SHORT QUESTIONS

7.1 Name two characteristics of simple harmonic motion.

Ans. Two characteristics of simple Harmonic Motion are given as:

(i) Acceleration of a vibrating body is directly proportional to the displacement and is always directed towards the mean position i.e.,

$$a \propto -x$$

(ii) Total energy of the particle executing simple harmonic motion remains conserved.

$$E_{total} = K.E + P.E = \text{Constant}$$

(iii) Simple harmonic motion can be represented by function of sine or cosine in the form of equation i.e.,

$$x = x_o \sin(\omega t + \phi)$$

and $$x = x_o \cos(\omega t + \phi)$$

where ϕ is a measure of phase.

7.2 Does frequency depend on amplitude for harmonic oscillators?

Ans. No, the frequency of oscillator is independent of the amplitude of oscillator.

(i) In case of mass-spring system, the frequency of mass is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

According to this relation, the frequency of oscillator depends upon mass and spring constant but it does not depend upon the amplitude of oscillator.

(ii) In case of simple pendulum, the frequency of the harmonic oscillator is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

This relation shows that the frequency does not depend upon the amplitude but it depends upon the length of pendulum and acceleration due to gravity.

7.3 Can we realize an ideal simple pendulum?

Ans. No we cannot realize an ideal simple pendulum because ideal simple pendulum consists of heavy but small mass suspended from a frictionless support by means of an inextensible string. As there conditions are impossible to attain therefore we cannot realize an ideal simple pendulum.

7.4 What is the total distance traveled by an object moving with SHM in a time equal to its period, if its amplitude is A?

Ans. As we know that time period of a simple harmonic motion is the time required to complete one vibration. If A is the amplitude of vibration then the distance travelled by an object in a time equal to its period is 4A.

7.5 What happens to the period of simple pendulum if its length is doubled? What happens if the suspended mass is doubled?

Ans. We know that the time period of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Let T′ be the time period of simple pendulum when length becomes double.

i.e., $l' = 2l$

Then $T' = 2\pi\sqrt{\frac{2l}{g}}$

$$= 2\pi\sqrt{\frac{l}{g}} \times \sqrt{2}$$

$$T' = \sqrt{2}\,T$$

$$= 1.41\ T$$

If the length of pendulum is doubled then its time period increases by 1.41 times the original time period.

Mass: If mass of pendulum is doubled, there is no change in time period because it is independent of mass.

7.6 Does the acceleration of a simple harmonic oscillator remain constant during its motion? Is the acceleration ever zero? Explain.

Ans. No, the acceleration of a simple harmonic oscillator does not remain constant. The acceleration of harmonic oscillator varies with displacement because:

$$a = -\omega^2 x$$

where ω^2 is constant

So $a = -\text{Constant} \times x$

$$a \propto -x$$

This shows that acceleration is directly proportional to displacement.

As displacement is changing during motion, therefore acceleration is also changing.

The acceleration will be zero at the mean position i.e., $x = 0$.

So, $a = -\omega^2(0)$

$$a = 0$$

7.7 What is meant by phase angle? Does it define angle between maximum displacement and the driving force?

Ans. **Phase Angle:** The angle which specifies the displacement as well as the direction of motion of the point executing SHM is called phase angle.

The phase gives the information about the state of motion of the vibrating point. We can get the waveform of SHM by applying the concept of phase.

The phase angle does not define angle between maximum displacement and the driving force.

7.8 Under what conditions does the addition of two simple harmonic motions produce resultant, which is also simple harmonic?

Ans. In order to produce resultant SHM by the addition of two simple harmonic motions following conditions must be required:

(i) Two SHMs are parallel i.e., in same direction.

(ii) Two SHMs are in phase.

(iii) Two SHMs vibrate with same frequency.

7.9 Show that in SHM, the acceleration is zero when the velocity is greatest and the velocity is zero when the acceleration is greatest?

Ans. As the velocity and acceleration of a SHM are

$$a = -\omega^2 x \quad \text{and} \quad v = \omega\sqrt{x_o^2 - x^2}$$

At the mean position x = 0

Therefore;

$$a = -\omega^2(0) \quad \text{and} \quad v = \omega x_o = \text{Maximum value}$$

$$a = 0$$

At the extreme position $x = x_o$. Therefore

$$a = -\omega^2(x_o) \quad \text{and} \quad v = \omega\sqrt{x_o^2 - x_o^2}$$

$$a = -\omega^2 x_o \quad \text{and} \quad v = 0 = \text{Minimum value}$$

Thus it is clear that in SHM, acceleration is zero when then velocity is greatest and the velocity is zero when the acceleration is greatest.

7.10 In relation to SHM, explain the equations:

(i) $y = A\sin(\omega t + \varphi)$ (ii) $a = -\omega^2 x$

Ans. (i) $y = A\sin(\omega t + \phi)$

(ii) $a = -\omega^2 x$

Here, y = Instantaneous displacement

A = Amplitude

ϕ = Initial phase

ωt = Angle subtended in time 't'

This equation represents instantaneous the acceleration of an object executing SHM in which "a" represents acceleration, "ω" is the angular frequency and x represents its instantaneous displacement.

7.11 Explain the relation between total energy, potential energy and kinetic energy for body oscillating with SHM.

Ans. According to law of conservation of energy, the total energy of a body executing SHM remains constant. The K.E is maximum at the mean position and zero at the extreme position while the potential energy is maximum at the extreme position and zero at the mean position.

7.12 Describe some common phenomena in which resonance plays an important role.

Ans. The phenomenon resonance plays a very important role in:

(i) Musical instrument.

(ii) Producing electrical resonance in radio set with transmission of a particular radio frequency.

(iii) **In microwave oven:** The waves produced in this type of oven have a wavelength of 12 cm at a frequency of 2450 MHz.

7.13 If a mass spring system is hung vertically and set into oscillations, why does the motion eventually stop?

Ans. If a mass spring system is hung vertically and set into oscillations, the motion eventually stops due to air resistance and friction. Because of these frictional forces, energy is dissipated into heat and the system does not oscillate.

PROBLEMS WITH SOLUTIONS

PROBLEM 7.1

A 100.0 g body hung on a spring elongates the spring by 4.0 cm. When a certain object is hung on the spring and set vibrating, its period is 0.568 s. What is the mass of the object pulling the spring?

Data

Mass of body	$= m_1$	$= 100\ \text{g}$
		$= 0.1\ \text{kg}$
Extension in the string	$= x$	$= 4.0\ \text{cm}$
		$= 0.04\ \text{m}$
Time period	$= T$	$= 0.568\ \text{sec.}$
Amplitude	$= x_o$	$= 10\ \text{cm}$
		$= 0.1\ \text{m}$

To Find

Mass of object pulling the spring $= m_2 = ?$

SOLUTION

By using the formula

$$T = 2\pi\sqrt{\frac{m_2}{K}}$$

But for the value of K

$$F = Kx \quad \text{and} \quad F = m_1 g$$

$$K = \frac{F}{x} = \frac{m_1 g}{x}$$

$$K = \frac{0.1 \times 9.8}{0.04}$$

$$= 24.5\ \text{N/m}$$

Therefore; $T = 2\pi\sqrt{\dfrac{m_2}{K}}$

Squaring

$$T^2 = 4\pi^2 \frac{m_2}{K}$$

$$\boxed{m_2 = \frac{T^2 \times K}{4\pi^2}}$$

Putting the values

$$m_2 = \frac{(0.568)^2 \times 24.5}{4(3.14)^2}$$

$$m_2 = 0.200 \text{ kg}$$

$$m_2 = 200 \text{ g}$$

Result

Mass of object pulling the spring = m_2 = 200 g

PROBLEM 7.2

A load of 15.0 g elongates a spring by 2.00 cm. If body of mass 294 g is attached to the spring and is set into vibration with an amplitude of 10.0 cm, what will be its (i) period (ii) spring constant (iii) maximum speed of its vibration.

Data

Mass elongates = m_1 = 15.0 g
= 0.015 kg

Extension in the spring = x = 2.00 cm
= 0.02 m

Mass attached with spring = m_2 = 294 g
= 0.294 kg

To Find

(i) Time period = T = ?

(ii) Spring constant = K = ?

(iii) Maximum speed = v_o = ?

SOLUTION

(i) For time period

$$T = 2\pi\sqrt{\frac{m_2}{K}}$$

But $K = \frac{m_1 g}{x}$

$$= \frac{0.015 \times 9.8}{0.02}$$

$$= 7.35 \text{ N/m}$$

Therefore; $T = 2(3.14)\sqrt{\frac{0.294}{7.35}}$

$$T = 1.256 \text{ sec.}$$

(ii) Spring constant $= K = \frac{F}{x} = \frac{m_1 g}{x}$

$$K = \frac{0.015 \times 9.8}{0.02}$$

$$= 7.35 \text{ N/m}$$

(iii) For maximum speed

$$v_o = x_o \sqrt{\frac{K}{m_2}}$$

$$= 0.1 \sqrt{\frac{7.35}{0.294}}$$

$$= 0.5 \text{ m/s}$$

$$v_o = 50 \text{ cm/s}$$

Result

(i) Time period $= T = 1.256$ sec.

(ii) Spring constant $= K = 7.35$ N/m

(iii) Maximum speed $= v_o = 50$ cm/s

PROBLEM 7.3

An 8.0 kg body executes SHM with amplitude 30 cm. The restoring force is 60 N when the displacement is 30 cm. Find

(i) Period

(ii) Acceleration, speed, kinetic energy and potential energy when the displacement is 12 cm.

Data

Mass of body $= m = 8.0$ kg

Amplitude $= x_o = 30$ cm $= 0.3$ m

Restoring force $= F = 60$ N

Displacement $= x = 30$ cm $= 0.3$ m

To Find

(i) Time period $= T = ?$

(ii) Acceleration $= a = ?$

Speed $= v = ?$

Kinetic energy $= K.E = ?$

Potential energy $= P.E = ?$

When

Displacement $= x_1 = 12$ cm $= 0.12$ m

SOLUTION

(i) For time period

$$T = 2\pi\sqrt{\frac{m}{K}}$$

As $K = \frac{F}{x}$

$$K = \frac{60}{0.3}$$

$$= 200 \text{ N/m}$$

So $T = 2(3.14)\sqrt{\frac{8.0}{200}}$

$$= 1.256 \text{ sec.}$$

$$T = 1.3 \text{ sec.}$$

(ii) For acceleration

$$a = -\frac{K}{m}x$$

$$= -\frac{200}{8.0} \times 0.12$$

$$a = 3 \text{ m/s}^2$$

For speed

$$v = \omega\sqrt{x_o^2 - x_1^2}$$

But $\omega = \frac{2\pi}{T}$

$$= \frac{2(3.14)}{1.3}$$

$$= 4.83 \text{ rad/s}$$

Therefore; $v = 4.83\sqrt{(0.3)^2 - (0.12)^2}$

$$v = 1.33 \text{ m/s}$$

For kinetic energy

$$K.E = \frac{1}{2}K(x_o^2 - x_1^2)$$

$$= \frac{1}{2} \times 200\,(0.3^2 - 0.12^2)$$

$$= 100(0.0756)$$

$$= 7.56 \text{ J}$$

For potential energy

$$P.E = \frac{1}{2}Kx_1^2$$

$$= \frac{1}{2} \times 200(0.12)^2$$

$$= 1.44\ J$$

Result

(i)	Time period	=	T	=	1.3 sec.
(ii)	Acceleration	=	a	=	3 m/s^2
	Speed	=	v	=	1.33 m/s
	Kinetic energy	=	K.E	=	7.56 J
	Potential energy	=	P.E	=	1.44 J

PROBLEM 7.4

A block of mass 4.0 kg is dropped from a height of 0.80 m on to a spring of spring constant K = 1960 Nm^{-1}. Find the maximum distance through which the spring will be compressed.

Data

Mass of block	=	m	=	4.0 kg
Height	=	h	=	0.80 m
Spring constant	=	K	=	1960 N/m

To Find

Maximum distance = x_o = ?

SOLUTION

By formula

$$P.E = \frac{1}{2}Kx_o^2$$

$$\boxed{x_o^2 = \frac{2P.E}{K}} \qquad \ldots\ldots \text{(i)}$$

But $P.E = mgh$

$$= 4.0 \times 9.8 \times 0.80$$

$$= 31.36\ J$$

Putting in eq. (i)

So $$x_o^2 = \frac{2 \times 31.36}{1960}$$

$$x_o = 0.178$$

$$= 0.18\ m$$

Result

Maximum distance = x_o = 0.18 m

= 18 cm

PROBLEM 7.5

A simple pendulum is 50.0 cm long. What will be its frequency of vibration at a place where g = 9.8 ms^{-2}?

Data

Length of simple pendulum = l = 50.0 cm

= 0.50 m

Value of g = 9.8 m/s^2

To Find

Frequency of vibration = f = ?

SOLUTION

By formula

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T = 2(3.14)\sqrt{\frac{0.5}{9.8}}$$

$$T = 1.41 \text{ sec.}$$

But $$f = \frac{1}{T}$$

$$= \frac{1}{1.41}$$

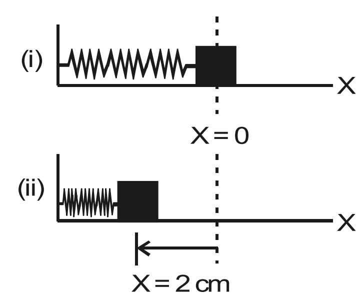
$$f = 0.70 \text{ Hz}$$

Result

Frequency of vibration = f = 0.70 Hz

PROBLEM 7.6

A block of mass 1.6 kg is attached to a spring with spring constant 1000 Nm^{-1}, as shown in Fig. The spring is compressed through a distance of 2.0 cm and the block is released from rest. Calculate the velocity of the block as it passes through the equilibrium position, x = 0, if the surface is frictionless.



Data

Mass of block = m = 1.6 kg

Spring constant = K = 1000 N/m

Maximum displacement = x_o = 2.0 cm

= 0.02 m

To Find

Velocity of the block = v_o = ?

SOLUTION

By formula

$$v_o = x_o\sqrt{\frac{K}{m}}$$

$$= 0.02\sqrt{\frac{100}{1.6}}$$

$$v_o = 0.50 \text{ m/s}$$

Result

Velocity of block = v_o = 0.50 m/s

PROBLEM 7.7

A car of mass 1300 kg is constructed using a frame supported by four springs. Each spring has a spring constant 20,000 Nm^{-1}. If two people riding in the car have a combined mass of 160 kg, find the frequency of vibration of the car, when it is driven over a pot hole in the road. Assume the weight is evenly distributed.

Data

Mass of car = m_1 = 1300 kg

Spring constant for each = K = 20,000 N/m

Mass of persons = m_2 = 160 kg

Spring constant for 4 springs = 4 × 20,000 = 80,000 N/m

To Find

Frequency of vibration = f = ?

SOLUTION

By formula

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

But $m = m_1 + m_2$

$$= 1300 + 160$$

$$= 1460 \text{ kg}$$

Therefore

$$f = \frac{1}{2(3.14)}\sqrt{\frac{80000}{1460}}$$

$$f = 1.18 \text{ Hz}$$

Result

Frequency of vibration = f = 1.18 Hz

PROBLEM 7.8

Find the amplitude, frequency and period of an object vibrating at the end of a spring, if the equation for its position, as a function of time, is

$$x = 0.25 \cos\left(\frac{\pi}{8}\right) t$$

What is the displacement of the object after 2.0 s?

Data

The given equation is

$$x = 0.25 \cos\left(\frac{\pi}{8}\right) t$$

Time = t = 2.0 sec.

To Find

Amplitude = x_o = ?

Frequency = f = ?

Time period = T = ?

SOLUTION

As we know that the equation for SHM is

$$x = x_o \cos \omega t \qquad \ldots\ldots \text{(i)}$$

and the given equation is

$$x = 0.25 \cos\left(\frac{\pi}{8}\right) t \qquad \ldots\ldots \text{(ii)}$$

Comparing the eq. (i) and (ii)

$$x_o = 0.25$$

So

Amplitude = x_o = 0.25 m

$$\omega = \frac{\pi}{8}$$

But

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\frac{\pi}{8}}$$

$$T = 2\pi \times \frac{8}{\pi}$$

Time period = T = 16 sec.

$$\text{Frequency} = f = \frac{1}{T}$$

$$f = \frac{1}{16}\ \text{Hz}$$

For displacement after 2 sec. is

$$x = 0.25 \cos \frac{\pi}{8} t$$

$$= 0.25 \cos \left(\frac{\pi}{8} \times 2\right)$$

$$= 0.25 \cos \left(\frac{\pi}{4}\right)$$

$$= 0.25 \times 0.707$$

$$= 0.18\ \text{m}$$

Result

Amplitude = x_o = 0.25 m

Time period = T = 16 sec.

Frequency = f = $\frac{1}{16}$ Hz

Displacement = x = 0.18 m