\cos(\theta + p) = cos\theta, \ \ \forall\theta \in R \rightarrow (i)$$

$$put\ \theta = \frac{\pi}{2}$$

$$\Longrightarrow cos\left(\frac{\pi}{2} + p\right) = \cos\frac{\pi}{2}$$

$$\Rightarrow -sinp = 0 \Rightarrow p = \sin^{-1}(0)$$

$$\Rightarrow p = 0, \pm\pi, \pm2\pi, \pm3\pi, \ldots$$

$$\boldsymbol{if\ p = \pi\ \ then}$$

$$(i) \Rightarrow \cos(\theta + \pi) = cos\theta$$

False because $\cos(\pi + \theta) - cos\theta$

$if\ \ p = 2\pi\ \ then$

$$(i) \Rightarrow \cos(\theta + 2\pi) = cos\theta$$

Which is true because $\cos(2\pi + \theta) = cos\theta$

Hence cosine is a periodic functions of periodic function of period 2π

Theorem -3

Tangent is a periodic function and its is π.

Proof:

Suppose p is period of tangent function such that

$$\tan(\theta + p) = tan\theta, \ \ \forall\theta \in R \rightarrow (i)$$

$$put\ \theta = 0$$

$$\Longrightarrow tan(0 + p) = tan0$$

$$\Rightarrow tanp = 0 \Rightarrow p = \tan^{-1}(0)$$

$$\Rightarrow p = 0, \pm\pi, \pm2\pi, \pm3\pi$$

$$\boldsymbol{if\ p = \pi\ \ then}$$

$$(i) \Rightarrow \tan(\theta + \pi) = tan\theta$$

Which is true because $\tan(\pi + \theta) = tan\theta$

Hence tangent is a periodic functions of periodic function of period π

Theorem -4

Cosecant is a periodic functions and it period is 2π

Proof:

Suppose p is period of cosecant function such that

$$\text{cosec}(\theta + p) = \text{cosec}\theta, \ \ \forall\theta \in R \rightarrow (i)$$

$$put\ \theta = \frac{\pi}{2}$$

$$\Longrightarrow \text{cosec}\left(\frac{\pi}{2} + p\right) = \text{cosec}\frac{\pi}{2}$$

$$\frac{1}{\sin\left(\frac{\pi}{2}+p\right)} = 1 \quad \because \left(\text{cosec}\frac{\pi}{2} = \frac{1}{\sin\frac{\pi}{2}} = \frac{1}{1} = 1\right)$$

$$\Rightarrow 1 = \sin\left(\frac{\pi}{2}+p\right)$$

$$\Rightarrow cosp = 1 \Rightarrow p = \cos^{-1}(1)$$

$$\Rightarrow p = 0, \pm\pi, \pm 2\pi, \pm 3\pi$$

if p = π then

$$(i) \Rightarrow \text{cosec}(\theta+\pi) = \text{cosec}\theta$$

False because $\text{cosec}(\pi+\theta) = \frac{1}{\sin(\pi+\theta)} = \frac{1}{-sin\theta}$

$$\Rightarrow \text{cosec}(\theta+\pi) = -\text{cosec}\theta$$

if $p = 2\pi$ *then*

$$(i) \Rightarrow \text{cosec}(\theta+2\pi) = \text{cosec}\theta$$

Which is true because $\text{cosec}(2\pi+\theta) = \frac{1}{\sin(2\pi+\theta)} = \frac{1}{sin\theta}$

$$\Rightarrow \text{cosec}(2\pi+\theta) = \text{cosec}\theta$$

Hence cosecent is a periodic functions of periodic function of period 2π

Theorem -5

Secant is a periodic function and its period is 2π.

Proof:

Suppose p is period of secant function such that

$$\sec(\theta+p) = \sec\theta, \quad \forall\theta \in R \rightarrow (i)$$

$$put\ \theta = 0$$

$$\Rightarrow \sec(0+p) = \sec 0$$

$$\Rightarrow secp = 1 \quad \because \left(sec0 = \frac{1}{cos0} = \frac{1}{1} = 1\right)$$

$$\Rightarrow p = \sec^{-1}(1)$$

$$p = 0, \pm\pi, \pm 2\pi, \pm 3\pi$$

if p = π then

$$(i) \Rightarrow \sec(\theta+\pi) = \sec\theta$$

False because $\sec(\pi+\theta) = \frac{1}{\cos(\pi+\theta)} = \frac{1}{-cos\theta}$

$$\Rightarrow \text{cosec}(\theta+\pi) = -\sec\theta$$

if $p = 2\pi$ *then*

$$(i) \Rightarrow \sec(\theta+2\pi) = \sec\theta$$

Which is true because $\sec(2\pi+\theta) = \frac{1}{\cos(2\pi+\theta)} = \frac{1}{cos\theta}$

$$\Rightarrow \sec(2\pi+\theta) = \sec\theta$$

Hence secent is a periodic functions of periodic function of period 2π.

Theorem -6

Tangent is a periodic function and its period is π.

Proof:

Suppose p is period of cotangent function such that

$$\cot(\theta+p) = cot\theta, \quad \forall\theta \in R \rightarrow (i)$$

$$put\ \theta = \frac{\pi}{2}$$

$$\Rightarrow cot\left(\frac{\pi}{2}+p\right) = \cot\frac{\pi}{2}$$

$$\Rightarrow -tanp = 0 \qquad \because \left(\cot\left(\frac{\pi}{2}+\theta\right) = -tan\theta\right)$$

$$\Rightarrow tanp = 0$$

$$\Rightarrow p = \tan^{-1}(0)$$

$$\Rightarrow p = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$$

if p = π then

$$(i) \Rightarrow \cot(\theta+\pi) = cot\theta$$

Which is true because $\cot(\pi+\theta) = cot\theta$

Hence cotangent is a periodic functions of periodic function of period π

Now remember that

Functions	Period	Take angle θ
sin	2π	0
cos	2π	$\frac{\pi}{2}$
tan	π	0
cosec	2π	$\frac{\pi}{2}$
sec	2π	0
cot	π	$\frac{\pi}{2}$

Exercise 11.1

Find the periods of the following functions.

Q.NO.1. $sin3x$

Solution:

$$\because periodof\ sinx\ is\ 2\pi. so$$

$$\sin(3x+2\pi) = sin3x \quad \because sin(\theta+2\pi) = sin\theta$$

$$\Rightarrow \sin 3\left(x+\frac{2\pi}{3}\right) = sin3x$$

$$\Rightarrow period\ of\ sin3x\ is\ \frac{2\pi}{3}$$

Q.NO.2. $cos2x$

Solution:

$$\because periodof\ cos2x\ is\ 2\pi. so$$

$$\cos(2x+2\pi) = cos2x \quad \because cos(\theta+2\pi) = cos\theta$$

$$\Rightarrow cos2(x+\pi) = cos2x$$

$$\Rightarrow period\ of\ cos2x\ is\ \pi$$

Q.NO.3. $tan4x$

Solution:

$$\because periodof\ tanx\ is\ \pi. so$$

$$\tan(4x+\pi) = tan4x \quad \because tan(\theta+\pi) = tan\theta$$

$$\Rightarrow tan4\left(x+\frac{\pi}{4}\right) = tan4x$$

$$\Rightarrow period\ of\ tan4x\ is\ \frac{\pi}{4}$$

Q.NO.4. $\cot\frac{x}{2}$

Solution :

$$\because periodof\ cotx\ is\ \pi. so$$

$$\cot\left(\frac{x}{2}+\pi\right) = \cot\frac{x}{2} \quad \because cot(\theta+\pi) = cot\theta$$

$$\Rightarrow cot\frac{1}{2}(x+2\pi) = \cot\frac{x}{2}$$

$$\Rightarrow period\ of\ \cot\frac{x}{2}\ is\ 2\pi$$

Q.NO.5. $\sin\left(\frac{x}{3}\right)$

Solution:

$$\because \text{periodof } sinx \text{ is } 2\pi. so$$
$$\sin\left(\frac{x}{3}+2\pi\right)=\sin\frac{x}{3} \quad \because \sin(\theta+2\pi)=\sin\theta$$
$$\Rightarrow \sin\frac{1}{3}(x+6\pi)=\sin\frac{x}{2}$$
$$\Rightarrow \text{period of } \sin\frac{x}{3} \text{ is } 6\pi$$

Q.NO.6. $cosec\frac{x}{4}$

Solution:

$$\because \text{periodof } cosecx \text{ is } 2\pi. so$$
$$\text{cosec}\left(\frac{x}{4}+2\pi\right)=\text{cosec}\frac{x}{4} \quad \because cosec(\theta+2\pi)=cosec\theta$$
$$\Rightarrow cosec\frac{1}{4}(x+8\pi)=\text{cosec}\frac{x}{4}$$
$$\Rightarrow \text{period of cosec}\frac{x}{4} \text{ is } 8\pi$$

Q.NO.7. $\sin\frac{x}{5}$

Solution:

$$\because \text{periodof } sinx \text{ is } 2\pi. so$$
$$\sin\left(\frac{x}{5}+2\pi\right)=\sin\frac{x}{5} \quad \because \sin(\theta+2\pi)=\sin\theta$$
$$\Rightarrow \sin\frac{1}{5}(x+10\pi)=\sin\frac{x}{5}$$
$$\Rightarrow \text{period of } \sin\frac{x}{5} \text{ is } 10\pi$$

Q.NO.8. $\cos\frac{x}{6}$

Solution:

$$\because \text{periodof } cosx \text{ is } 2\pi. so$$
$$\cos\left(\frac{x}{6}+2\pi\right)=\cos\frac{x}{6} \quad \because \cos(\theta+2\pi)=\cos\theta$$
$$\Rightarrow \cos\frac{1}{6}(x+12\pi)=\cos\frac{x}{6}$$
$$\Rightarrow \text{period of } \cos\frac{x}{6} \text{ is } 12\pi$$

Q.NO.9. $\tan\frac{x}{7}$

Solution:

$$\because \text{periodof } tanx \text{ is } \pi. so$$
$$\tan\left(\frac{x}{7}+\pi\right)=\tan\frac{x}{7} \quad \because \tan(\theta+\pi)=\tan\theta$$
$$\Rightarrow \tan\frac{1}{7}(x+7\pi)=\tan\frac{x}{7}$$
$$\Rightarrow \text{period of } \tan\frac{x}{7} \text{ is } 7\pi$$

Q.NO.10. $cot8xx$

Solution:

$$\because \text{periodof } cotx \text{ is } \pi. so$$
$$\cot(8x+\pi)=\cot 8x \quad \because \cot(\theta+\pi)=\cot\theta$$
$$\Rightarrow cot8\left(x+\frac{\pi}{8}\right)=\cot 8x$$
$$\Rightarrow \text{period of } \cot 8x \text{ is } \frac{\pi}{8}$$

Q.NO.11. $sec9x$

Solution:

$$\because \text{periodof } secx \text{ is } 2\pi. so$$
$$\sec(9x+2\pi)=sec9x \quad \because \sec(\theta+\pi)=\sec\theta$$
$$\Rightarrow sec9\left(x+\frac{2\pi}{9}\right)=sec9x$$
$$\Rightarrow \text{period of } sec9x \text{ is } \frac{2\pi}{9}$$

Q.NO.12. $cosec10xx$

Solution:

$$\because \text{periodof } cosecx \text{ is } 2\pi. so$$
$$\text{cosec}(10x+2\pi)=cosec10x \quad \because cosec(\theta+\pi)=cosec\theta$$
$$\Rightarrow cosec10\left(x+\frac{2\pi}{10}\right)=cosec10x$$
$$\Rightarrow cosec10\left(x+\frac{\pi}{5}\right)=cosec10x$$
$$\Rightarrow \text{period of } cosec10x \text{ is } \frac{\pi}{5}$$

Q.NO.13. $3sinx$

Solution:

$$\because \text{periodof } sinx \text{ is } 2\pi. so$$
$$3\sin(x+2\pi)=3sinx \quad \because \sin(\theta+2\pi)=\sin\theta$$
$$\Rightarrow \text{period of } 3sinx \text{ is} 2\pi$$

Q.NO.14. $2cosx$

Solution:

$$\because \text{periodof } cosx \text{ is } 2\pi. so$$
$$2\cos(x+2\pi)=2cosx \quad \because \cos(\theta+2\pi)=\cos\theta$$
$$\Rightarrow \text{period of } 2cosx \text{ is} 2\pi$$

Q.NO.15. $3\cos\frac{x}{5}$

Solution:

$$\because \text{periodof } cosx \text{ is } 2\pi. so$$
$$3\cos\left(\frac{x}{5}+2\pi\right)=3\cos\frac{x}{5} \quad \because \cos(\theta+2\pi)=\cos\theta$$
$$\Rightarrow 3\cos\frac{1}{5}(x+10\pi)=3\cos\frac{x}{5}$$
$$\Rightarrow \text{period of } 3\cos\frac{x}{5} \text{ is } 10\pi$$

With best wishes