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MATHEMATICS 11 INTERMEDIATE PART 1

Bilal Article

Chapter 10.

TRIGONOMETRIC IDENTITIES

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Chapter#10	Class 1st	Trigonometric Identities
Contacts:	Definitions →Theory → Exercise	

Distance formula:

let $P(x_1, y_1)$ *and* $Q(x_2, y_2)$ *be two points*. If d *denotes* the distance between them then

$$\boldsymbol{d = |PQ| = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}}$$

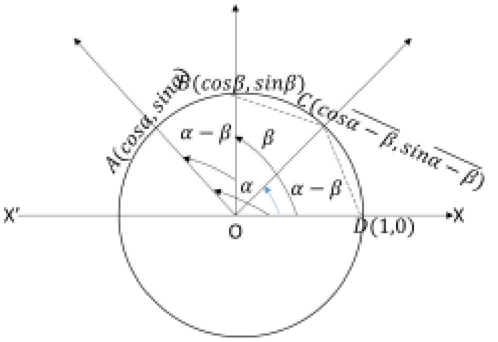
Fundamental law of trigonometry:

let α *and* β be any two angles (real numbers) then

$$\boldsymbol{cos(\alpha - \beta) = cos\alpha cos\beta + sin\alpha sin\beta}$$

Which is called the fundamental law of trigonometry.

Proof:



Consider a unit circle at O.

$$\text{where } \angle AOD = \alpha, \angle BOD = \beta$$

$$\angle AOB = \angle COD = \alpha - \beta$$

Now ΔAOB *and* ΔCOB are congruent.

Then $|AB| = |CD|$

$$\Rightarrow |AB|^2 = |CD|^2$$

Using distance formula we have

$$(cos\alpha - cos\beta)^2 + (sin\alpha - sin\beta)^2$$

$$= \left(cos\overline{\alpha - \beta} - 1\right)^2 + \left(sin\overline{\alpha - \beta} - 0\right)^2$$

$$\cos^2\alpha + cos^2\beta - 2cos\alpha cos\beta + sin^2\alpha + sin^2\beta - 2sin\alpha sin\beta$$

$$\cos^2\overline{\alpha - \beta} + 1 - 2\cos\overline{\alpha - \beta} + \sin^2\overline{\alpha - \beta}$$

$$cos^2\alpha + sin^2\alpha + cos^2\beta - 2(cos\alpha cos\beta + sin\alpha sin\beta)$$

$$= \boldsymbol{cos^2}\overline{\alpha - \beta} + \sin^2(\overline{\alpha - \beta}) + 1 - 2cos(\alpha - \beta)$$

$$\Rightarrow 2 - 2(cos\alpha cos\beta + sin\alpha sin\beta) = 2 - 2cos(\alpha - \beta)$$

Subtract 2 from both sides

$$\Rightarrow -2(cos\alpha cos\beta + sin\alpha sin\beta) = -2cos(\alpha - \beta)$$

$$\Rightarrow (cos\alpha cos\beta + sin\alpha sin\beta) = cos(\alpha - \beta) \div -2$$

Or

$$cos(\alpha - \beta) = (cos\alpha cos\beta + sin\alpha sin\beta)$$

Note:

We have proved this law for $\boldsymbol{\alpha > \beta > 0}$, *it is true for all* valves of $\boldsymbol{\alpha}$ *and* $\boldsymbol{\beta}$

Deduction from fundamental law:

1) we know that

$$\cos(\alpha - \beta) = cos\alpha cos\beta + sin\alpha sin\beta$$

$$put\ \alpha = \frac{\pi}{2}\ \ we\ get$$

$$\cos\left(\frac{\pi}{2} - \beta\right) = \cos\frac{\pi}{2}cos\beta + \sin\frac{\pi}{2}sin\beta$$

$$= (0)cos\beta + (1)sin\beta$$

$$\Rightarrow \cos\left(\frac{\pi}{2} - \beta\right) = sin\beta$$

$$(\because \cos\frac{\pi}{2} = 0\ \ \sin\frac{\pi}{2} = 1)$$

2) we know that

$$\cos(\alpha - \beta) = cos\alpha cos\beta + sin\alpha sin\beta$$

$$put\ \beta = -\frac{\pi}{2}\ \ we\ get$$

$$\cos\left(\alpha - \left(-\frac{\pi}{2}\right)\right) = \cos\alpha cos\left(-\frac{\pi}{2}\right) + \sin\alpha\, sin\left(-\frac{\pi}{2}\right)$$

$$\cos\left(\alpha + \frac{\pi}{2}\right) = \cos\alpha cos\frac{\pi}{2} - \sin\alpha\, sin\frac{\pi}{2}$$

$$= cos\alpha(0) - sin\alpha(1)$$

$$\Rightarrow \cos\left(\alpha + \frac{\pi}{2}\right) = -sin\alpha$$

$$(\because \cos(-\frac{\pi}{2}) = \cos\frac{\pi}{2} = 0)$$

$$\because \boldsymbol{sin\left(-\frac{\pi}{2}\right) = -sin\frac{\pi}{2} = -1}$$

3) we know that

$$\cos\left(\frac{\pi}{2} - \beta\right) = sin\beta$$

Put $\beta = \frac{\pi}{2} + \alpha$ *we get*

$$\Rightarrow \cos\left(\frac{\pi}{2} - \left(\frac{\pi}{2} + \alpha\right)\right) = \sin\left(\frac{\alpha}{2} + \alpha\right)$$

$$\Rightarrow \cos\left(\frac{\pi}{2} - \frac{\pi}{2} - \alpha\right) = \sin\left(\frac{\pi}{2} + \alpha\right)$$

$$\Rightarrow \cos(-\alpha) = \sin\left(\frac{\pi}{2} + \alpha\right)$$

$$\Rightarrow cos\alpha = \sin\left(\frac{\pi}{2} + \alpha\right)$$

$$\Rightarrow \sin\left(\frac{\pi}{2} + \alpha\right) = cos\alpha$$

4) we know that

$$\cos(\alpha - \beta) = cos\alpha cos\beta + sin\alpha sin\beta$$

$$replacing\ \beta\ \ \ by - \beta\ \ we\ get$$

$$\cos[\alpha - (-\beta)] = cos\alpha cos(-\beta) + sin\alpha sin(-\beta)$$

$$\Rightarrow \cos(\alpha + \beta) = cos\alpha cos\beta - sin\alpha sin\beta$$

$$(\because\ \cos(-\beta) = cos\beta, \quad \sin(-\beta) = -sin\beta$$

5) we know that

$$\cos(\alpha + \beta) = cos\alpha cos\beta - sin\alpha sin\beta$$

Replacing α *by* $\frac{\pi}{2} + \alpha$

$$\cos\left(\frac{\pi}{2} + \alpha + \beta\right) = cos\frac{\pi}{2} + \alpha cos\beta - sin\frac{\pi}{2} + \alpha sin\beta$$

$$cos\left(\left(\frac{\pi}{2} + \alpha\right) + \beta\right) = -sin\alpha cos\beta - cos\alpha sin\beta$$

$$-sin(\alpha + \beta) = -(sin\alpha cos\beta + cos\alpha sin\beta)$$

$$sin(\alpha + \beta) = sin\alpha cos\beta + cos\alpha sin\beta$$

$$(\because sin\left(\frac{\pi}{2} + \alpha\right) = -cos\alpha, cos\left(\frac{\pi}{2} + \alpha\right) = -sin\alpha$$

6) we know that

$$sin(\alpha + \beta) = sin\alpha cos\beta + cos\alpha sin\beta$$

Replacing $\boldsymbol{\beta}$ *by*$(\boldsymbol{-\beta})$ *we get*

$$sin(\alpha + (\boldsymbol{-\beta})) = sin\alpha cos(\boldsymbol{-\beta}) + cos\alpha sin(\boldsymbol{-\beta})$$

$$sin(\alpha - \beta) = sin\alpha cos\beta - cos\alpha sin\beta$$

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$\because \boldsymbol{\cos(-\beta) = \cos\beta}, \quad \boldsymbol{\sin(-\beta) = -\sin\beta}$

7) **we know that**

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\boldsymbol{let\ \alpha = 2\pi \ \ and \ \ \beta = \theta}$$
$$\cos(2\pi-\theta) = \cos2\pi\cos\theta + \sin2\pi\sin\theta$$
$$\cos(2\pi-\theta) = (1)\cos\theta + (0)\sin\theta$$
$$\Rightarrow \cos(2\pi-\theta) = \cos\theta \quad \because \cos2\pi = 1 \ and \ \sin2\pi = 0$$

8) **we know that**

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$let\ \alpha = 2\pi \ \ and \ \ \beta = \theta$$
$$\sin(2\pi-\theta) = \sin2\pi\cos\theta - \cos2\pi\sin\theta$$
$$\sin(2\pi-\theta) = (0)\cos\theta - (1)\sin\theta$$
$$\sin(2\pi-\theta) = -\sin\theta$$

9) $\because \ \boldsymbol{\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}}$

$$= \frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta + \sin\alpha\sin\beta}$$

During up and down by $\boldsymbol{\cos\alpha\cos\beta}$

$$= \frac{\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} + \frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} + \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$$

Thus,

$$= \boldsymbol{\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}}$$

10) $\because \ \boldsymbol{\tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}}$

$$= \frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta + \sin\alpha\sin\beta}$$

During up and down by $\boldsymbol{\cos\alpha\cos\beta}$

$$= \frac{\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} - \frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} + \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$$

Thus,

$$= \boldsymbol{\frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}}$$

Trigonometric Ratio of allied Angle:

Allied angles:

The angle associated with basic angle of measure θ *to* a right angle or multiple are called allied angles.

Examples:

$$90^0 \pm \theta, \quad 180^0 \pm \theta, \quad 270^0 \pm \theta, \qquad 360^0 \pm \theta \ \ etc.$$

Remember some basic results of allied angles:

1) **If $\boldsymbol{\theta}$ *is add to or subtracted* from odd multiple of right angle, trigonometric ratio change into co-ratio and versa. i.e**

$$\boldsymbol{\sin \rightleftharpoons \cos, \quad \tan \leftrightharpoons \cot, \qquad \sec \leftrightharpoons cosec}$$

$\boldsymbol{\sin \to \cos}$

$$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta, \qquad \sin\left(\frac{\pi}{2}+\theta\right) = \cos\theta$$
$$\sin\left(\frac{3\pi}{2}-\theta\right) = -\cos\theta$$
$$\sin\left(\frac{3\pi}{2}+\theta\right) = -\cos\theta$$

$\boldsymbol{\cos \to \sin}$

$$\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta, \qquad \cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$$
$$\cos\left(\frac{3\pi}{2}-\theta\right) = -\sin\theta$$
$$\cos\left(\frac{3\pi}{2}+\theta\right) = \sin\theta$$

$\boldsymbol{\tan \to \cot}$

$$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta, \qquad \tan\left(\frac{\pi}{2}+\theta\right) = -\cot\theta$$
$$\tan\left(\frac{3\pi}{2}-\theta\right) = \cot\theta$$
$$\tan\left(\frac{3\pi}{2}+\theta\right) = -\cot\theta$$

2) if θ *is add* or subtracted from an even multiple of $\frac{\pi}{2}$, the trigonometric ratio shall remain the same.

3) so far as the sign of result is concerned, it is determined by the quadrant in which the terminal arm of the angle lies.

$\boldsymbol{\sin \to \sin}$

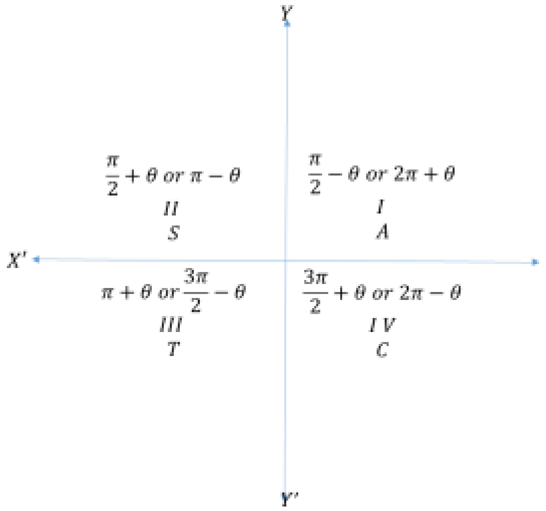
$$\sin(\pi-\theta) = \sin\theta, \qquad \sin(\pi+\theta) = -\sin\theta$$
$$\sin(2\pi-\theta) = -\sin\theta, \quad \sin(2\pi+\theta) = \sin\theta$$

$\boldsymbol{\cos \to \cos}$

$$\cos(\pi-\theta) = -\cos\theta, \qquad \cos(\pi+\theta) = -\cos\theta$$
$$\cos(2\pi-\theta) = \cos\theta, \quad \cos(2\pi+\theta) = \cos\theta$$

$\boldsymbol{\tan \to \tan}$

$$\boldsymbol{\tan(\pi-\theta) = -\tan\theta, \quad \tan(\pi+\theta) = \tan\theta}$$
$$\boldsymbol{\tan(2\pi-\theta) = -\tan\theta, \tan(2\pi+\theta) = \tan\theta}$$



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Exercise 10.1

Question # 1
Without using the tables, find the value of :
(i). $\boldsymbol{Sin(-780°)}$
Solution.
$Sin(-780°) = -Sin(780°)$
$Sin(-780°) = -Sin(8(90°) + 60°)$
$Sin(-780°) = -Sin(60°)$
$Sin(-780°) = -\frac{\sqrt{3}}{2}$
Answer.
(ii). $\boldsymbol{Cot(-885°)}$
Solution.
$Cot(-885°) = -Cot(885°)$
$Cot(-885°) = -Cot(9(90°) + 45°)$
$Cot(-885°) = -\tan(45°)$
$Cot(-885°) = -1$
Answer.
(iii). $\boldsymbol{Csc(2040°)}$
Solution.
$Csc(2040°) = Csc(22(90)° + 60°)$
$Csc(2040°) = -Csc(60°)$
$Cot(-885°) = -\frac{2}{\sqrt{3}}.$
Answer.
(iv). $\boldsymbol{Sec(-960°)}$
Solution.
$Sec(960°) = Sec(10(90)° + 60°)$
$Sec(960°) = -Sec(60°)$
$Sec(-960°) = -2.$
Answer.
(v). $\boldsymbol{\tan(1110°)}$
Solution.
$tan(1110°) = tan(12(90)° + 30°)$
$tan(1110°) = tan(30°)$
$tan(1110°) = \frac{1}{\sqrt{3}}.$
Answer.
(vi). $\boldsymbol{Sin(-300°)}$
Solution.
$$\because sin(-\theta) = -sin\theta$$
$$= -sin300^0$$
$$= -sin(360^0 - 60^0)$$
$$= -sin\left(4\frac{\pi}{2} - 60^0\right)$$
$$= -(-sin60^0) = sin60^0 = \frac{\sqrt{3}}{2}$$
$$\because sin\left(\frac{4\pi}{2} - \theta\right) = -sin\theta$$

Question # 2
Express each of the following as a trigonometric function of an angle of positive degree measure of less than 45°.
(i). $\boldsymbol{Sin196°}$
Solution:
$Sin(196°) = Sin(180° + 16°)$
$Sin(196°) = Sin180°\,Cos16° + Cos180°Sin16°$
$Sin(196°) = (0)\,Cos16° + (-1)Sin16°$
$Sin(196°) = -Sin16°.$
Answer.
(ii). $\boldsymbol{Cos147°}$
Solution.
$Cos(147°) = Cos(180° - 33°)$
$Cos(147°) = Cos180°\,Cos33° + Sin180°Sin33°$
$Cos(147°) = (-1)\,Cos33° + (0)Sin33°$
$Cos(147°) = -Cos33°.$
Answer.
(iii). $\boldsymbol{Sin319°}$
Solution.
$Sin(319°) = Sin(360° - 41°)$
$Sin(319°) = Sin360°\,Cos41° - Cos360°Sin41°$
$Sin(319°) = (0)Cos41° - (1)Sin41°$
$Sin(319°) = -Sin41°.$
Answer.
(iv). $\boldsymbol{Cos254°}$
Solution.
$Cos(254°) = Cos(270° - 16°)$
$Cos(254°) = Cos270°\,Cos16° + Sin270°Sin16°$
$Cos(254°) = (0)Cos16° + (-1)Sin16°$
$Cos(254°) = -Sin16°.$
Answer.
(v). $\boldsymbol{tan294°}$
Solution.
$$tan294° = \frac{Sin294°}{Cos294°}$$
$$tan294° = \frac{Sin(270° + 24°)}{Cos(270° + 24°)}$$
$$tan294° = \frac{Sin270°\,Cos24° + Cos270°Sin24°}{Cos270°\,Cos24° - Sin270°Sin24°}$$
$$tan294° = \frac{(-1)\,Cos24° + (0)Sin24°}{(0)\,Cos24° - (-1)Sin24°}$$
$$tan294° = -\frac{Cos24°}{Sin24°}$$
$tan294° = -Cot24°.$
Answer.
(vi). $\boldsymbol{Cos(728°)}$
Solution.
$Cos(728°) = Cos(720° + 8°)$
$Cos(728°) = Cos720°\,Cos8° - Sin720°Sin8°$
$Cos(728°) = (1)Cos8° + (0)Sin8°$
$Cos(728°) = Cos8°.$
Answer.
(vii). $\boldsymbol{Sin(-625°)}$

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Solution.

$Sin(-625°) = -Sin(625°)$
$Sin(-625°) = -Sin(630° - 5°)$
$Sin(-625°) = -(Sin630°\,Cos5° - Cos630°Sin5°)$
$Sin(-625°) = -((-1)Cos5° - (0)Sin5°)$
$Sin(-625°) = Cos5°.$

Answer.

(viii). $\boldsymbol{Cos(-435°)}$

Solution.

$Cos(-435°) = Cos(435°)$
$Cos(-435°) = Cos(450° - 15°)$
$Cos(-435°) = Cos450°\,Cos15° + Sin450°Sin15°$
$Cos(-435°) = (0)Cos15° + (1)Sin15°$
$Cos(-435°) = Sin15°.$

Answer.

(ix) $sin150^0$

$$= sin(180^0 - 30^0)$$
$$= sin(\pi - 30^0)$$
$$= sin30^0 \quad \because \begin{pmatrix} sin(\pi - \theta) \\ = sin\theta \end{pmatrix}$$

Question # 3

Prove the following:

(i). $\boldsymbol{Sin(180° + \alpha)Sin(90° - \alpha) = -Sin\alpha Cos\alpha}$

Solution.

$L.H.S = Sin(180° + \alpha)Sin(90° - \alpha)$
$L.H.S = (Sin180°\,Cos\alpha + Cos180°Sin\alpha)(Sin90°\,Cos\alpha - Cos90°Sin\alpha)$
$L.H.S = ((0)Cos\alpha + (-1)Sin\alpha)((1)Cos\alpha - (0)Sin\alpha)$
$L.H.S = (-Sin\alpha)(Cos\alpha)$
$L.H.S = -Sin\alpha Cos\alpha$
$L.H.S = R.H.S$

Hence Proved.

(ii). $\boldsymbol{Sin780°Sin480° + Cos120°Sin30° = \frac{1}{2}}$

Solution.

$L.H.S = Sin780°Sin480° + Cos120°Sin30°$
$L.H.S = Sin(720° + 60°)Sin(450° + 30°) + Cos120°Sin30°$
$L.H.S = Sin(60°)Sin(30°) + Cos120°Sin30°$
$L.H.S = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)$
$L.H.S = \frac{3}{4} - \frac{1}{4}$
$L.H.S = \frac{2}{4}$
$L.H.S = \frac{1}{2}$
$L.H.S = R.H.S$

Hence Proved.

(iii). $\boldsymbol{Sin306° + Cos234° + Cos162° + Cos18° = 0}$

Solution.

$L.H.S = Sin306° + Cos234° + Cos162° + Cos18°$
$L.H.S = Sin(3(90°) + 36°) + Cos(3(90°) - 36°) + Cos(180° - 18°) + Cos18°$
$L.H.S = Cos(36°) - Sin(36°) - Cos(18°) + Cos18°$
$L.H.S = 0$

$L.H.S = R.H.S$

Hence Proved .

(iv). $\boldsymbol{Cos330°Sin600° + Cos120°Sin150° = -1}$

Solution.

$L.H.S = Cos330°Sin600° + Cos120°Sin150°$
$L.H.S = Cos(360° - 30°)Sin(3(180°) + 60°) + Cos(90° + 30°)Sin(90° + 60°)$
$L.H.S = Cos(30°)(-Sin(60°)) + (-Sin(30°))Cos(60°)$
$L.H.S = \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)$
$L.H.S = -\frac{3}{4} - \frac{1}{4}$
$L.H.S = -1$
$L.H.S = R.H.S$

Hence Proved.

Question # 4

Prove that;

(i). $\boldsymbol{\frac{Sin^2(\pi+\theta)\tan(\frac{3\pi}{2}+\theta)}{Cot^2\left(\frac{3\pi}{2}-\theta\right)Cos^2(\pi-\theta)Cosec(2\pi-\theta)} = Cos\theta}$

Solution.

$$L.H.S = \frac{Sin^2(\pi+\theta)\tan(\frac{3\pi}{2}+\theta)}{Cot^2\left(\frac{3\pi}{2}-\theta\right)Cos^2(\pi-\theta)Cosec(2\pi-\theta)}$$
$$L.H.S = \frac{(-Sin\theta)^2(-\text{Cot}(\theta))}{(tan\theta)^2(-Cos\theta)^2(-Cosec\theta)}$$
$$L.H.S = \frac{Sin^2\theta\frac{Cos\theta}{Sin\theta}}{\frac{Sin^2\theta}{Cos^2\theta}Cos^2\theta\frac{1}{Sin\theta}}$$
$$L.H.S = \frac{Sin\theta Cos\theta}{Sin\theta}$$
$L.H.S = Cos\theta$
$L.H.S = R.H.S$

Hence Proved.

(ii). $\boldsymbol{\frac{Cos(90°+\theta)Sec(-\theta)\tan(180°-\theta)}{Sec(360°-\theta)Sin(180°+\theta)Cot(90°-\theta)} = -1}$

Solution.

$$L.H.S \frac{Cos(90°+\theta)Sec(-\theta)\tan(180°-\theta)}{Sec(360°-\theta)Sin(180°+\theta)Cot(90°-\theta)}$$
$$L.H.S = \frac{-Sin(\theta)Sec(\theta)\,(-\tan(\theta))}{Sec(\theta)(-Sin(\theta))tan\theta}$$
$$L.H.S \frac{-Sin(\theta)(-1)}{(-Sin(\theta))}$$
$L.H.S = -1$
$L.H.S = R.H.S$

Hence Proved.

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Question # 5

If α, β, γ are the angles of a triangle ABC, then prove that;

(i). $\boldsymbol{Sin(\alpha+\beta) = Sin\gamma}$

Solution.

Since α, β, γ are the angles of a triangle ABC, then

$\alpha + \beta + \gamma = 180°$

$\Longrightarrow \alpha + \beta = 180° - \gamma$

$Sin(\alpha + \beta) = Sin(180° - \gamma)$

$Sin(\alpha + \beta) = Sin180°Cos\gamma - Cos180°Sin\gamma$

$Sin(\alpha + \beta) = (0)Cos\gamma - (-1)Sin\gamma$

$Sin(\alpha + \beta) = -Sin\gamma$

Hence Proved.

(ii). $\boldsymbol{Cos\left(\frac{\alpha+\beta}{2}\right) = Sin\frac{\gamma}{2}}$

Solution.

Since α, β, γ are the angles of a triangle ABC, then

$\alpha + \beta + \gamma = 180°$

$\Longrightarrow \alpha + \beta = 180° - \gamma$

$\frac{\alpha + \beta}{2} = \frac{180° - \gamma}{2}$

$Cos\left(\frac{\alpha + \beta}{2}\right) = Cos\left(\frac{180° - \gamma}{2}\right)$

$Cos\left(\frac{\alpha + \beta}{2}\right) = Cos\left(90° - \frac{\gamma}{2}\right)$

$Cos\left(\frac{\alpha + \beta}{2}\right) = Cos\left(90° - \frac{\gamma}{2}\right)$

$Cos\left(\frac{\alpha + \beta}{2}\right) = Sin\left(\frac{\gamma}{2}\right)$

Hence Proved.

(iii). $\boldsymbol{Cos(\alpha + \beta) = -Cos\gamma}$

Solution.

Since α, β, γ are the angles of a triangle ABC, then

$\alpha + \beta + \gamma = 180°$

$\Longrightarrow \alpha + \beta = 180° - \gamma$

$Cos(\alpha + \beta) = Cos(180° - \gamma)$

$Cos(\alpha + \beta) = Cos180°Cos\gamma + Sin180°Sin\gamma$

$Cos(\alpha + \beta) = (-1)Cos\gamma - (0)Sin\gamma$

$Cos(\alpha + \beta) = -Cos\gamma$

Hence Proved.

(iv). $\boldsymbol{tan(\alpha + \beta) = tan\gamma}$

Solution.

Since α, β, γ are the angles of a triangle ABC, then

$\alpha + \beta + \gamma = 180°$

$\Longrightarrow \alpha + \beta = 180° - \gamma$

$tan(\alpha + \beta) = tan(180° - \gamma)$

$tan(\alpha + \beta) = -tan\gamma$

$tan(\alpha + \beta) + tan\gamma = 0$

Hence Proved.

Further Application of Basic identities

Exercise 10.2

Question#1 Prove that:

(i) $\boldsymbol{\sin(180+\theta) = -\sin\theta}$

L.H.S=sin(180+θ)

$=\sin180\cos\theta + \cos180\sin\theta$ ∴ sin180=0

$=(0)\cos\theta + (-1)\sin\theta$ ∴ cos180=-1

$= -\sin\theta$

(ii) $\boldsymbol{\cos(180+\theta) = -\cos\theta}$

L.H.S= $\cos(180+\boldsymbol{\theta})$

$=\cos180\cos\boldsymbol{\theta} - \sin180\sin\boldsymbol{\theta}$ ∴ sin180=0

$= (-1)\cos\theta - (0)\sin\theta$ ∴ cos180=-1

$= -cos\theta$

(iii) $\tan(270^0 - \theta) = cot\theta$

Solution:

$$L.H.S = \tan(270^0 - \theta)$$
$$= \frac{\sin(270^0 - \theta)}{cos(270^0 - \theta)}$$
$$= \frac{sin270^0cos\theta - cos270^0sin\theta}{cos270^0cos\theta + cos270^0cos\theta}$$
$$= \frac{(-1)cos\theta - (0)sin\theta}{(0)cos\theta + (-1)sin\theta} = \frac{-cso\theta}{-sin\theta}$$
$$= cot\theta = R.H.S$$

hence proved.

(iv) $\cos(\theta - 180^0) = -cos\theta$

Solution:

$$L.H.S = \cos(\theta - 180^0)$$
$$= cos\theta cos180^0 - sin\theta sin180^0$$
$$= \cos(-1) - sin\theta(0)$$
$$= -cos\theta = R.H.S$$

hence proved.

(v) $\boldsymbol{\cos(270+\theta) = \sin\theta}$

L .H.S= $\cos(270+\boldsymbol{\theta})$ ∴ sin270=-1

$=\cos270\cos\theta - \sin270\sin\theta$ ∴ cos270=0

$= (0)\cos270 - (-1)\sin\boldsymbol{\theta}$

$= \sin\boldsymbol{\theta}$ = R.H.S

(vi) $\sin(\theta + 270^0) = -cos\theta$

Solution:

$$L.H.S = \sin(\theta + 270^0)$$
$$= sin\theta cos270^0 + cos\theta sin270^0$$
$$= sin\theta(0) + cos\theta(-1)$$
$$= -cos\theta = R.H.S$$

hence proved

(vii) $\tan(180^0 + \theta)\, tan\theta$

Solution:

$$L.H.S = \tan(180^0 + \theta)$$
$$= \frac{sin(180^0 - \theta)}{\cos(180^0 + \theta)}$$

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$$=\frac{sin180^0cos\theta + cos180^0sin\theta}{cos180^0cos\theta - sin180^0sin\theta}$$
$$=\frac{(0)cos\theta + (-1)sin\theta}{(-!)cos\theta - (0)sin\theta}$$
$$=\frac{-sin\theta}{-cos\theta}=\frac{sin\theta}{cos\theta}=tan\theta = R.H.S$$

Hence proved.

(viii) $\boldsymbol{Cos(360-\theta) = cos\theta}$

L.H.S= Cos(360−θ)

= cos360cosθ + sin360sinθ ∴ cos360=1

= $(1)cos\theta + (0)sin\theta$ ∴ sin360=0

= $cos\theta$ =R.H.S

Question#2 Find the values of the following

Note

We use $15^0=60^0-45^0$ and $105^0=60^0+45^0$

(i) Sin15

⟹sin(60-45)

=sin60cos45-cos60sin45

$=\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)-\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$=\frac{\sqrt{3}}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}$

$=\frac{\sqrt{3}-1}{2\sqrt{2}}$

(iv) Sin105

⟹sin(60+45)

=sin60cos45−cos60sin45

$=\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$=\frac{\sqrt{3}}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}$

$=\frac{\sqrt{3}+1}{2\sqrt{2}}$

(ii) Cos15

⟹cos(60-45)

= cos60cos45 –sin60 sin45

$=\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)-\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$=\frac{1}{2\sqrt{2}}-\frac{\sqrt{3}}{2\sqrt{2}}$

$\frac{1-\sqrt{3}}{2\sqrt{2}}$

(v) Cos105

⟹cos(60-45)

= cos60cos45 + sin60 sin45

$=\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)+\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$=\frac{1}{2\sqrt{2}}+\frac{\sqrt{3}}{2\sqrt{2}}$

$\frac{1+\sqrt{3}}{2\sqrt{2}}$

(iii) tan 15^0

Solution:

$$tan15^0 = \tan(45^0-30^0)$$
$$=\frac{tan45^0 - tan30^0}{1+tan45^0tan30^0}$$
$$=\frac{1-\frac{1}{\sqrt{3}}}{1+(1)\left(\frac{1}{\sqrt{3}}\right)}$$
$$=\frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}=\frac{\sqrt{3}-1}{\sqrt{3}+1}$$

(vi) $tan105^0$

Solution:

$$tan105^0 = \tan(60^0+45^0)$$
$$=\frac{tan60^0 - tan45^0}{1+tan60^0tan45^0}$$
$$=\frac{\sqrt{3}+1}{1-\sqrt{3}(1)}=\frac{\sqrt{3}+1}{1-\sqrt{3}}$$

Question#3 Prove the following

i. Sin(45+α)=$\frac{1}{\sqrt{2}}(sin\alpha + cos\alpha)$

L.H.S= Sin(45+α)

= sin45cos α + cos45sin α

$=\left(\frac{1}{\sqrt{2}}\right)$ cos α + $\left(\frac{1}{\sqrt{2}}\right)$ sin α

$=\frac{1}{\sqrt{2}}(cos\alpha + sin\alpha)$=R.H.S

ii. cos(α+45)=$\frac{1}{\sqrt{2}}(sin\alpha - cos\alpha)$

= cosαcos45 –sinα sin45

= $\left(\frac{1}{\sqrt{2}}\right)$ cos α − $\left(\frac{1}{\sqrt{2}}\right)$ sin α

= $\frac{1}{\sqrt{2}}(cos\alpha - sin\alpha)$=R.H.S

Question#4 Prove that

(i) tan(45+A) tan(45-A)=1

solution:-L.H.S= tan(45+A) tan(45-A)

$=\left(\frac{tan45+tanA}{1-tan45tanA}\right)\left(\frac{tan45-tanA}{1+tan45tanA}\right)$

$=\left(\frac{1+tanA}{1-(1)tanA}\right)\left(\frac{1-tanA}{1+(1)tanA}\right)$ ∴ tan45=1

$=\left(\frac{1+tanA}{1-tanA}\right)\left(\frac{1-tanA}{1+tanA}\right)$

=1 (after cancellation)

=R.H.S

ii:- tan($\frac{\pi}{4}$ -θ)+ tan($\frac{3\pi}{4}$ + θ)=0

solution:-L.H.S= tan($\frac{\pi}{4}$ -θ)+ tan($\frac{3\pi}{4}$ + θ)

= tan(45-θ)+tan(135+θ)

$=\left(\frac{tan45-tan\theta}{1+tan45tan\theta}\right)+\left(\frac{tan135+tan\theta}{1-tan135tan\theta}\right)$

$=\left(\frac{(1)-tan\theta}{1+(1)tan\theta}\right)+\left(\frac{(-1)+tan\theta}{1-(-1)tan\theta}\right)$

$=\left(\frac{1-tan\theta}{1+tan\theta}\right)+\left(\frac{-1+tan\theta}{1+tan\theta}\right)$

$=\left(\frac{1-tan\theta}{1+tan\theta}\right)-\left(\frac{1-tan\theta}{1-tan\theta}\right)$ (cancellation)

=0 R.H.S

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iii. $\sin\left(\theta+\frac{\pi}{6}\right)+\cos\left(\theta+\frac{\pi}{3}\right)=\cos\theta$

L.H.S:- $\sin\left(\theta+\frac{\pi}{6}\right)+\cos\left(\theta+\frac{\pi}{3}\right)$

$\left(\sin\theta\cos\frac{\pi}{6}+\cos\theta\sin\frac{\pi}{6}\right)+\left(\cos\theta\cos\frac{\pi}{3}-\sin\theta\sin\frac{\pi}{3}\right)$

$\left(\sin\theta\left(\frac{\sqrt{3}}{2}\right)+\cos\theta\left(\frac{1}{2}\right)\right)+\left(\cos\theta\left(\frac{1}{2}\right)-\sin\theta\left(\frac{\sqrt{3}}{2}\right)\right)$

$\left(\frac{\sqrt{3}}{2}\right)\sin\theta+\left(\frac{1}{2}\right)\cos\theta+\left(\frac{1}{2}\right)\cos\theta-\left(\frac{\sqrt{3}}{2}\right)\sin\theta$(cancellation)

$\left(\frac{1}{2}\right)\cos\theta+\left(\frac{1}{2}\right)\cos\theta=\cos\theta=$R.H.S

iv:- $\frac{\sin\theta-\cos\theta\tan\frac{\theta}{2}}{\cos\theta+\sin\theta\tan\frac{\theta}{2}}=\tan\frac{\theta}{2}$

L.H.S $=\frac{\sin\theta-\cos\theta\tan\frac{\theta}{2}}{\cos\theta+\sin\theta\tan\frac{\theta}{2}}$

$=\frac{\sin\theta-\cos\theta\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}}{\cos\theta+\sin\theta\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}}=\frac{\frac{\sin\theta\cos\frac{\theta}{2}-\cos\theta\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}}{\frac{\cos\theta\cos\frac{\theta}{2}+\sin\theta\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}}$

$=\frac{\sin\theta\cos\frac{\theta}{2}-\cos\theta\sin\frac{\theta}{2}}{\cos\theta\cos\frac{\theta}{2}+\sin\theta\sin\frac{\theta}{2}}=\frac{\sin\left(\theta-\frac{\theta}{2}\right)}{\cos\left(\theta-\frac{\theta}{2}\right)}$

$=\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}=\tan\frac{\theta}{2}$

(v): $\frac{1-\tan\theta\tan\emptyset}{1+\tan\theta\tan\varphi}=\frac{\cos(\theta+\emptyset)}{\cos(\theta-\emptyset)}$

$=\frac{1-\frac{\sin\theta}{\cos\theta}\frac{\sin\emptyset}{\cos\emptyset}}{1+\frac{\sin\theta}{\cos\theta}\frac{\sin\emptyset}{\cos\emptyset}}$

$=\frac{\frac{\cos\theta\cos\emptyset-\sin\theta\sin\emptyset}{\cos\theta\cos\emptyset}}{\frac{\cos\theta\cos\emptyset+\sin\theta\sin\emptyset}{\cos\theta\cos\emptyset}}$

$=\frac{\cos\theta\cos\emptyset-\sin\theta\sin\emptyset}{\cos\theta\cos\emptyset+\sin\theta\sin\emptyset}$

$=\frac{\cos(\theta+\emptyset)}{\cos(\theta-\emptyset)}=$R.H.S

Question #5 prove that $\cos(\alpha+\beta)\cos(\alpha-\beta)=\cos^2\alpha-\sin^2\beta=\cos^2\beta-\sin^2\alpha$

L.H.S= $\cos(\alpha+\beta)\cos(\alpha-\beta)$

$=(\cos\alpha\cos\beta-\sin\alpha\sin\beta)(\cos\alpha\cos\beta+\sin\alpha\sin\beta)$ [Difference between two square]

$=\cos^2\alpha\cos^2\beta-\sin^2\alpha\sin^2\beta$

$=\cos^2\alpha(1-\sin^2\beta)-(1-\cos^2\alpha)\sin^2\beta$

$=\cos^2\alpha-\cos^2\alpha\sin^2\beta-\sin^2\beta+\cos^2\alpha\sin^2\beta$ (cancellation)

$=\cos^2\alpha-\sin^2\beta$=Middle side

$=(1-\sin^2\alpha)-(1-\cos^2\beta)$

$=1-\sin^2\alpha-1+\cos^2\beta$

$=\cos^2\beta-\sin^2\alpha$ =R.H.S

Question #6 show that $\frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)}=\tan\alpha$

Solution:-L.H.S$=\frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)}$

$=\frac{(\sin\alpha\cos\beta+\cos\alpha\sin\beta)+(\sin\alpha\cos\beta-\cos\alpha\sin\beta)}{(\cos\alpha\cos\beta-\sin\alpha\sin\beta)+(\cos\alpha\cos\beta+\sin\alpha\sin\beta)}$

$=\frac{2\sin\alpha\cos\beta}{2\cos\alpha\cos\beta}=\frac{\sin\alpha}{\cos\alpha}$

=tanα =R.H.S

Question #7 show that

i. $\cot(\alpha+\beta)=\frac{\cot\alpha\cot\beta-1}{\cot\alpha+\cot\beta}$

Solution:-R.H.S$=\frac{\cot\alpha\cot\beta-1}{\cot\alpha+\cot\beta}$

$\frac{\frac{\cos\alpha}{\sin\alpha}\frac{\cos\beta}{\sin\beta}-1}{\frac{\cos\alpha}{\sin\alpha}+\frac{\cos\beta}{\sin\beta}}=\frac{\frac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{\sin\alpha\sin\beta}}{\frac{\cos\alpha\sin\beta+\sin\alpha\cos\beta}{\sin\alpha\sin\beta}}$ (cancellation)

$=\frac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{\cos\alpha\sin\beta+\sin\alpha\cos\beta}$

$=\frac{\cos(\alpha+\beta)}{\sin(\alpha+\beta)}$

$=\cot(\alpha+\beta)=$ R.H.S

ii. $\cot(\alpha-\beta)=\frac{\cot\alpha\cot\beta+1}{\cot\alpha-\cot\beta}$

Solution:-R.H.S$=\frac{\cot\alpha\cot\beta+1}{\cot\alpha-\cot\beta}$

$=\frac{\frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta}+1}{\frac{\cos\alpha}{\sin\alpha}-\frac{\cos\beta}{\sin\beta}}=\frac{\frac{\cos\alpha\cos\beta+\sin\alpha\sin\beta}{\sin\alpha\sin\beta}}{\frac{\cos\alpha\sin\beta-\sin\alpha\cos\beta}{\sin\alpha\sin\beta}}$ (cancellation)

$=\frac{\cos\alpha\cos\beta+\sin\alpha\sin\beta}{\cos\alpha\sin\beta-\sin\alpha\cos\beta}$

$=\frac{\cos(\alpha-\beta)}{\sin(\alpha-\beta)}$

$=\cot(\alpha-\beta)=$ R.H.S

iii. $\frac{\tan\alpha+\tan\beta}{\tan\alpha-\tan\beta}=\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$

Solution:-$\frac{\frac{\sin\alpha}{\cos\alpha}+\frac{\sin\beta}{\cos\beta}}{\frac{\sin\alpha}{\cos\alpha}-\frac{\sin\beta}{\cos\beta}}$

$=\frac{\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\sin\alpha\cos\beta-\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}$ (cancellation)

$=\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\sin\alpha\cos\beta-\cos\alpha\sin\beta}$

$=\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}=R.H.S$

Question #8 if $\sin\alpha=\frac{4}{5}$ and $\cos\beta=\frac{40}{41}$, where $0<\alpha<\frac{\pi}{2}$ and $0<\beta<\frac{\pi}{2}$. show that $\sin(\alpha-\beta)=\frac{133}{205}$

Solution:- As $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$ so we first need to find $\cos\alpha$ and $\sin\beta$

$\Rightarrow\cos^2\alpha=1-\sin^2\alpha$

$=\cos^2\alpha=1-\left(\frac{4}{5}\right)^2$

$=\cos^2\alpha=1-\frac{16}{25}$

$=\cos^2\alpha=\frac{25-16}{25}$

$=\cos^2\alpha=\frac{9}{25}$ taking square root

$=\cos\alpha=\pm\frac{3}{5}$ (only take positive because of first quadrant)

So $\cos\alpha=+\frac{3}{5}$

Also

$\Rightarrow\sin^2\beta=1-\cos^2\beta$

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$=\sin^2\beta = 1-\left(\frac{40}{41}\right)^2$

$=\sin^2\beta = 1-\frac{1600}{1681}$

$=\sin^2\beta = \frac{81}{1681}$

$=\sin^2\beta = \frac{81}{1681}$ taking square root

$=\sin\beta = \pm\frac{9}{41}$ (only take positive because of first quadrant)

So $\sin\beta = +\frac{9}{41}$

$\Longrightarrow \sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$=\sin(\alpha-\beta) = \left(\frac{4}{5}\right)\left(\frac{40}{41}\right) - \left(\frac{3}{5}\right)\left(\frac{9}{41}\right)$

$=\frac{160-27}{205} = \frac{133}{205}$

$= \sin(\alpha-\beta) = \frac{133}{205}$ (proved)

Question #9 if $\sin\alpha = \frac{4}{5}$ and $\sin\beta = \frac{12}{13}$ where $\frac{\pi}{2} < \alpha < \pi$ and $\frac{\pi}{2} < \beta < \pi$. Find

Note:- first of all we find cos**β** and cos**β** and use all these four values to find all values given below and quadrant is 2nd so only sin (cosec) is positive

$\Longrightarrow \cos^2\alpha = 1-\sin^2\alpha$

$=\cos^2\alpha = 1-\left(\frac{4}{5}\right)^2$

$=\cos^2\alpha = 1-\frac{16}{25}$

$=\cos^2\alpha = \frac{25-16}{25}$

$=\cos^2\alpha = \frac{9}{25}$ taking square root

$=\cos\alpha = \pm\frac{3}{5}$ (only sin is positive because of 2nd quadrant)

So $\cos\alpha = -\frac{3}{5}$

$\Longrightarrow \cos^2\beta = 1-\sin^2\beta$

$=\cos^2\beta = 1-\left(\frac{12}{13}\right)^2$

$=\cos^2\beta = 1-\frac{144}{169}$

$=\cos^2\beta = \frac{169-144}{169}$

$=\cos^2\beta = \frac{25}{169}$ taking square root

$=\cos\beta = \pm\frac{5}{13}$ (only sin is positive because of 2nd quadrant)

So $\cos\beta = -\frac{5}{13}$

From above calculations we have $\sin\alpha = \frac{4}{5}$, $\sin\beta = \frac{12}{13}$,

$\cos\alpha = -\frac{3}{5}$ and $\cos\beta = -\frac{5}{13}$

i. **sin(α+β)**

solution:- As $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha+\beta) = \left(\frac{4}{5}\right)\left(-\frac{5}{13}\right) + \left(-\frac{3}{5}\right)\left(\frac{12}{13}\right)$

$=\left(-\frac{20}{65}\right) + \left(-\frac{36}{65}\right)$

$= -\frac{20}{65} - \frac{36}{65}$

$=\frac{-20-36}{65} = -\frac{56}{65}$

ii. **cos(α+β)**

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\alpha+\beta) = \left(-\frac{3}{5}\right)\left(-\frac{5}{13}\right) - \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$

$=\left(\frac{15}{65}\right) - \left(\frac{48}{65}\right)$

$= \frac{15}{65} - \frac{48}{65}$

$= \frac{15-48}{65}$

$= -\frac{33}{65}$

iii. **tan(α+β)**

solution:- To find the value of tan(α+β) we use

$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$

$\tan(\alpha+\beta) = \frac{-\frac{56}{65}}{-\frac{33}{65}} = \frac{56}{33}$

iv. **sin(α−β)**

solution:- As $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\sin(\alpha+\beta) = \left(\frac{4}{5}\right)\left(-\frac{5}{13}\right) - \left(-\frac{3}{5}\right)\left(\frac{12}{13}\right)$

$=\left(-\frac{20}{65}\right) - \left(-\frac{36}{65}\right)$

$= -\frac{20}{65} + \frac{36}{65}$

$=\frac{-20+36}{65} = \frac{16}{65}$

v. **cos(α−β)**

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos(\alpha+\beta) = \left(-\frac{3}{5}\right)\left(-\frac{5}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$

$=\left(\frac{15}{65}\right) + \left(\frac{48}{65}\right)$

$= \frac{15}{65} + \frac{48}{65}$

$= \frac{15+48}{65}$

$= \frac{63}{65}$

vi. **tan(α−β)**

solution:- To find the value of tan(α-β) we use

$\tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}$

$\tan(\alpha-\beta) = \frac{\frac{16}{65}}{\frac{63}{65}} = \frac{16}{63}$

Question#10 find sin(α+β) and cos(α+β), given that

(i) $\tan\alpha = \frac{3}{4}, \cos\beta = \frac{5}{13}$ and neither the terminal side of the angle of measure α nor that of β is the I quadrant.

Solution:

$$\tan\alpha = \frac{3}{4} (\alpha \text{ is not in } I \text{ quadrant})$$

so α lies in III quadrant.

$\cos\beta = \frac{5}{13}$ (*β not in I quadrant so β lies in IV quadr.*)

$$\because \sec^2\alpha = 1+\tan^2\alpha = 1+\left(\frac{3}{4}\right)^2$$

$$\sec^2\alpha = 1+\tan^2\alpha = 1+\frac{9}{16} = \frac{16+9}{16} = \frac{25}{16}$$

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$$\Rightarrow sec\alpha = -\frac{5}{4} (\because \alpha \text{ is in } IV \text{ quadrant})$$

$$or \quad cos\alpha = -\frac{4}{5}$$

$$also \quad sin\alpha = \pm\sqrt{1-\cos^2\alpha}$$

$$sin\alpha = \pm\sqrt{1-\left(-\frac{4}{5}\right)^2}$$

$$sin\alpha = \pm\sqrt{1-\frac{16}{25}} = \pm\sqrt{\frac{25-16}{25}}$$

$$sin\alpha = \pm\sqrt{\frac{9}{25}} = \pm\frac{3}{5}$$

$$\Rightarrow sin\alpha = -\frac{3}{5} (\because \alpha \text{ is in } III \text{ quadrant})$$

$$\because \quad sin\beta = \pm\sqrt{1-\cos^2\beta}$$

$$= \pm\sqrt{1-\left(\frac{5}{13}\right)^2}$$

$$= \pm\sqrt{1-\frac{25}{169}}$$

$$= \pm\sqrt{\frac{169-25}{169}} = \pm\sqrt{\frac{144}{169}}$$

$$= \pm\frac{12}{13}$$

$$\Rightarrow sin\beta = -\frac{12}{13} (\because \beta \text{ is in } IV \text{ quad})$$

Now

$$\sin(\alpha+\beta) = sin\alpha cos\beta + cos\alpha sin\beta$$

$$= \left(-\frac{3}{5}\right)\left(\frac{5}{13}\right) - \left(-\frac{3}{5}\right)\left(-\frac{12}{13}\right)$$

$$= -\frac{15}{65} + \frac{48}{65} = \frac{-15+48}{65} = \frac{33}{65}$$

$$\cos(\alpha+\beta) = cos\alpha cos\beta - sin\alpha sin\beta$$

$$= \left(-\frac{4}{5}\right)\left(\frac{5}{13}\right) + \left(-\frac{4}{5}\right)\left(-\frac{12}{13}\right)$$

$$= -\frac{20}{65} - \frac{36}{65} = \frac{-20-36}{65} = -\frac{56}{65}$$

(ii) tanα=$-\frac{15}{8}$ and sinβ=$-\frac{7}{25}$ and neither the terminal arm of the angle of measure nor of β is in forth quadrant.

Solution:-

NOTE:-from above signs of α and β we have α is in 2nd quadrant and β is in 3rd quadrant.

We first find sinα and cosα by using tanα and we also find cosβ using by sinβ

$\Rightarrow \sec^2\alpha = 1+\tan^2\alpha$

$= \sec^2\alpha = 1+\left(\frac{15}{8}\right)^2$

$= \sec^2\alpha = 1+\frac{225}{64}$

$=\sec^2\alpha = \frac{64+225}{64} = \frac{289}{64}$ (taking square root)

$=sec\alpha = -\frac{17}{8}$ (-ve sign is due to 2nd quadrant)

Also cos**α**= $-\frac{8}{17}$

$\sin^2\alpha$ =1-$\cos^2\alpha$

$\sin^2\alpha = 1-\left(-\frac{8}{17}\right)^2$

$\sin^2\alpha = 1-\frac{64}{289} = \frac{289-64}{169}$

$\sin^2\alpha = \frac{225}{169}$ (square root both sides)

Sinα=$+\frac{15}{17}$ (2nd quadrant)

$\Rightarrow \cos^2\beta$ =1-$\sin^2\beta$

$=\cos^2\beta = 1-\left(-\frac{7}{25}\right)^2$

$=\cos^2\beta = 1-\frac{49}{625}$

$=\cos^2\beta = \frac{625-49}{625}$

$=\cos^2\beta = \frac{576}{625}$ (taking square root)

$=\cos\beta = -\frac{24}{25}$ (Because of 3rd quadrant)

So $\cos\beta = -\frac{24}{25}$

Now sin(α+β)= $\sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha+\beta) = \left(\frac{15}{17}\right)\left(-\frac{24}{25}\right) + \left(-\frac{8}{17}\right)\left(-\frac{7}{25}\right)$

$=\left(-\frac{360}{425}\right) + \left(\frac{56}{425}\right)$

$=\frac{-360+56}{425} = -\frac{304}{425}$

$\sin(\alpha+\beta) = -\frac{304}{425}$

Now we find cos(α+β)

cos(α+β)= $cos\alpha cos\beta - sin\alpha sin\beta$

$\cos(\alpha+\beta) = \left(-\frac{8}{17}\right)\left(-\frac{24}{25}\right) - \left(\frac{15}{17}\right)\left(-\frac{7}{25}\right)$

$=\frac{192}{425} + \frac{105}{425}$

$=\frac{192+105}{425} = \frac{292}{425}$

$\cos(\alpha+\beta) = \frac{292}{425}$

Question#11:- Prove that: $\frac{cos8^0 - sin8^0}{cos8^0 + sin8^0} = tan37^0$

Solution:- R.H.S= $tan37^0$

We put $37^0 = 45^0 - 8^0$

$\Rightarrow \tan(45^0 - 8^0)$

$= \frac{tan45^0 - tan8^0}{1+tan45^0 tan8^0}$ ∴ tan45=1

$= \frac{(1)-tan8^0}{1+(1)tan8^0}$

$= \frac{1-tan8^0}{1+tan8^0} = \frac{1-\frac{sin8^0}{cos8^0}}{1+\frac{sin8^0}{sin8^0}}$

$= \frac{\frac{cos8^0 - sin8^0}{cos8^0}}{\frac{sin8^0 - sin8^0}{sin8^0}}$ (cancellation)

$= \frac{cos8^0 - sin8^0}{cos8^0 + sin8^0}$ =R.H.S

Question#12:-If α, and γ are the angles of a triangle ABC, sho[w] that $\cot\frac{\alpha}{2} + \cot\frac{\beta}{2} + \cot\frac{\gamma}{2} = \cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$

Solution:- if **α**,**β** and **γ** are the angles of a triangle

⇒ **α**+**β** +**γ** =180

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$= \alpha+\beta = 180-\gamma$

Dividing B.S by 2

$\frac{\alpha+\beta}{2} = \frac{180-\gamma}{2}$

$\frac{\alpha}{2} + \frac{\beta}{2} = 90 - \frac{\gamma}{2}$

Apply tan on both side

$\tan\left(\frac{\alpha}{2} + \frac{\beta}{2}\right) = \tan\left(90 - \frac{\gamma}{2}\right)$

$= \frac{\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}}{1-\tan\frac{\alpha}{2}\tan\frac{\beta}{2}} = \cot\frac{\gamma}{2}$ (put $\tan = \frac{1}{\cot}$)

$= \frac{\frac{1}{\cot\frac{\alpha}{2}}+\frac{1}{\cot\frac{\beta}{2}}}{1-\frac{1}{\cot\frac{\alpha}{2}}\frac{1}{\cot\frac{\beta}{2}}} = \cot\frac{\gamma}{2}$

$= \frac{\frac{\cot\frac{\beta}{2}+\cot\frac{\alpha}{2}}{\cot\frac{\alpha}{2}\cot\frac{\beta}{2}}}{\frac{\cot\frac{\alpha}{2}\cot\frac{\beta}{2}-1}{\cot\frac{\alpha}{2}\cot\frac{\beta}{2}}} = \cot\frac{\gamma}{2}$ (cancellation)

$= \frac{\cot\frac{\beta}{2}+\cot\frac{\alpha}{2}}{\cot\frac{\alpha}{2}\cot\frac{\beta}{2}-1} = \cot\frac{\gamma}{2}$ (by cross multiplication)

$= \cot\frac{\beta}{2} + \cot\frac{\alpha}{2} = \cot\frac{\gamma}{2}\left(\cot\frac{\alpha}{2}\cot\frac{\beta}{2} - 1\right)$

$= \cot\frac{\beta}{2} + \cot\frac{\alpha}{2} = \cot\frac{\gamma}{2}\cot\frac{\alpha}{2}\cot\frac{\beta}{2} - \cot\frac{\gamma}{2}$

$= \cot\frac{\alpha}{2} + \cot\frac{\beta}{2} + \cot\frac{\gamma}{2} = \cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$ (Proved)

Question#13 If $\alpha+\beta+\gamma = 180$,

show that $\cot\alpha\cot\beta + \cot\beta\cot\gamma + \cot\gamma\cot\alpha = 1$

Solution:-

$\alpha+\beta = 180-$ apply tan on both sides

$\tan(\alpha+\beta) = \tan(180-\gamma)$

Apply formula of "tan"

$\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta} = \frac{\tan 180-\tan\gamma}{1+\tan 180\tan\gamma}$ $\therefore \tan 180 = 0$

$\Rightarrow \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta} = -\tan\gamma$ (put $\tan = \frac{1}{\cot}$)

$= \frac{\frac{1}{\cot\alpha}+\frac{1}{\cot\beta}}{1-\frac{1}{\cot\alpha}\frac{1}{\cot\beta}} = -\frac{1}{\cot\gamma}$ (L.C.M)

$= \frac{\frac{\cot\beta+\cot\alpha}{\cot\alpha\cot\beta}}{\frac{\cot\alpha\cot\beta-1}{\cot\alpha\cot\beta}} = -\frac{1}{\cot\gamma}$ (cancellation)

$= \frac{\cot\beta+\cot\alpha}{\cot\alpha\cot\beta-1} = -\frac{1}{\cot\gamma}$ (cross multiplication)

$= \cot\gamma(\cot\beta + \cot\alpha) = -(\cot\alpha\cot\beta - 1)$

$= \cot\gamma\cot\beta + \cot\gamma\cot\alpha = -\cot\alpha\cot\beta + 1$

$= \cot\gamma\cot\beta + \cot\gamma\cot\alpha + \cot\alpha\cot\beta = 1$

OR

$\cot\alpha\cot\beta + \cot\beta\cot\gamma + \cot\gamma\cot\alpha = 1$ (Proved)

Question#14:-Express the following in the form of r sin(θ+∅) where terminal arm of the angle lies in first quadrant

i. $12\sin\theta + 5\cos\theta$

Solution:-

As $r\sin(\theta+\emptyset) = r[\sin\theta\cos\emptyset + \cos\theta\sin\emptyset] \rightarrow (1)$

By comparing we have $\cos\emptyset = 12$ and $\sin\emptyset = 5$

As $r = \sqrt{12^2 + 5^2}$

$\Rightarrow r = \sqrt{169} = +13$ (because of 1st quadrant)...........(2)

Now $\tan\emptyset = \frac{\sin\emptyset}{\cos\emptyset}$

$= \tan\emptyset = \frac{5}{12}$

$\Rightarrow \emptyset = \tan^{-1}\frac{5}{12}$....................(3)

Put (2) and (3) in (1)

$\Rightarrow r\sin(\theta+\emptyset) = 13[\sin\theta\cos\emptyset + \cos\theta\sin\emptyset]$

Where $\emptyset = \tan^{-1}\frac{5}{12}$

ii. $3\sin\theta - 4\cos\theta$

Solution:-

As $r\sin(\theta+\emptyset) = r[\sin\theta\cos\emptyset + \cos\theta\sin\emptyset] \rightarrow (1)$

By comparing we have $\cos\emptyset = 3$ and $\sin\emptyset = -4$

As $r = \sqrt{3^2 + (-4)^2}$

$\Rightarrow r = \sqrt{25} = +5$ (because of 1st quadrant)...........(2)

Now $\tan\emptyset = \frac{\sin\emptyset}{\cos\emptyset}$

$= \tan\emptyset = \frac{-4}{3}$

$\Rightarrow \emptyset = \tan^{-1}\frac{-4}{3}$....................(3)

Put (2) and (3) in (1)

$\Rightarrow r\sin(\theta+\emptyset) = 5[\sin\theta\cos\emptyset + \cos\theta\sin\emptyset]$

Where $\emptyset = \tan^{-1}\frac{5}{12}$

(iii) $\sin\theta - \cos\theta$

solution:

let $1 = r\cos \rightarrow (i)$

$-1 = r\sin\emptyset \rightarrow (ii)$

By $(i)^2 + (ii)^2$

$\Rightarrow (1)^2 + (-1)^2 = r^2\cos^2\emptyset r^2\sin^2\emptyset$

$\Rightarrow 1 + 1 = r^2(\cos^2\emptyset + \sin^2\emptyset)$

$\Rightarrow r^2 = 2 \Rightarrow r = \sqrt{2}$

$by \frac{ii}{i} \Rightarrow \frac{r\sin\emptyset}{r\cos\emptyset} = \frac{-1}{1} \Rightarrow \tan\emptyset = -1$

$\Rightarrow \emptyset = \tan^{-1}(-1)$

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Now

$\boldsymbol{sin\theta - cos\theta = rcos\emptyset sin\theta + rsin\emptyset . cos\theta}$

$\boldsymbol{= r(sin\theta cos\emptyset + cos\theta sin\emptyset)}$

$\boldsymbol{= rsin(\theta + \emptyset) \text{ where } r = \sqrt{2}}$

$\boldsymbol{and\ \emptyset = tan^{-1}(-1)}$

(iv) $\boldsymbol{5sin\theta - 4cos\theta}$

Solution:

$\boldsymbol{let\ 5 = rcos\emptyset \to (i)}$

$\boldsymbol{-4 = rsin\emptyset \to (ii)}$

$\boldsymbol{by\ (i)^2 + (ii)^2}$

$\boldsymbol{\Rightarrow (5)^2 + (-4)^2 = r^2cos^2\emptyset + r^2sin^2\emptyset}$

$\boldsymbol{\Rightarrow 25 + 16 = r^2(cos^2\emptyset + sin^2\emptyset)r^2(1)}$

$\boldsymbol{\Rightarrow r^2 = 41 \Rightarrow r = \sqrt{41}}$

$\boldsymbol{by \frac{(ii)}{(i)} \Rightarrow -\frac{4}{5} = \frac{rsin\emptyset}{rcos\emptyset} \Rightarrow tan\emptyset = -\frac{4}{5}}$

$\boldsymbol{or\quad \emptyset = tan^{-1}\left(-\frac{4}{5}\right)}$

$\boldsymbol{5sin\theta - 4cos\theta = rcos\emptyset sin\theta + rsin\emptyset cos\theta}$

$\boldsymbol{= r(sin\theta cos\emptyset + cos\theta sin\emptyset)}$

$\boldsymbol{= rsin(\theta + \emptyset), \quad where\ r = \sqrt{41}}$

And $\boldsymbol{\emptyset = tan^{-1}\left(-\frac{4}{5}\right)}$

(v) $\boldsymbol{sin\theta + cos\theta}$

Solution:

$let\ 1 = rcos\emptyset \to (i)$

$1 = rsin\emptyset \to (ii)$

$by\ (i)^2 + (ii)^2$

$\Rightarrow (1)^2 + (1)^2 = r^2cos^2\emptyset + r^2sin^2\emptyset$

$\Rightarrow 1 + 1 = r^2(cos^2\emptyset + sin^2\emptyset)r^2(1)$

$\Rightarrow r^2 = 2 \Rightarrow r = \sqrt{2}$

$by \frac{(ii)}{(i)} \Rightarrow \frac{1}{1} = \frac{rsin\emptyset}{rcos\emptyset} \Rightarrow tan\emptyset = 1$

$or\quad \emptyset = tan^{-1}(1)$

Now

$sin\theta + cos\theta = rcos\emptyset sin\theta + rsin\emptyset cos\theta$

$= r(sin\theta cos\emptyset + cos\theta sin\emptyset)$

$= rsin(\theta + \emptyset), \quad where\ r = \sqrt{2}$

And $\emptyset = tan^{-1}(1)$

(vi) $\boldsymbol{3sin\theta - 5cos\theta}$

Solution:

$let\ 3 = rcos\emptyset \to (i)$

$-5 = rsin\emptyset \to (ii)$

$by\ (i)^2 + (ii)^2$

$\Rightarrow (3)^2 + (-5)^2 = r^2cos^2\emptyset + r^2sin^2\emptyset$

$\Rightarrow 9 + 25 = r^2(cos^2\emptyset + sin^2\emptyset)r^2(1)$

$\Rightarrow r^2 = 34 \Rightarrow r = \sqrt{34}$

$by \frac{(ii)}{(i)} \Rightarrow \frac{-5}{3} = \frac{rsin\emptyset}{rcos\emptyset} \Rightarrow tan\emptyset = -\frac{5}{3}$

$or\quad \emptyset = tan^{-1}\left(-\frac{5}{3}\right)$

Now

$3sin\theta - 5cos\theta = rcos\emptyset sin\theta + rsin\emptyset cos\theta$

$= r(sin\theta cos\emptyset + cos\theta sin\emptyset)$

$= rsin(\theta + \emptyset), \quad where\ r = \sqrt{34}$

And $\emptyset = tan^{-1}\left(-\frac{5}{3}\right)$

Double angle identities

i) $\boldsymbol{sin2\alpha = 2sin\alpha cos\alpha}$

we know that

$sin(\alpha + \beta) = sin\alpha cos\beta + cos\alpha sin\beta$

put $\beta = \alpha\quad we\ get$

$sin(\alpha + \alpha) = sin\alpha cos\alpha + cos\alpha sin\alpha$

$\Rightarrow \boldsymbol{sin2\alpha} = 2sin\alpha cos\alpha$

ii) $\boldsymbol{cos2\alpha = 2sin\alpha cos\alpha}$

we know that

$\boldsymbol{cos}(\alpha + \beta) = \boldsymbol{cos}\alpha cos\beta + sin\alpha \boldsymbol{sin}\beta$

put $\beta = \alpha\quad we\ get$

$cos(\alpha + \alpha) = cos\alpha cos\alpha + sin\alpha sin\alpha$

$\Rightarrow cos2\alpha = \cos^2\alpha - \sin^2\alpha$

$\Rightarrow cos2\alpha = cos^2\alpha - (1 - cos^2\alpha)$

$= cos^2\alpha - 1 + cos^2\alpha$

$\Rightarrow cos2\alpha = 2cos^2\alpha - 1$

$\Rightarrow cos2\alpha = 2(1 - sin^2\alpha) - 1$

$= 2 - 2sin^2\alpha - 1$

$\Rightarrow cos2\alpha = 1 - 2sin^2\alpha$

(iii) $\boldsymbol{tan2\alpha = \frac{2tan\alpha}{1-\tan^2\alpha}}$

we know that

$\tan(\alpha + \beta) = \frac{tan\alpha + tan\beta}{1 - tan\alpha tan\beta}$

Put $\beta = \alpha,\ we\ get$

$\Rightarrow \tan(\alpha + \beta) = \frac{tan\alpha + tan\alpha}{1 - tan\alpha tan\alpha}$

$\Rightarrow \tan(\alpha + \beta) = \frac{2tan\alpha}{1 - \tan^2\alpha}$

Half angle identities

i $\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+cos\alpha}{2}}$

we know that

$\boldsymbol{cos2\alpha = 2cos^2\alpha - 1}$

Similarly

$\boldsymbol{cos\alpha = 2cos^2\frac{\alpha}{2} - 1\quad in\ form\ half\ angle}$

$\boldsymbol{\Rightarrow\ 2cos^2\frac{\alpha}{2} = 1 + cos\alpha}$

$\boldsymbol{\Rightarrow cos^2\frac{\alpha}{2} = \frac{1 + cos\alpha}{2}}$

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$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$

$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$

We know that

$\boldsymbol{\cos 2\alpha = 1 - 2\sin^2\alpha}$

Similarly

$\boldsymbol{\cos\alpha = 1 - 2\sin^2\frac{\alpha}{2}}$ ***in form half angle***

$\Rightarrow \boldsymbol{2\sin^2\frac{\alpha}{2} = 1 - \cos\alpha}$

$\Rightarrow \boldsymbol{\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2}}$

$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$

iii $\boldsymbol{\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}}$

We know that

$\boldsymbol{\tan\alpha = \frac{\sin\alpha}{\cos\beta}}$

Similarly

$\boldsymbol{\tan\frac{\alpha}{2} = \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}}$ ***(inform of half angle)***

$\Rightarrow \boldsymbol{\tan\frac{\alpha}{2} = \pm\frac{\sqrt{\frac{1-\cos\alpha}{2}}}{\sqrt{\frac{1+\cos\alpha}{2}}}}$

$\Rightarrow \boldsymbol{\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}}$

Triple angle Identities

i $\boldsymbol{\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha}$

$\boldsymbol{L.H.S} = \sin 3\alpha$

$= \sin(2\alpha + \alpha)$

$= 2\sin\alpha\cos\alpha + \cos 2\alpha\sin\alpha$

$= 2\sin\alpha\cos\alpha\cos\alpha + (1 - 2\sin^2\alpha)\sin\alpha$

$= 2\sin\alpha\cos^2\alpha + \sin\alpha - 2\sin^3\alpha$

$= 2\sin\alpha(1 - \sin^2\alpha) + \sin\alpha - 2\sin^3\alpha$

$= 2\sin\alpha - 2\sin^3\alpha + \sin\alpha - 2\sin^3\alpha$

$= 3\sin\alpha - 4\sin^3\alpha = R.H.S$

Hence $\boldsymbol{\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha}$

ii $\boldsymbol{\cos 3\alpha} = 4\cos^3\alpha - 3\cos\alpha$

L.H.S= $\boldsymbol{\cos 3\alpha}$

$= \boldsymbol{\cos}(2\alpha + \alpha)$

$= \boldsymbol{\cos} 2\alpha\cos\alpha - \sin 2\alpha\sin\alpha$

$(2\boldsymbol{\cos^2}\alpha - 1)\cos\alpha - 2\sin\alpha\cos\alpha\sin\alpha$

$= 2\boldsymbol{\cos^3}\alpha - \cos\alpha - 2\sin^2\alpha\cos\alpha$

$= 2\boldsymbol{\cos^3}\alpha - \cos\alpha - 2(1 - \cos^2\alpha)\boldsymbol{\cos\alpha}$

$= 2\boldsymbol{\cos^3}\alpha - \cos\alpha - 2\cos\alpha + 2\cos^3\alpha$

$= 4\boldsymbol{\cos^3}\alpha - 3\cos\alpha = R.H.S$

iii $\tan 3\alpha = \frac{3\tan\alpha - \tan^3\alpha}{1 - 3\tan^2\alpha}$

$L.H.S = \tan 3\alpha$

$= \tan(2\alpha + \alpha)$

$= \frac{\boldsymbol{\tan 2}\alpha + \tan\alpha}{\boldsymbol{1 - \tan 2}\alpha\tan\alpha}$

$= \frac{\frac{2\tan\alpha}{1-\tan^2\alpha} + \tan\alpha}{1 - \frac{2\tan\alpha}{1-\tan^2\alpha}.\tan\alpha}$

$= \frac{\frac{2\tan\alpha + \tan\alpha - \tan^3\alpha}{1-\tan^2\alpha}}{\frac{1-\tan^2\alpha - 2\tan^2\alpha}{1-\tan^2\alpha}}$

$= \frac{3\tan\alpha - \tan^3\alpha}{1 - 3\tan^2\alpha} = \boldsymbol{R.H.S}$

Hence

$\tan 3\alpha = \frac{3\tan\alpha - \tan^3\alpha}{1 - 3\tan^2\alpha}$

Exercise 10.3

1. Find the values of sin2α, cos2α and tan2α, when where 0<α<$\frac{\pi}{2}$

i. Sinα=$\frac{12}{13}$

Solution : - To find all the above values we first calculate cosα by using sin $\boldsymbol{\alpha}$

$\Rightarrow \boldsymbol{\cos^2\alpha = 1 - \sin^2\alpha}$

$= \boldsymbol{\cos^2\alpha = 1 - (\frac{12}{13})^2}$

$= \boldsymbol{\cos^2\alpha = 1 - \frac{144}{169}}$

$= \boldsymbol{\cos^2\alpha = \frac{169-144}{169}}$

$= \boldsymbol{\cos^2\alpha = \frac{25}{169}}$ **(taking square root)**

⇒ cos α =+$\frac{5}{13}$(+is because of first quadrant)

Now sin2α=2sin αcos α =2$\left(\frac{12}{13}\right)\left(\frac{5}{13}\right)$

sin2α =$\frac{120}{169}$

⇒cos2α=$\cos^2\alpha - \sin^2\alpha$

cos2α=$\left(\frac{5}{13}\right)^2 - \left(\frac{12}{13}\right)^2$

cos2α=$\frac{25}{169} - \frac{144}{169}$

cos2α=$\frac{25-144}{169}$

cos2α=$\frac{-119}{169}$

⇒tan2α=$\frac{\sin 2\alpha}{\cos 2\alpha} = \frac{\frac{120}{169}}{\frac{-119}{169}}$

tan2α=$-\frac{120}{119}$

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ii. $\cos\alpha = \frac{3}{5}$

(Same as above)

Prove the following identities

2. $\cot\alpha - \tan\alpha = 2\cot2\alpha$

L .H.S = $\cot\alpha - \tan\alpha$

$\frac{\cos\alpha}{\sin\alpha} - \frac{\sin\alpha}{\cos\alpha}$

$= \frac{\cos^2\alpha - \sin^2\alpha}{\sin\alpha\cos\alpha}$

$= \frac{\cos2\alpha}{\sin\alpha\cos\alpha}$ (× and ÷ by 2)

$= \frac{2\cos2\alpha}{2\sin\alpha\cos\alpha} = \frac{2\cos2\alpha}{\sin2\alpha}$

$= 2\cot2\alpha$ = R.H.S

3. $\frac{\sin2\alpha}{1+\cos2\alpha} = \tan\alpha$

L .H.S $= \frac{\sin2\alpha}{1+\cos2\alpha}$

$= \frac{2\sin\alpha\cos\alpha}{1+2\cos^2\alpha-1}$

$= \frac{2\sin\alpha\cos\alpha}{2\cos^2\alpha}$

$= \frac{\sin\alpha}{\cos\alpha} = \tan\alpha$ = R.H.S

4. $\frac{1-\cos\alpha}{\sin\alpha} = \tan\frac{\alpha}{2}$

L.H.S $= \frac{2\sin^2\frac{\alpha}{2}}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}$ (cancellation)

$= \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}$

$= \tan\frac{\alpha}{2}$ R.H.S

5. $\frac{\cos\alpha-\sin\alpha}{\cos\alpha+\sin\alpha} = \sec2\alpha - \tan2\alpha$

L.H.S $= \frac{\cos\alpha-\sin\alpha}{\cos\alpha+\sin\alpha}$

$= \frac{\cos\alpha-\sin\alpha}{\cos\alpha+\sin\alpha} \times \frac{\cos\alpha-\sin\alpha}{\cos\alpha-\sin\alpha}$

$= \frac{(\cos\alpha-\sin\alpha)^2}{\cos^2\alpha-\sin^2\alpha}$

$= \frac{\cos^2\alpha+\sin^2\alpha+2\sin\alpha\cos\alpha}{\cos2\alpha}$ $\quad \therefore \cos^2\alpha + \sin^2\alpha = 1$

$= \frac{1+\sin2\alpha}{\cos2\alpha} = \frac{1}{\cos2\alpha} + \frac{\sin2\alpha}{\cos2\alpha}$

$= \sec2\alpha - \tan2\alpha$ = R.H.S

6. $\sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}} = \frac{\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}-\cos\frac{\alpha}{2}}$

$L.H.S = \sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}}$

$= \sqrt{\frac{\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}-2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}}$ $\quad \therefore \cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2} = 1$

$\sqrt{\frac{\left(\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\right)^2}{\left(\sin\frac{\alpha}{2}-\cos\frac{\alpha}{2}\right)^2}}$ $\quad \therefore \sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$

$= \frac{\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}-\cos\frac{\alpha}{2}}$ R.H.S

7. $\frac{\operatorname{cosec}\theta+\operatorname{cosec}2\theta}{\sec\theta} = \cot\frac{\theta}{2}$

$L.H.S = \frac{\operatorname{cosec}\theta+\operatorname{cosec}2\theta}{\sec\theta}$

$= \frac{\frac{1}{\sin\theta}+\frac{2}{\sin2\theta}}{\frac{1}{\cos\theta}}$

$= \left(\frac{1}{\sin\theta} + \frac{2}{\sin2\theta}\right)\cos\theta$

$= \left(\frac{1}{\sin\theta} + \frac{2}{2\sin\theta\cos\theta}\right)\cos\theta$

$= \left(\frac{1}{\sin\theta} + \frac{1}{\sin\theta\cos\theta}\right)\cos\theta$

$= \left(\frac{\cos\theta+1}{\sin\theta\cos\theta}\right)\cos\theta$

$= \frac{\cos\theta+1}{\sin\theta}$

$= \frac{2\cos^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}$

$= \cot\frac{\theta}{2}$ R.H.S

8. $1+\tan\alpha\tan2\alpha = \sec2\alpha$

L.H.S $= 1+\tan\alpha\tan2\alpha$

$= 1+\frac{\sin\alpha\ \sin2\alpha}{\cos\alpha\ \cos2\alpha}$

$= \frac{\cos2\alpha\cos\alpha+\sin2\alpha\ \sin\alpha}{\cos\alpha\ \cos2\alpha}$

$= \frac{\cos(2\alpha-\alpha)}{\cos\alpha\cos2\alpha} = \frac{\cos\alpha}{\cos\alpha\cos2\alpha}$

$= \frac{1}{\cos2\alpha} = \sec2$ R.H.S

9. $\frac{2\sin\theta\sin2\theta}{\cos\theta+\cos3\theta} = \tan2\theta\tan\theta$

L.H.S $= \frac{2\sin\theta\sin2\theta}{\cos\theta+\cos3\theta}$

$= \frac{2\sin\theta\sin2\theta}{\cos3\theta+\cos\theta}$ (using formula from 10.4)

$= \frac{2\sin\theta\sin2\theta}{2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)}$

$= \frac{2\sin\theta\sin2\theta}{2\cos\left(\frac{4\theta}{2}\right)\cos\left(\frac{2\theta}{2}\right)}$

$= \frac{2\sin\theta\sin2\theta}{2\cos2\theta\cos\theta} = \frac{\sin2\theta}{\cos2\theta}\ \frac{\sin\theta}{\cos\theta}$

$= \tan2\theta\tan\theta$ = R.H.S

10. $\frac{\sin3\theta}{\sin\theta} - \frac{\cos3\theta}{\cos\theta} = 2$

L.H.S $= \frac{\sin3\theta}{\sin\theta} - \frac{\cos3\theta}{\cos\theta}$

$= \frac{\sin3\theta\cos\theta-\cos3\theta\sin\theta}{\sin\theta\cos\theta}$

$= \frac{\sin(3\theta-\theta)}{\sin\theta\cos\theta} = \frac{\sin(2\theta)}{\sin\theta\cos\theta}$

$= \frac{2\sin\theta\ \cos\theta}{\sin\theta\cos\theta}$ (cancellation)

$= 2$ R.H.S

11. $\frac{\cos3\theta}{\cos\theta} + \frac{\sin3\theta}{\sin\theta} = 4\cos2\theta$

$L.H.S = \frac{\cos3\theta}{\cos\theta} + \frac{\sin3\theta}{\sin\theta}$

$= \frac{\cos3\theta\sin\theta+\sin3\theta\cos\theta}{\cos\theta\sin\theta}$

$= \frac{\sin(3\theta+\theta)}{\cos\theta\sin\theta} = \frac{\sin4\theta}{\cos\theta\sin\theta}$ (× and ÷ by 2)

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$=\frac{2\sin4\theta}{2cos\theta sin\theta}$

$=\frac{2\sin4\theta}{sin2\theta}$ $\therefore \sin4\theta = 2\sin2\theta\cos2\theta$

$=\frac{2.\ 2\sin2\theta\cos2\theta}{sin2\theta}$ (cancellation)

$= 4\cos2\theta$ R.H.S

12. $\frac{tan\frac{\theta}{2}+cot\frac{\theta}{2}}{cot\frac{\theta}{2}-tan\frac{\theta}{2}} = \sec\theta$

$L.H.S = \frac{tan\frac{\theta}{2}+cot\frac{\theta}{2}}{cot\frac{\theta}{2}-tan\frac{\theta}{2}}$

$\frac{\frac{sin\frac{\theta}{2}}{cos\frac{\theta}{2}}+\frac{cos\frac{\theta}{2}}{sin\frac{\theta}{2}}}{\frac{cos\frac{\theta}{2}}{sin\frac{\theta}{2}}-\frac{sin\frac{\theta}{2}}{cos\frac{\theta}{2}}} = \frac{\frac{\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}}{cos\frac{\theta}{2}sin\frac{\theta}{2}}}{\frac{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}}{cos\frac{\theta}{2}sin\frac{\theta}{2}}}$

$=\frac{\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}}$ $\therefore \sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2} = 1$

$=\frac{1}{cos\theta} = \sec\theta$

13. $\frac{sin3\theta}{cos\theta}+\frac{cos3\theta}{sin\theta} = 2\cot2\theta$

$L.H.S = \frac{sin3\theta}{cos\theta}+\frac{cos3\theta}{sin\theta}$

$=\frac{sin3\theta sin\theta+cos3\theta cos\theta}{cos\theta sin\theta}$

$=\frac{\cos(3\theta-\theta)}{cos\theta sin\theta} = \frac{\cos2\theta}{cos\theta sin\theta}$ (× and ÷ by 2)

$=\frac{2\cos2\theta}{2cos\theta sin\theta}$

$=\frac{2\cos2\theta}{sin2\theta}$

$=\frac{2\cos2\theta}{sin2\theta}$ (cancellation)

$= 2\cot2\theta$ R.H.S

14. Reduce $sin^4\theta$ to an expression involving function of θ raised to the first power.

Solution:- $(\sin^2\theta)^2$

$=\left(\frac{1-cos2\theta}{2}\right)^2$

$=\frac{1-2cos2\theta+cos^2 2\theta}{4}$

$=\frac{1}{4}(1-2cos2\theta+cos^2 2\theta)$

$=\frac{1}{4}\left(1-2cos2\theta+\frac{1+cos4\theta}{2}\right)$

$=\frac{1}{4}\left(\frac{2-4cos2\theta+1+cos4\theta}{2}\right)$

$=\frac{1}{8}(2-4cos2\theta+1+cos4\theta)$

$=\frac{1}{8}(3-4cos2\theta+cos4\theta)$

Question No.15

Find the value of $sin\theta$ and $cos\theta$ without using table or calculator where θ

$(i)\ 18^0$ $(ii)\ 36^0$ $(iii)\ 54^0$ $(iv)\ 72^0$

Hence prove that

$cos36^0cos72^0cos108^0cos144^0 = \frac{1}{16}$

Hint

$$\left(\begin{array}{c} let\ \theta = 18^0 \\ \Rightarrow 5\theta = 90^0 \\ 3\theta+2\theta = 90^0 \\ 3\theta+2\theta = 90^0 \\ 3\theta = 90^0-2\theta \\ let\ \theta = 36^0 \\ 5\theta = 180^0 \quad \Rightarrow 3\theta+2\theta = 180^0 \\ sin3\theta = sin(180^0-2\theta) e.t.c \end{array}\right)$$

Solution:

$(i)\ 18^0$

$let\ \theta = 18^0$

$\Rightarrow 5\theta = 90^0$

$\Rightarrow 2\theta+3\theta = 90^0$

$\Rightarrow 2\theta = 90^0-3\theta$

$\Rightarrow sin2\theta = sin(90-3\theta)$

$2sin\theta cos\theta = 4cos^3\theta-3cos\theta$

$2sin\theta cos\theta = cos\theta(4cos^2\theta-3)$

$2sin\theta = 4(1-sin^2\theta)$

$= 4-4sin^2\theta-3$

$2sin\theta = 1-4sin^2\theta$

$\Rightarrow 4sin^2\theta+2sin\theta-1 = 0$

Here $a = 4,\ b = 2,\ c = -1$

$$sin\theta = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$sin\theta = \frac{-2\pm\sqrt{(2)^2-4(4)(-1)}}{2(4)}$$

$$sin\theta = \frac{-2\pm\sqrt{4+16}}{8}$$

$$sin\theta = \frac{-2\pm\sqrt{20}}{8} = \frac{-2\pm2\sqrt{5}}{8}$$

$$sin\theta = \frac{-1\pm\sqrt{5}}{4}$$

$put\ \theta = 18^0\ so$

$$sin\theta = \frac{-1+\sqrt{5}}{4} (\because 18^0 lies\ in\ I\ quad.)$$

$$\because\ cos^2\theta = (1-sin^2\theta)$$

$$cos^2\theta = 1-\left(\frac{-1+\sqrt{5}}{4}\right)^2 = 1-\left(\frac{\sqrt{5}-1}{4}\right)^2$$

$$cos^2\theta = 1-\left(\frac{5+1-2\sqrt{5}}{16}\right) = \frac{16-6+2\sqrt{5}}{16}$$

$$cos^2\theta = \frac{10+2\sqrt{5}}{16}$$

$$cos\theta = \pm\sqrt{\frac{10+2\sqrt{5}}{16}}$$

$$\Rightarrow cos\theta = \pm\frac{\sqrt{10+2\sqrt{5}}}{4}$$

$$\Rightarrow cos18^0 = \pm\frac{\sqrt{10+2\sqrt{5}}}{4} \quad (\because 18^0\ lies\ in\ I\ quad)$$

(ii)

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36^0

Let $\theta = 36^0$

$$cos2\theta = 2cos^2\theta - 1$$

Put $\theta = 18^0 \Rightarrow cos2(18^0) = 2cos^2 18^0 - 1$

$$\Rightarrow cos36^0 = 2(cos18^0) = 2cos^2 18^0 - 1$$

$$\Rightarrow cos36^0 = 2(cos18^0)^2 - 1$$

$$= 2\left(\frac{\sqrt{10+2\sqrt{5}}}{4}\right)^2 - 1$$

$$cos36^0 = 2\left(\frac{\sqrt{10+2\sqrt{5}}}{4}\right)^2 - 1$$

$$cos36^0 = 2\left(\frac{10+2\sqrt{5}}{16}\right) - 1$$

$$cos36^0 = \frac{10+2\sqrt{5}}{8} - 1 = \frac{10+2\sqrt{5}-8}{8}$$

$$cos36^0 = \frac{2+2\sqrt{5}}{8} = \frac{2(1+\sqrt{5})}{8}$$

$$\Rightarrow cos36^0 = \frac{1+\sqrt{5}}{4}$$

$$\because \quad sin^2\theta = 1 - cos^2\theta$$

$$\Rightarrow sin^2 36^0 = 1 - cos^2 36^0$$

$$= 1 - (cos36^0)^2$$

$$= 1 - (cos36^0)^2$$

$$= 1 - \left(\frac{1+\sqrt{5}}{4}\right)^2$$

$$= 1 - \frac{1+5+2\sqrt{5}}{16}$$

$$sin^2 36^0 = \frac{16-6-2\sqrt{5}}{16}$$

$$sin^2 36^0 = \frac{10-2\sqrt{5}}{16}$$

$$\Rightarrow sin36^0 = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

(iii) 54^0

Let $\theta = 54^0$

$$let\ cos54^0$$

$$\because \quad cos54^0 = cos(90^0 - 54^0)$$

$$cos54^0 = sin36^0 = \frac{10-2\sqrt{5}}{4}$$

$$cos54^0 = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

$$\because \quad sin54^0 = sin(90^0 - 54^0)$$

$$= cos36^0 = \frac{1+\sqrt{5}}{4}$$

$$\Rightarrow sin54^0 = 1 + \frac{\sqrt{1+\sqrt{5}}}{4}$$

(iv) 72^0

$$let\ \theta = 72^0$$

$$\because \quad sin72^0 = sin(90^0 - 18^0)$$

$$= cos18^0 = \frac{\sqrt{10+2\sqrt{2}}}{4}$$

$$\Rightarrow sin72^0 = \frac{10+2\sqrt{2}}{4}$$

$$cos72^0 = cos(90^0 - 18^0)$$

$$= sin18^0 = \frac{-1+\sqrt{5}}{4}$$

$$\Rightarrow cos72^0 = \frac{-1+\sqrt{5}}{4}$$

Now

$$L.H.S = cos36^0 cos72^0 cos108^0 cos144^0$$

$$= cos36^0 cos72^0 cos(180^0 - 72^0) cos(180^0 - 36^0)$$

$$= cos36^0 cos72^0 (-cos72^0)(-cos36^0)$$

$$= cos^2 36^0 cos^2 72^0$$

$$= \left(\frac{1+\sqrt{5}}{4}\right)^2 \left(\frac{\sqrt{5}-1}{4}\right)^2$$

$$= \left(\frac{1+5+2\sqrt{5}}{16}\right)\left(\frac{5+1-2\sqrt{5}}{16}\right)$$

$$= \left(\frac{6+2\sqrt{5}}{16}\right)\left(\frac{6-2\sqrt{5}}{16}\right)$$

$$= \frac{(6)^2 - (2\sqrt{5})^2}{(16)^2}$$

$$= \frac{36-4(5)}{16\times16} = \frac{36-20}{16\times16}$$

$$= \frac{16}{16\times16} = \frac{1}{16} = R.H.S$$

Sum, difference and products of sine and cosines

We know that

$$sin\ (\alpha+\beta) = sin\alpha cos\beta + cos\alpha sin\beta \rightarrow (i)$$

$$sin(\alpha-\beta) = sin\alpha cos\beta - cos\alpha sin\beta \rightarrow (ii)$$

$$cos(\alpha+\beta) = cos\alpha cos\beta - sin\alpha sin\beta \rightarrow (iii)$$

$$cos(\alpha-\beta) = cos\alpha cos\beta - sin\alpha sin\beta \rightarrow (iv)$$

$eq(i) + eq(ii) \Rightarrow$

$sin\ (\alpha+\beta) + sin\ (\alpha-\beta) = 2sin\alpha cos\beta$

$eq(i) - eq(ii) \Rightarrow$

$sin\ (\alpha+\beta) - sin\ (\alpha-\beta) = 2cos\alpha cos\beta$

$by(iii) + (iv) \Rightarrow$

$$cos(\alpha+\beta) + cos(\alpha-\beta) = 2cos\alpha cos\beta$$

$by(iii) - (iv) \Rightarrow$

$cos(\alpha+\beta) - cos(\alpha-\beta) = -2sin\alpha sin\beta$

So we get four identities as

$$sin\ (\alpha+\beta) + sin\ (\alpha-\beta) = 2sin\alpha cos\beta$$

$$sin\ (\alpha+\beta) - sin\ (\alpha-\beta) = 2cos\alpha cos\beta$$

$$cos(\alpha+\beta) + cos(\alpha-\beta) = 2cos\alpha cos\beta$$

$$cos(\alpha+\beta) - cos(\alpha-\beta) = -2sin\alpha sin\beta$$

Now put $\alpha + \beta = p \rightarrow (1)$

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And $\alpha - \beta = Q \to (2)$

By (1)+(2) $\Rightarrow 2\alpha = P + Q$

$$\Rightarrow \alpha = \frac{P+Q}{2}$$

$$by\ (1)-(2) \Rightarrow 2\beta = P - Q \Rightarrow \beta = \frac{P-Q}{2}$$

Note:

$$sinP + sinQ = 2sin\frac{P+Q}{2}\ cos\frac{P-Q}{2}$$

$$sinP - sinQ = 2cos\frac{P+Q}{2}\ sin\frac{P-Q}{2}$$

$$cosP + cosQ = 2cos\frac{P+Q}{2}\ cos\frac{P-Q}{2}$$

$$cosP - cosQ = -2sin\frac{P+Q}{2}\ sin\frac{P-Q}{2}$$

Exercise 10.4

1. **Express the following product as sum or differences:**

i. 2 sin3θ cosθ

Solution:- $2\sin3\theta\cos\theta$

∴ using formula

$=\sin(3\theta+\theta) + \sin(3\theta-\theta)$

$=\sin4\theta + \sin2\theta$

ii. 2 cos5θ sin3θ

Solution:- 2 cos5θ sin3θ

∴ using formula

$=\sin(5\theta+3\theta) - \sin(5\theta-3\theta)$

$=\sin8\theta - \sin2\theta$

iii. Sin5θ cos2θ

Solution:- Sin5θ cos2θ

$=\frac{1}{2}[2\ \text{Sin}5\theta\cos2\theta]$

∴ using formula

$=\frac{1}{2}[\text{Sin}(5\theta+2\theta)+\sin(5\theta-2\theta)]$

$=\frac{1}{2}[\text{Sin}7\theta+\sin3\theta]$

iv. 2 sin7θ sin2θ

Solution:- 2 sin7θ sin2θ

$=-[-2\ \text{Sin}5\theta\sin2\theta]$ ∴ using formula

$=-[\cos(7\theta+2\theta)-\sin(7\theta-2\theta)]$

$=-[\text{Sin}9\theta-\sin5\theta]$

v. Cos(x+y) sin(x-y)

Solution:- $\frac{1}{2}[2\text{Cos}(x+y)\sin(x-y)]$

∴ using formula

$=\frac{1}{2}[\sin(x+y+x-y) - \sin(x+y-x+y)]$

$=\frac{1}{2}[\sin(x+x) - \sin(y+y)]$

$=\frac{1}{2}[\sin(2x) - \sin(2y)]$

vi. Cos(2x+30) cos(2x-30)

Solution:- $\frac{1}{2}[2\text{Cos}(2x+30)\cos(2x-30)]$

using formula

$=\frac{1}{2}\begin{bmatrix}\text{Cos}(2x+30+2x-30)+\\ \cos(2x+30-2x+30)\end{bmatrix}$

$=\frac{1}{2}[\text{Cos}(2x+2x)+\cos(30+30)]$

$=\frac{1}{2}[\text{Cos}(4x)+\cos(60)]$

vii. Sin 12 sin46

Solution: $-\frac{1}{2}[-2\sin(12)\sin(46)]$

∴ using formula

$=-\frac{1}{2}[\cos(12+46)-\cos(12-46)]$

$=-\frac{1}{2}[\cos(58)-\cos(-34)]$

∴ cos(-α)=cos α

$=-\frac{1}{2}[\cos(58)-\cos(34)]$

viii. Sin (x+45) sin(x-45)

Solution: $-\frac{1}{2}[-2\sin(x+45)\sin(x-45)]$

∴ using formula

$=-\frac{1}{2}\begin{bmatrix}\cos(x+45+x-45)-\\ \cos(x+45-x+45)\end{bmatrix}$

$=-\frac{1}{2}[\cos(x+x)-\cos(45+45)]$

$=-\frac{1}{2}[\cos(2x)-\cos(90)]$

2. **Express the following product as sum or differences:**

i. Sin5 θ + sin 3θ

Solution:- Sin5 θ +sin 3θ

$=2\sin\left(\frac{5\theta+3\theta}{2}\right)\text{Cos}\left(\frac{5\theta-3\theta}{2}\right)$

$=2\sin\left(\frac{8\theta}{2}\right)\text{Cos}\left(\frac{2\theta}{2}\right)$

$=2\sin(4\theta)\ \text{Cos}(\theta)$

ii. Sin8θ – sin4θ

Solution:- Sin8θ – sin4θ

$=2\cos\left(\frac{8\theta+4\theta}{2}\right)\sin\left(\frac{8\theta-4\theta}{2}\right)$

$=2\cos\left(\frac{12\theta}{2}\right)\sin\left(\frac{4\theta}{2}\right)$

$=2\cos(6\theta)\sin(2\theta)$

iii. Cos6θ + cos3θ

Solution:- Cos6θ + cos3θ

$=2\cos\left(\frac{6\theta+3\theta}{2}\right)\text{Cos}\left(\frac{6\theta-3\theta}{2}\right)$

$=2\cos\left(\frac{9\theta}{2}\right)\text{Cos}\left(\frac{3\theta}{2}\right)$

iv. Cos7 θ – cos θ

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Solution:- Cos7 θ – cos θ

$=-2\sin\left(\frac{7\theta-\theta}{2}\right)\sin\left(\frac{7\theta+\theta}{2}\right)$

$=-2\sin\left(\frac{6\theta}{2}\right)\sin\left(\frac{8\theta}{2}\right)$

$=-2\sin(3\theta)\sin(4\theta)$

v. Cos12 + cos 48

Soltion:- Cos12 + cos 48

$=2\cos\left(\frac{12+48}{2}\right)\cos\left(\frac{12-48}{2}\right)$

$=2\cos\left(\frac{60}{2}\right)\cos\left(\frac{-36}{2}\right)$

$=2\cos(30)\cos(-18)$ ∴ cos(-α)=cos α

$=2\cos(30)\cos(18)$

vi. Sin(x+30)+sin(x-30)

Solution:- Sin(x+30)+sin(x-30)

$=2\sin\left(\frac{x+30+x-30}{2}\right)\cos\left(\frac{x+30-x+30}{2}\right)$

$=2\sin\left(\frac{x+x}{2}\right)\cos\left(\frac{30+30}{2}\right)$

$=2\sin\left(\frac{x+x}{2}\right)\cos\left(\frac{30+30}{2}\right)$

$=2\sin\left(\frac{2x}{2}\right)\cos\left(\frac{60}{2}\right)$

$=2\sin(x)\cos(30)^0$

3. Prove the following identities.

i. $\frac{\sin 3x-\sin x}{\cos x-\cos 3x}=\cot 2x$

$L.H.S$:- $\frac{\sin 3x-\sin x}{\cos x-\cos 3x}$

$=\frac{2\cos\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)}{-2\sin\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}$

$=\frac{2\cos\left(\frac{4x}{2}\right)\sin\left(\frac{2x}{2}\right)}{-2\sin\left(\frac{4x}{2}\right)\sin\left(\frac{-2x}{2}\right)}$

$=\frac{2\cos(2x)\sin(x)}{-2\sin(2x)\sin(-x)}$ ∴ sin(-α)=−sin α

$=\frac{2\cos(2x)\sin(x)}{2\sin(2x)\sin(x)}$

$=\frac{\cos 2x}{\sin 2x}$

$=\cot 2x$ R.H.S

ii. $\frac{\sin 8x+\sin 2x}{\cos 8x+\cos 2x}=\tan 5x$

L.H.S = $\frac{2\sin\left(\frac{8x+2x}{2}\right)\cos\left(\frac{8x-2x}{2}\right)}{2\cos\left(\frac{8x+2x}{2}\right)\cos\left(\frac{8x-2x}{2}\right)}$

$=\frac{2\sin\left(\frac{10x}{2}\right)\cos\left(\frac{6x}{2}\right)}{2\cos\left(\frac{10x}{2}\right)\cos\left(\frac{6x}{2}\right)}$

$=\frac{2\sin(5x)\cos(3x)}{2\cos(5x)\cos(3x)}$ (cancellation)

$=\frac{\sin 5x}{\cos 5x}$

$=\tan 5x$ R.H.S

iii. $\frac{\sin\alpha-\sin\beta}{\sin\alpha+\sin\beta}=\tan\frac{\alpha-\beta}{2}\cot\frac{\alpha+\beta}{2}$

Solution:- $\frac{\sin\alpha-\sin\beta}{\sin\alpha+\sin\beta}$

$=\frac{2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}{2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}$

$=\frac{\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}{\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}$

$=\frac{\cos\left(\frac{\alpha+\beta}{2}\right)}{\sin\left(\frac{\alpha+\beta}{2}\right)}\frac{\sin\left(\frac{\alpha-\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}$

$=\tan\frac{\alpha-\beta}{2}\cot\frac{\alpha+\beta}{2}$ =R.H.S

4. Prove that:

i. Cos20 + cos100 + cos140 = 0

L.H.S= Cos20 + cos100 + cos140 (re-arranging)

= Cos140 + cos100 + cos20

$=\cos 140+2\cos\left(\frac{100+20}{2}\right)\cos\left(\frac{100-20}{2}\right)$

$=\cos 140+2\cos\left(\frac{120}{2}\right)\cos\left(\frac{80}{2}\right)$

$=\cos 140+2\cos(60)\cos(40)$

$=\cos 140+2\left(\frac{1}{2}\right)\cos(40)$

$=\cos 140+\cos 40$

$=2\cos\left(\frac{140+40}{2}\right)\cos\left(\frac{140-40}{2}\right)$

$=2\cos\left(\frac{180}{2}\right)\cos\left(\frac{120}{2}\right)$

$=2\cos(90)\cos(60)$

$=2\ (0)\cos 60$

=0 =R.H.S

ii. $\sin\left(\frac{\pi}{4}-\theta\right)\sin\left(\frac{\pi}{4}+\theta\right)=\frac{1}{2}\cos 2\theta$

Solution:- $\sin\left(\frac{\pi}{4}-\theta\right)\sin\left(\frac{\pi}{4}+\theta\right)$

$=\left(\sin\frac{\pi}{4}\cos\theta-\cos\frac{\pi}{4}\sin\theta\right)\left(\sin\frac{\pi}{4}\cos\theta+\cos\frac{\pi}{4}\sin\theta\right)$

$=\left(\left(\frac{1}{\sqrt{2}}\right)\cos\theta-\left(\frac{1}{\sqrt{2}}\right)\sin\theta\right)\left(\left(\frac{1}{\sqrt{2}}\right)\cos\theta+\left(\frac{1}{\sqrt{2}}\right)\sin\theta\right)$

$=\left(\frac{1}{\sqrt{2}}\cos\theta\right)^2-\left(\frac{1}{\sqrt{2}}\sin\theta\right)^2$

$=\left(\frac{1}{\sqrt{2}}\cos\theta\right)^2-\left(\frac{1}{\sqrt{2}}\sin\theta\right)^2$

$=\frac{1}{2}(\cos^2\theta-\sin^2\theta)$

$=\frac{1}{2}\cos 2\theta$

iii. $\frac{\sin\theta+\sin 3\theta+\cos 5\theta+\sin 7\theta}{\cos\theta+\cos 3\theta+\cos 5\theta+\cos 7\theta}=\tan 4\theta$

$$L.H.S=\frac{\sin\theta+\sin 3\theta+\cos 5\theta+\sin 7\theta}{\cos\theta+\cos 3\theta+\cos 5\theta+\cos 7\theta}$$

$$=\frac{\left[2\sin\frac{\theta+3\theta}{2}\cos\frac{\theta-3\theta}{2}\right]+\left[2\sin\frac{5\theta+7\theta}{2}\cos\frac{5\theta-7\theta}{2}\right]}{\left[2\cos\frac{\theta+3\theta}{2}\cos\frac{\theta-3\theta}{2}\right]+\left[2\cos\frac{5\theta+7\theta}{2}\cos\frac{5\theta-7\theta}{2}\right]}$$

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$$=\frac{\left[2sin\frac{4\theta}{2}cos\left(-\frac{2\theta}{2}\right)\right]+\left[2sin\frac{12\theta}{2}cos\left(-\frac{2\theta}{2}\right)\right]}{\left[2cos\frac{4\theta}{2}cos\left(-\frac{2\theta}{2}\right)\right]+\left[2cos\frac{12\theta}{2}cos\frac{-2\theta}{2}\right]}$$

$$=\frac{2sin2\theta cos(-\theta)\_2sin6\theta cos(-\theta)}{2cos2\theta cos(-\theta)+2cos6\theta cos(-\theta)}$$

$$=\frac{2sin2\theta cos\theta+2sin6\theta cos(-\theta)}{2cos2\theta cos(-\theta)+2cos6\theta cos(-\theta)}$$

$$=\frac{2cos(sin2\theta+sin\theta)}{2cos\theta(cos2\theta+cos60)}$$

$$=\frac{sin2\theta+sin6\theta}{cos2\theta+cos6\theta}$$

$$=\frac{2sin\frac{2\theta+6\theta}{2}cos\frac{2\theta-6\theta}{2}}{2cos\frac{2\theta+6\theta}{2}cos\frac{2\theta-6\theta}{2}}$$

$$=\frac{2sin\left(\frac{8\theta}{2}\right)cos\left(-\frac{4\theta}{2}\right)}{2cos\left(\frac{8\theta}{2}\right)cos\left(-\frac{4\theta}{2}\right)}$$

$$=\frac{sin4\theta}{cos4\theta}=tan4\theta=R.H.S$$

Hence proved.

5. Prove that:

i. $Cos20^0\ cos40^0\ cos60^0\ cos80^0=\frac{1}{16}$

Solution:

$$L.H.S=Cos20^0\ cos40^0\ cos60^0\ cos80^0$$
$$=cos20^0cos40^0\left(\frac{1}{2}\right)cos80^0$$
$$=\frac{1}{2}(cos20^0cos40^0)cos80^0$$
$$=\frac{1}{4}(2cos20^0cos40^0)cos80^0$$
$$=\frac{1}{4}[cos(20^0+40^0)+co(20^0-40^0)]cos80^0$$
$$=\frac{1}{4}[cos60^0+cos(-20)^0]cos80^0$$
$$=\frac{1}{4}\left[\frac{1}{2}+cos20^0\right]cos80^0$$
$$=\frac{1}{8}cos80^0+\frac{1}{4}cos20^0cos80^0$$
$$=\frac{1}{8}cos80^0+\frac{1}{8}(2cos20^0cos80^0)$$
$$=\frac{1}{8}cos80^0+\frac{1}{8}[cos(20^0+80^0)+cos(20^0-80^0)]$$
$$=\frac{1}{8}cos80^0+\frac{1}{8}[cos100^0+cos60^0]$$
$$=\frac{1}{8}cos80^0+\frac{1}{8}cos100^0+\frac{1}{8}\left(\frac{1}{2}\right)$$
$$=\frac{1}{8}cos80^0+\frac{1}{8}cos(180^0-80^0)+\frac{1}{16}$$
$$=\frac{1}{8}cos80^0-\frac{1}{8}cos80^0+\frac{1}{16}$$
$$=\frac{1}{16}R.H.S\ \because\ \ cos(180^0-\theta)=-cos\theta$$

Hence proved.

ii. $\sin\frac{\pi}{9}\sin\frac{2\pi}{9}\sin\frac{\pi}{3}\sin\frac{4\pi}{9}=\frac{3}{16}$

solution:

$$L.H.S=\sin\frac{\pi}{9}\sin\frac{2\pi}{9}\sin\frac{\pi}{3}\sin\frac{4\pi}{9}$$
$$=sin20^0sin40^0sin60^0sin80^0$$
$$=sin20^0sin40^0\left(\frac{\sqrt{3}}{2}\right)sin80^0$$
$$=\frac{\sqrt{3}}{2}(sin20^0sin40^0)sin80^0$$
$$=-\frac{\sqrt{3}}{2}[cos(20^0+40^0)-cos(20^0-40^0)]sin80^0$$
$$=-\frac{\sqrt{3}}{4}(cos60^0-cos(-20^0)sin80^0$$
$$=-\frac{\sqrt{3}}{4}\left(\frac{1}{2}-cos20^0\right)sin80^0$$
$$=\left(-\frac{\sqrt{3}}{8}+\frac{\sqrt{3}}{4}cos20^0\right)sin80^0$$
$$=-\frac{\sqrt{3}}{8}sin80^0+\frac{\sqrt{3}}{4}cos20^0sin80^0$$
$$=-\frac{\sqrt{3}}{8}sin80^0+\frac{\sqrt{3}}{8}(2cos20^0sin80^0)$$
$$=-\frac{\sqrt{3}}{8}sin80^0+\frac{\sqrt{3}}{8}(sin(20^0+80^0)-sin(20^0-80^0))$$
$$=-\frac{\sqrt{3}}{8}sin80^0+\frac{\sqrt{3}}{8}(sin100^0-sin(-60^0))$$
$$=-\frac{\sqrt{3}}{8}sin180^0+\frac{\sqrt{3}}{8}(sin100^0+sin60^0)$$
$$=-\frac{\sqrt{3}}{8}sin80^0+\frac{\sqrt{3}}{8}sin100^0+\frac{\sqrt{3}}{8}sin60^0$$
$$=-\frac{\sqrt{3}}{8}sin80^0+\frac{\sqrt{3}}{8}sin(180^0-80^0)+\frac{\sqrt{3}}{8}\left(\frac{\sqrt{3}}{2}\right)$$
$$=-\frac{\sqrt{3}}{8}sin80^0+\frac{\sqrt{3}}{8}sin80^0+\frac{3}{16}$$
$$=\frac{3}{16}=R.H.S$$

iii. $sin10^0\ sin30^0\ sin50^0\ sin70^0=\frac{1}{16}$

solution:

$$L.H.S=sin10^0\ sin30^0\ sin50^0\ sin70^0$$
$$=sin10^0\left(\frac{1}{2}\right)sin50^0sin70^0$$
$$=\frac{1}{2}(sin10^0sin50^0)sin70^0$$
$$=-\frac{1}{4}(-2sin10^0sin50^0)sin70^0$$
$$=-\frac{1}{4}\big(cos(10^0+50^0)-cos(10^0-50^0)\big)sin70^0$$
$$=-\frac{1}{4}\big(cos60^0-cos(-40^0)\big)sin70^0$$
$$=-\frac{1}{4}\left(\frac{1}{2}-cos40^0\right)sin70^0$$
$$=\left(-\frac{1}{8}+\frac{1}{4}cos40^0\right)sin70^0$$
$$=-\frac{1}{8}sin70^0+\frac{1}{4}cos40^0sin70^0$$
$$=-\frac{1}{8}sin70^0+\frac{1}{8}(2cos40^0sin70^0)$$
$$=-\frac{1}{8}sin70^0+\frac{1}{8}(sin(40^0+70^0)-sin(40^0-70^0))$$

$$= -\frac{1}{8}sin70^0 + \frac{1}{8}\left(sin110^0 - sin(-30^0)\right)$$

$$= -\frac{1}{8}sin70^0 + \frac{1}{8}sin110^0 + \frac{1}{8}sin30^0$$

$$= -\frac{1}{8}sin70^0 + \frac{1}{8}sin(180^0 - 70^0) + \frac{1}{8}sin30^0$$

$$= -\frac{1}{8}sin70^0 + \frac{1}{8}sin70^0 + \frac{1}{8}\left(\frac{1}{2}\right)$$

$$= \frac{1}{16} = R.H.S$$

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