SHORT QUESTIONS

5.1 Explain the difference between tangential velocity and the angular velocity, if one of these is given for a wheel of known radius, how will you find the other?

Ans. **Tangential Velocity:** The tangential velocity v_T is the linear velocity of a particles moving along a curve or a circle directed along the tangent at any point on the curve.

Angular Velocity: The angular velocity ω is the rate of change of angular displacement moving along a curved path.

Relation: The tangential velocity v_T, angular velocity ω and the radius r of the wheel are related by the relation

$$v_T = r\omega$$

If one of these is given for a wheel of known radius r then the other one can be find by using the above relation.

5.2 Explain what is meant by centripetal force and why it must be furnished to an object if the object to follow a circular path?

Ans. The force which is required to keep the body moving in a circular path and directed towards the centre of the circular path is called centripetal force. It is denoted by F_c and mathematically, it can be written as

$$F_c = \frac{mv^2}{r}$$

For uniform circular motion, it must be under the influence of a force which changes the direction of motion continuously. So the centripetal force is always needed if a body is to maintain its circular motion.

5.3 What is meant by moment of inertia? Explain its significance.

Ans. **Moment of Inertia:** It is defined as the product of mass of particle and square of its perpendicular distance from axis of rotation. It is denoted by I. Mathematically it can be written as

$$I = mr^2$$

Where m is the mass of the particle and r is the perpendicular distance of particle from the axis of rotation.

Significance: The moment of inertia plays the same role in angular motion as the mass in linear motion. As mass of a body is called measure of inertia but moment of inertia is the property of a body which resists to change its uniform circular motion.

5.4 What is meant by angular momentum? Explain the law of conservation of angular momentum.

Ans. **Angular Momentum:** The angular momentum of an object is defined as the cross product of position vector $\vec{r}$ with respect to the axis of rotation and the linear momentum $\vec{P}$. It is denoted by $\vec{L}$. Mathematically

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$$\vec{L} = \vec{r} \times \vec{P}$$

$$= rP \sin\theta \, \hat{n}$$

It is a vector quantity so the direction of angular momentum $\vec{L}$ is perpendicular to the plane contained by $\vec{r}$ and $\vec{P}$.

Law of Conservation of Angular Momentum: This law states that if no external torque acts on a system, the total angular momentum of a system remains constant.

Mathematically

$$\vec{L} = \vec{L_1} + \vec{L_2} + \vec{L_3} \ \ldots\ldots = \text{Constant}$$

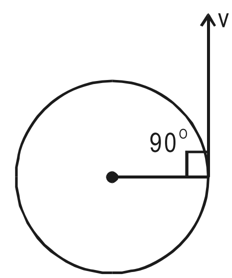
5.5 Show that orbital angular momentum $L_o = mvr$.

Ans. From the definition of angular momentum

$$\vec{L_o} = \vec{r} \times \vec{P}$$

$$L_o = rP \sin\theta = mvr \sin\theta$$

where θ is the angle between position vector $\vec{r}$ and velocity $\vec{v}$. In case of circular orbital motion, the angle between radius and tangential velocity is 90°. Hence



$$L_o = rP \sin\theta = mvr \sin 90° \quad \text{where} \quad \sin 90° = 1$$

$$L_o = mrv \ (1)$$

Hence $$L_o = mvr$$

5.6 Describe what should be the minimum velocity, for a satellite, to orbit close to the Earth around it.

Ans. When a satellite is moving in a circular orbit, it has an acceleration $a_c = \frac{v^2}{r}$ which is supplied by gravity. The low flying earth satellite have an acceleration $a_c = g = 9.8 \text{ m/s}^2$. Thus

$$g = \frac{v^2}{r}$$

$$v^2 = gr$$

$$v = \sqrt{gr}$$

where r is the radius of earth = R = 6400 km.

So $$v = \sqrt{9.8 \times 6400}$$

$$= 7.9 \text{ km/s}$$

This is the minimum velocity necessary to put a satellite into orbit around the earth.

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5.7 State the direction of the following vectors in simple situations; angular momentum and angular velocity.

Ans. **Direction of Angular Momentum:** As we know that the angular momentum is always directed perpendicular to the plane containing $\vec{v}$ and $\vec{r}$ according to right hand rule.

$$\vec{L} = \vec{r} \times \vec{P} = (\vec{r} \times \vec{v})m$$

Direction of Angular Velocity: The direction of angular velocity is taken along the axis of rotation given by right hand rule. If rotation is along the curl of the fingers of right hand, then the outward thumb gives the direction of angular velocity. According to this rule, for counter clockwise rotation, the direction of angular velocity is outward along the axis of rotation.

5.8 Explain why an object, orbiting the Earth, is said to be freely falling. Use you explanation to point out why objects appear weightless under certain circumstances.

Ans. An object such as an artificial satellite orbits the Earth due to force of gravity. Its centripetal acceleration is equal to the acceleration due to gravity directed towards the centre of the Earth. Hence, a satellite is always falling towards the centre of Earth with acceleration equal to "g" and is a freely falling object. It does not hit the ground because of its tangential velocity and curvature of the Earth.

In the fame of reference of a freely falling object the bodies appears weightless because both, the body inside and the frame of reference, are falling with the same acceleration. The relative acceleration of inside body with respect to its frame of reference is zero. Hence, it seems that no force is acting on the body and when it is suspended to a spring balance, the spring balance shows its weight as zero.

5.9 When mud flies off the tyre of a moving bicycle, in what direction does it fly? Explain.

Ans. The mud flies off the tyre of a moving bicycle along the tangent as it leaves the path of circular motion. Actually the mud is sticked with the surface of tyre due to adhesive force (the attraction force existing between the molecules of different surfaces). This adhesive force provides the necessary centripetal force to the mud to move in a circular path. If the speed of rotation is increased then at a certain instant this adhesive force becomes unable to meet the demand of centripetal force and hence, it will no longer move in circular path. As, in the circular path mud has linear velocity along the tangent at every point so when it leaves its circular path it moves along tangent at that point.

5.10 A disc and a hoop start moving down from the top of an inclined plane at the same time. Which one will be moving faster on reaching the bottom?

Ans. The velocity of the disc on reaching the bottom of the inclined plane is given as

$$v_{Disc} = \sqrt{\frac{4gh}{3}} = \sqrt{\frac{4}{3}} \times \sqrt{gh}$$

and for a hoop, the velocity of hoop on reaching the bottom of the inclined plane is given as

$$v_{Hoop} = \sqrt{gh}$$

$$v_{Disc} = \sqrt{\frac{4}{3}} \times \sqrt{gh}$$

$$v_{Disc} = \sqrt{\frac{4}{3}}\, V_{Hoop}$$

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As $\sqrt{\frac{4}{3}} > 1$

$\therefore$ $v_{Disc} > V_{Hoop}$

From these relations, it is clear that the velocity of disc is greater than the velocity of hoop so the disc moves faster as compared to hoop.

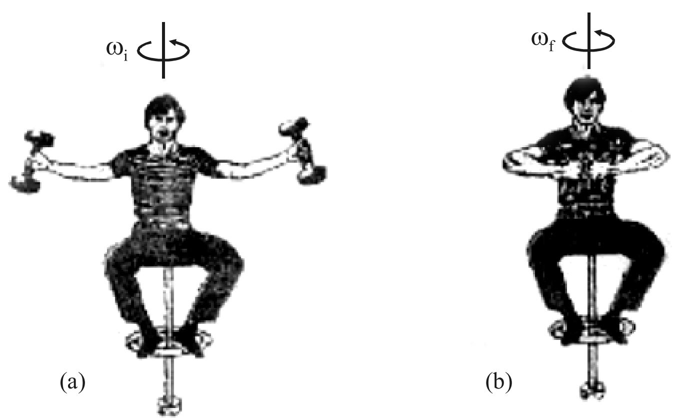
5.11 Why does a diver change his body position before diving in the pool?

Ans. Before lifting off the diving board, the diver's legs and arms are fully extended which means that the diver has a large moment of inertia I_1 about an axis. The moment of inertia is considerably reduced to a new value I_2, when the legs and arms are drawn into closed tuck position. As angular momentum is conserved, so

$$I_1\omega_1 = I_2\omega_2$$

where ω_1 and ω_2 are the angular speeds before and after diving. Hence the diver spins faster when its moment of inertia becomes smaller.

5.12 A student holds two dumb-bells without stretched arms while sitting on a turn table. He is given a push until he is rotating at certain angular velocity. The student then pulls the dumbbell towards his chest (Fig. 5.25). What will be the effect on rate of rotation?



Ans. By using the law of conservation of angular momentum

$$I_1\omega_1 = I_2\omega_2$$

With out-stretched arms, the moment of inertia is I_1 and his angular speed is ω_1. Their product $I_1\omega_1$ is equal to angular moment which remains constant. When he pulls the dumbbells towards his chest, his moment of inertia decrease and he spins faster as ω_2 increases to keep the product $I_2\omega_2$ constant.

5.13 Explain how many minimum number of geo-stationary satellites are required for global coverage of T.V transmission?

Ans. A satellite communication system can be set up by placing several geo stationary satellites in orbit over different point on the surface of the Earth. One such satellite covers 120^o of longitude, so that whole of the populated Earth's surface can be covered by three correctly positioned satellites.

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PROBLEMS WITH SOLUTIONS

PROBLEM 5.1

A tiny laser beam is directed from the Earth to the Moon. If the beam is to have a diameter of 2.50 m at the Moon, how small must divergence angle be for the beam? The distance of Moon from the Earth is 3.8×10^8m.

Data

Diameter of beam = Length of arc = S = 2.50 m

Distance of moon from the earth = r = 3.8×10^8 m

To Find

Divergence angle = θ = ?

SOLUTION

By formula

$$S = r\theta$$

$$\boxed{\theta = \frac{S}{r}}$$

$$\theta = \frac{2.50}{3.8 \times 10^8}$$

$$= 0.657 \times 10^{-8}$$

$$= 6.6 \times 10^{-9} \text{ rad}$$

Result

Divergence angle = θ = 6.6×10^{-9} rad

PROBLEM 5.2

A gramophone record turntable accelerate from rest to an angular velocity of 45.0 rev min^{-1} in 1.60s. What is its average angular acceleration?

Data

Initial angular velocity = ω_i = 0

Final angular velocity = ω_f = 45.0 rev/min.

$$= 45 \times \frac{2\pi}{60} \text{ rad/s}$$

$$= 1.5\pi \text{ rad/s}$$

Time interval = Δt = 1.60 sec.

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To Find

Angular acceleration $= \alpha = ?$

SOLUTION

By formula

$$\boxed{\alpha = \frac{\omega_f - \omega_i}{\Delta t}}$$

$$= \frac{1.5\pi - 0}{1.6}$$

$$= \frac{1.5 \times 3.14}{1.6}$$

$$\alpha = 2.9 \text{ rad/s}^2$$

Result

Angular acceleration $= \alpha = 2.9 \text{ rad/s}^2$

PROBLEM 5.3

A body of moment of inertia I = 0.80 kg m^2 about a fixed axis, rotates with a constant angular velocity of 100 rad s^{-1}. Calculate its angular momentum L and the torque to sustain this motion.

Data

Moment of inertia $= I = 0.80 \text{ kgm}^2$

Constant angular velocity $= \omega = 100 \text{ rad/s}$

To Find

Angular momentum $= \vec{L} = ?$

Torque $= \vec{\tau} = ?$

SOLUTION

For angular momentum

$$\boxed{L = I\omega}$$

$$L = 0.80 \times 100$$

$$= 80 \text{ kgm}^2/\text{s}^2 \qquad \text{As} \quad \text{kgm}^2/\text{s}^2 = \text{J.s}$$

So $L = 80 \text{ J.s}$

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For torque to sustain its motion

$$\boxed{\tau = I \times \alpha}$$

Since the body is moving with constant angular velocity so $\alpha = 0$

$$\tau = I \times 0$$

$$\tau = 0$$

Result

Angular momentum $= \vec{L} = 80$ J.s

Torque $= \vec{\tau} = 0$

PROBLEM 5.4

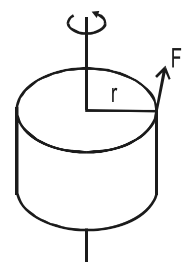

Fig. 5.26

Consider the rotating cylinder shown in Fig. 5.26. Suppose that m = 5.0 kg, F = 0.60 N and r = 0.20 m. Calculate (a) the torque acting on the cylinder, (b) the angular acceleration of the cylinder. (Moment of inertia of cylinder = $\frac{1}{2}mr^2$)

Data

Mass of cylinder $= m = 5.0$ kg

Force acting on cylinder $= F = 0.60$ N

Radius of cylinder $= r = 0.20$ m

To Find

(a) Torque acting on the cylinder $= \tau = ?$

(b) Angular acceleration $= \alpha = ?$

SOLUTION

(a) For torque acting on the cylinder

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = rF \sin\theta \qquad \text{But} \qquad \theta = 90°$$

$$\boxed{\tau = rF}$$

$$= 0.20 \times 0.60$$

$$= 0.12 \text{ Nm}$$

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(b) For angular acceleration

$$\tau = I\alpha$$

$$\alpha = \frac{\tau}{I} \qquad \text{But} \qquad I = \frac{1}{2}mr^2$$

$$\alpha = \frac{\tau}{\frac{1}{2}mr^2}$$

$$\boxed{\alpha = \frac{2\tau}{mr^2}}$$

Putting the values

$$\alpha = \frac{2 \times 0.12}{5.0 \times (0.20)^2}$$

$$= 1.2 \text{ rad/s}^2$$

Result

(a) Torque acting on the cylinder = τ = 0.12 N.m

(b) Angular acceleration = α = 1.2 rad/s^2

PROBLEM 5.5

Calculate the angular momentum of a star of mass 2.0 × 10^{30} kg and radius 7.0 × 10^{5}km. If it makes one complete rotation about its axis once in 20 days, what is its kinetic energy?

Data

Mass of star = M = 2.0 × 10^{30} kg

Radius of star = R = 7.0 × 10^5 km

= 7.0 × 10^8 m

Time period of star = T = 20 days

= 20 × 24 × 3600

= 1728000 sec.

To Find

Angular momentum = $\vec{L}$ = ?

Kinetic energy = $(K.E)_{rot}$ = ?

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SOLUTION

By using formula

$$L = I\omega \qquad \text{But} \qquad I = \frac{2}{5}MR^2$$

$$\text{and} \qquad \omega = \frac{2\pi}{T}$$

So

$$\boxed{L = \frac{2\pi}{T} \times \frac{2}{5}MR^2}$$

$$= \frac{2(3.14)}{1728000} \times \frac{2 \times 2.0 \times 10^{30} \times (7.0 \times 10^8)^2}{5}$$

$$= \frac{1230.80 \times 10^{46}}{8640000}$$

$$= 1.42 \times 10^{46-4}$$

$$= 1.42 \times 10^{42} \text{ J.s}$$

For rotational kinetic energy

$$(K.E)_{rot} = \frac{1}{2}I\omega^2$$

$$= \frac{1}{2} \times \frac{2}{5}MR^2 \times \left(\frac{2\pi}{T}\right)^2$$

$$= \frac{1}{5}MR^2 \times \frac{4\pi^2}{T^2}$$

$$= \frac{2.0 \times 10^{30} \times (7.0 \times 10^8)^2 \times 4(3.14)^2}{5 \times (1728000)^2}$$

$$= \frac{3864.9 \times 10^{30+16}}{1.49 \times 10^{13}}$$

$$= 2.58 \times 10^{46-10}$$

$$(K.E)_{rot} = 2.58 \times 10^{36} \text{ J}$$

Result

Angular momentum $= \vec{L} = 1.42 \times 10^{42}$ J.s

Kinetic energy $= (K.E)_{rot} = 2.58 \times 10^{36}$ J

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PROBLEM 5.6

A 1000 kg car traveling with a speed of 144 km h^{-1} rounds a curve of radius 100 m. Find the necessary centripetal force.

Data

Mass of car = m = 1000 kg

Speed of car = v = 144 km/h

$$= \frac{144 \times 1000}{3600} \text{ m/s}$$

= 40 m/s

Radius of curve = r = 100 m

To Find

Centripetal force = F_c = ?

SOLUTION

By using formula

$$\boxed{F_c = \frac{mv^2}{r}}$$

$$F_c = \frac{1000 \times (40)^2}{100}$$

= 16000 N

= 1.6×10^4 N

Result

Centripetal force = F_c = 1.6×10^4 N

PROBLEM 5.7

What is the least speed at which an aeroplane can execute a vertical loop of 1.0km radius so that there will be not tendency for the pilot to fall down at the highest point?

Data

Radius of loop = r = 1.0 km = 1000 m

Acceleration due to gravity = g = 9.8 m/s^2

To Find

Least speed of aeroplane = v = ?

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SOLUTION

By formula

$$\boxed{v = \sqrt{gr}}$$

$$= \sqrt{9.8 \times 1000}$$

$$= \sqrt{9800}$$

$$= 98.9 \text{ m/s}$$

$$= 99 \text{ m/s}$$

Result

Least speed of aeroplane = v = 99 m/s

PROBLEM 5.8

The Moon orbit the Earth so that the same side always faces the Earth. Determine the ratio of its spin angular momentum (about its own axis) and its orbital angular momentum. (In this case, treat Moon as a particle orbiting the Earth). Distance between the Earth and the Moon is 3.85×10^8 m. Radius of the Moon is 1.74×10^6 m.

Data

Distance between earth and moon = r = 3.85×10^8 m

Radius of moon = R_m = 1.74×10^6 m

To Find

Ratio $= \dfrac{L_s}{L_o} = ?$

SOLUTION

The spin angular momentum of the moon is

$$L_s = I\omega \quad \text{where} \quad I = \frac{2}{5}MR^2$$

$$\boxed{L_s = \frac{2}{5}MR_m^2\omega} \quad \ldots\ldots \text{(i)}$$

The orbital angular momentum is given by

$$\boxed{L_o = Mr^2\omega} \quad \ldots\ldots \text{(ii)}$$

where MR^2 = I (moment of inertia)

Divide eq. (i) by (ii)

$$\frac{L_s}{L_o} = \frac{\frac{2}{5}MR_m^2\omega}{Mr^2\omega}$$

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$$\boxed{\frac{L_s}{L_o} = \frac{2R_m^2}{5r^2}}$$

$$= \frac{2 \times (1.74 \times 10^6)^2}{5 \times (3.85 \times 10^8)^2}$$

$$= \frac{6.05 \times 10^{12}}{74.11 \times 10^{16}}$$

$$= 0.0816 \times 10^{12-16}$$

$$= 0.0816 \times 10^{-4}$$

$$\frac{L_s}{L_o} = 8.2 \times 10^{-6}$$

Result

$$\text{Ratio} = \frac{L_s}{L_o} = 8.2 \times 10^{-6}$$

PROBLEM 5.9

The Earth rotates on its axis once a day. Suppose, by some process the Earth contracts so that its radius is only half as large as at present. How fast will it be rotating then? (For sphere I = 2/5 MR^2).

Data

Time period = T_1 = 1 day

= 24 hr.

Moment of inertia = I = $\frac{2}{5}MR^2$

To Find

Time period = T_2 = ?

SOLUTION

According to law of angular momentum

$$\boxed{I_1\omega_1 = I_2\omega_2} \quad \ldots\ldots \text{(i)}$$

where $I_1 = \frac{2}{5}MR_1^2$ According to question

$I_2 = \frac{2}{5}MR_2^2$ $R_2 = \frac{R_1}{2}$

$$I_2 = \frac{2}{5}M\left(\frac{R_1}{2}\right)^2$$

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$$I_2 = \frac{2}{5} M \frac{R_1^2}{4} = \frac{1}{10} MR_1^2$$

and $\omega_1 = \frac{2\pi}{T_1}$ and $\omega_2 = \frac{2\pi}{T_2}$

Putting in eq. (i)

$$\frac{2}{5} MR_1^2 \times \frac{2\pi}{T_1} = \frac{1}{10} MR_1^2 \times \frac{2\pi}{T_2}$$

$$\frac{2}{5T_1} = \frac{1}{10T_2}$$

$$20T_2 = 5T_1$$

$$\boxed{T_2 = \frac{5T_1}{20}}$$

$$T_2 = \frac{5T_1}{20}$$

$$= \frac{5 \times 24}{20}$$

$$T_2 = 6 \text{ hr.}$$

Result

Earth completes its rotation = T_2 = 6 hr.

PROBLEM 5.10

What should be the orbiting speed to launch a satellite in a circular orbit 900 km above the surface of the Earth? (Take mass of the Earth as 6.0×10^{24} kg and its radius as 6400 km).

Data

Radius of earth = R = 6400 km

= 6400 × 1000

= 6400000 m

Height of circular orbit = h = 900 km

= 900 × 1000

= 900000 m

Mass of earth = M = 6.0×10^{24} kg

To Find

Orbit speed = v = ?

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SOLUTION

By formula

$$v = \sqrt{\frac{GM}{r}}$$

where $r = h + R$

$= 6400 + 900$

$= 7300\ \text{km}$

$= 7300 \times 1000$

$= 7300000\ \text{m} \quad = 7300 \times 10^3\ \text{m}$

Then $v = \sqrt{\dfrac{6.63 \times 10^{-11} \times 6.0 \times 10^{24}}{7300 \times 10^3}}$

$= \sqrt{5.44 \times 10^{-11-3-3+24}}$

$= \sqrt{5.44 \times 10^{7}}$

$= \sqrt{54.4 \times 10^{6}}$

$= 7.37 \times 10^3\ \text{m/s}$

$= 7.4 \times 10^3\ \text{m/s}$

Result

Orbital speed $= v = 7.4\ \text{km/s}$

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