SHORT QUESTIONS

3.1 What is the difference between uniform and variable velocity. From the explanation of variable velocity, define acceleration. Give SI units of velocity and acceleration.

Ans. **Uniform Velocity:** The velocity of a body is said to be uniform if it covers equal displacement in equal interval of time.

Variable Velocity: The velocity of a body is said to be variable if it covers unequal displacement in unequal interval of time.

Acceleration: From the variable velocity, the rate of change of velocity is called acceleration.

Let a body is moving with velocity $\vec{v}_i$. After small time Δt its velocity changes from $\vec{v}_i$ to $\vec{v}_f$ then the change in velocity $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$. So

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

SI Unit of Velocity: The SI unit of velocity is m/s or km/hr.

SI Unit of Acceleration: The SI unit of acceleration is m/s^2.

3.2 An object is thrown vertically upward. Discuss the sign of acceleration due to gravity, relative to velocity, while the object is in air.

Ans. When an object is thrown vertically upward, the sign of acceleration due to gravity is negative relative to velocity. But when the object is thrown downward, the sign of acceleration due to gravity is taken as positive because velocity and acceleration are in same direction.

3.3 Can the velocity of an object reverse direction when acceleration is constant? If so, give an example.

Ans. Yes, the velocity of an object can reverse its direction when acceleration is constant.

Example: When an object is thrown vertically upward then during upward motion its velocity decreases, the direction of velocity will be in upward while direction of acceleration due to gravity will be in downward and when it reach at the highest point its velocity become zero but during downward of object the direction of velocity will be in downward while direction of acceleration due to gravity will again in downward thus we see that in this case the velocity reverse the direction while acceleration is constant.

3.4 Specify the correct statements:

(a) An object can have a constant velocity even its speed is changing.

(b) An object can have a constant speed even its velocity is changing.

(c) An object can have a zero velocity even its acceleration is not zero.

(d) An object subjected to a constant acceleration can reverse its velocity.

Ans. (a) It is false statement because an object cannot have a constant velocity even its speed is changing.

(b) It is true when the object is moving along a circular path.

(c) It is true because when an object is thrown vertically upward, at maximum height, velocity is zero but acceleration is not zero, it is $a = g$.

(d) It is true. Yes an object subjected to a constant acceleration can reverse its velocity.

3.5 A man standing on the top of a tower throws a ball straight up with initial velocity v_i and at the same time throws a second ball straight downward with the same speed. Which ball will have larger speed when it strikes the ground? Ignore air friction.

Ans. Both the balls have the same speed on striking the ground but time is different. When the velocity of the ball thrown upward with initial velocity v_i, it will has same velocity v_i when it return back and passes the man so as the initial velocities of a ball is same for both cases, therefore the final velocities will also be same.

3.6 Explain the circumstances in which the velocity "v" and acceleration "a" of a car are:

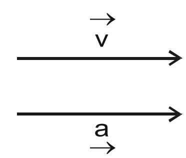
(i) Parallel **(ii) Anti-parallel**

(iii) Perpendicular to one another **(iv) "v" is zero but "a" is not**

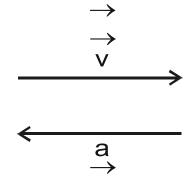
(v) "a" is zero but "v" is not zero

Ans. Following are the circumstances when velocity and acceleration of car:

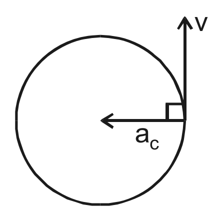
(i) Parallel: When the velocity of a car is increasing along a straight path then velocity and acceleration are parallel to each other.



(ii) Anti-parallel: When the velocity of car is decreasing along the straight line then velocity and acceleration are anti-parallel to each other.



(iii) Perpendicular to one another: The velocity and acceleration of a car are perpendicular to each other when the car is moving along a circular path.



(iv) v is zero but a is not zero: The velocity of a car becomes to zero when the brakes are applied and the car comes to rest due to acceleration in opposite direction.

(v) a is zero but v is not zero: Acceleration is zero when the car is moving with uniform acceleration.

3.7 Motion with constant velocity is a special case of motion with constant acceleration. Is this statement true? Discuss.

Ans. Yes, the motion with constant velocity is a special case of motion with constant acceleration. This statement is true.

Explanation: we know that when a body moves with constant velocity then its acceleration will be zero i.e., there is no rate of change of velocity so whenever it moves with constant velocity its acceleration will remain zero that is constant here zero is also a constant quantity. Therefore motion with constant velocity is a special case of motion with constant acceleration.

3.8 Find the change in momentum for an object subjected to a given force for a given time and state law of motion in terms of momentum.

Ans. Consider a body of mass "m" moving with velocity v_i. Let a force F is applied on the body which changes the velocity from v_i to v_f then according to 1st equation of motion.

$$v_f = v_i + at$$

$$a = \frac{v_f - v_i}{t}$$

But from Newton's second law of motion

$$F = ma$$

$$F = m\left(\frac{v_f - v_i}{t}\right)$$

$$F = \frac{mv_f - mv_i}{t}$$

Where mv_f is the final momentum and mv_i is the initial momentum so,

$$\frac{mv_f - mv_i}{t} = \text{Rate of change of momentum}$$

$$F = \text{Rate of changed momentum}$$

Newton's Second Law of Motion in Terms of Momentum: Newton's second law of motion in terms of momentum states, "the rate of change of momentum is equal to applied force".

3.9 Define impulse and show that how it is related to linear momentum?

Ans. **Impulse:** When a very large force acts on a body for a very short interval of time then the product of such a force and time is called impulse. It is a vector quantity

$$\text{Impulse} = I = \text{Force} \times \text{Time}$$

$$I = F \times \Delta t$$

As we know that

$$F \times \Delta t = mv_f - mv_i$$

So $$I = mv_f - mv_i$$

$$\vec{I} = m\Delta\vec{v}$$

$$\vec{I} = \Delta\vec{P} = \text{Change in momentum}$$

This shows that impulse is equal to change in momentum.

3.10 State the law of conservation of linear momentum, pointing out the importance of isolated system. Explain, why under certain conditions, the law is useful even though the system is not completely isolated?

Ans. **Law of Conservation of Linear Momentum:** This law states that the total linear momentum of an isolated system remains constant.

Importance of an Isolated System: This law holds good only for isolated system. An isolated system is one at which there is no external force acting. If the system is not isolated but the external forces are very small as compared to interacting forces so this law can also be applied on such a system.

3.11 Explain the difference between elastic and inelastic collisions. Explain how would a bouncing ball behave in each case? Give plausible reasons for the fact that K.E is not conserved in most cases?

Ans. **Elastic Collision:** These collision in which kinetic energy remains constant is called elastic collisions.

Inelastic Collision: These collision in which kinetic energy does not remain constant is called inelastic collisions.

In Case of Bouncing Ball: If the ideal bouncing ball returns to the same height where it is dropped then the collision is elastic collision. If the bouncing ball will not returned to the same height then the collision is inelastic. So due to change of energy, kinetic energy does not remain constant.

For example; when a heavy ball is dropped on to the surface of earth, it rebounds upto very little height because maximum K.E is lost due to friction and also changes into heat and sound energies. So in most cases, the K.E is not conserved. Thus momentum and K.E are conserved in all types of collisions. However the K.E is conserved only in elastic collision.

3.12 Explain what is meant by projectile motion. Derive expressions for

(a) The time of flight (b) The range of projectile.

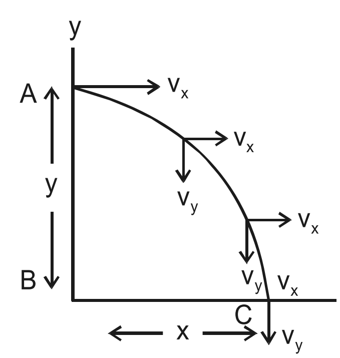
Show that the range of projectile is maximum when projectile is thrown at an angle of 45° with the horizontal.

Ans. **Projectile Motion:** When an object is thrown in air making a certain angle with horizontal, so that object moves under the action of gravity and moves along a curved path, is called as "projectile". Its motion is called "projectile motion". Its path is called trajectory. Its path is parabolic. **(OR)** Projectile motion is two dimensional motion under constant acceleration due to gravity.

The body thrown is called projectile and the curved path followed by it is called trajectory.

Examples:

1. Motion of football kicked off by a player.
2. A ball thrown by a cricketer.
3. Missile fired from launching pad.



Consider a body thrown in horizontal direction with horizontal velocity v_x from point A having vertical height 'y'. In the absence of horizontal force, the horizontal components v_x remain constant all along the motion. If the body hits at point 'C' the horizontal distance 'x' covered by the body is given by

$$x = v_x t$$

Where 't' is the time taken by body to move from A to C.

The body not only covers distance in forward direction but also moves down under the action of gravity. The downward vertical velocity of body under the action of gravity goes on increasing continuously. This vertical motion is same as for freely falling body. The distance covered by body in downward direction is AB = y and is given by

$$S = y = v_{iy}t + \frac{1}{2}at^2$$

As the ball at 'A' has only the horizontal velocity so

v_{iy} (initial vertical velocity) = 0 and a = g

So $y = \frac{1}{2} gt^2$

$$\boxed{y = \frac{1}{2} gt^2}$$

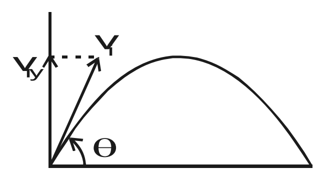
Time of Flight of Projectile: The time taken by body to cover the distance from place of projection to the place where it hits the ground, is called time of flight of projectile. The time of flight can be calculated by using 2nd equation of motion:

$$S = v_i t + \frac{1}{2} gt^2$$

As the ball returns to ground, so net vertical distance is zero. i.e.,

$S = 0$ and $v_i = v_{iy} = v_i \sin\theta$

The above equation becomes



$$0 = v_i \sin\theta t - \frac{1}{2} gt^2$$

$$\frac{1}{2} gt^2 = v_i \sin\theta t \quad \text{or} \quad \frac{1}{2} gt = v_i \sin\theta$$

or $$t = \frac{2v_i \sin\theta}{g}$$

Where 't' is the time of flight of projectile.

Range of Projectile: Max. distance which a projectile conversion the horizontal direction is called the range of projectile. In order to find R

$$R = v_{ix} \times t$$

$$= \frac{v_i \cos\theta \times 2v_i \sin\theta}{g}$$

$$= \frac{v_i^2}{g} 2 \sin\theta \cos\theta$$

$$R = \frac{v_i^2}{g} \sin 2\theta$$

The formula for the **range of projectile** is

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

For maximum horizontal range sin 2θ must have maximum value and we know that the maximum value of sin 2θ is 1.

$$\because \quad \sin 2\theta = 1$$

$$\sin 2\theta = \sin 90°$$

$$2\theta = 90°$$

$$\theta = 45°$$

So above equation becomes

$$R_{max.} = \frac{v_i^2 \sin 2(45°)}{g}$$

$$R_{max.} = \frac{v_i^2 \sin 90°}{g}$$

$$R_{max.} = \frac{v_i^2}{g}$$

So the range of projectile is maximum when projectile is thrown at an angle of 45° with the horizontal.

3.13 At what point or points in its path does a projectile have its minimum speed, its maximum speed?

Ans. The speed of the projectile is maximum at the point of projection and also at the point where it hits the ground. While the speed of projectile is minimum when it reaches the maximum height.

3.14 Each of the following questions is followed by four answers, one of which is correct answer. Identified that answer.

(i) What is meant by a ballistic trajectory?

(a) The paths followed by an un-powered and unguided projectile is called ballistic trajectory.

(b) The path followed by the powered and unguided projectile is called ballistic trajectory.

(c) The path followed by un-powered but guided projectile.

(d) The path followed by powered and guided projectile.

(ii) What happens when two-body system undergoes elastic collision?

(a) The momentum of the system changes.

(b) The momentum of the system does not change.

(c) The bodies come to rest after collision.

(d) The energy conservation law is violated.

Ans. **(i)** (a) is correct.

(ii) (b) is correct.

PROBLEMS WITH SOLUTIONS

PROBLEM 3.1

A helicopter is ascending vertically at the rate of 19.6 ms^{-1}. When it is at a height of 156.8 m above the ground, a stone is dropped. How long does the stone take to reach the ground?

METHOD-I

Data

Initial vertical velocity of helicopter = v_i = 19.6 m/s

Since initial velocity of the stone is upward and stone moves downward.

∴ Vertical distance travelled by stone = S = –156.8 m

To Find

Time taken by stone to reach the ground = t = ?

SOLUTION

By using 2nd equation of motion

$$S = v_i t + \frac{1}{2} g t^2$$

$$-156.8 = 19.6\,t + \frac{1}{2} \times -9.8\,t^2$$

$$-156.8 = 19.6\,t - 4.9\,t^2$$

$$4.9\,t^2 = -19.6\,t - 156.8 = 0$$

Dividing by 4.9

$$t^2 - 4t - 32 = 0$$

$$t^2 - 8t + 4t - 32 = 0$$

$$t(t - 8) + 4(t - 8) = 0$$

$$(t - 8)(t + 4) = 0$$

$$t - 8 = 0 \quad , \quad t + 4 = 0$$

$$t = 8 \text{ sec.} \quad , \quad t = -4 \text{ sec.}$$

Since time is always positive so ignoring the negative time hence.

Result

Time taken by stone = t = 8 sec.

METHOD-II

Data

Initial vertical velocity of helicopter = v_i = –19.6 m/s

Vertical distance = S = 156.8 m

To Find

Time taken by stone to reach the ground = t = ?

SOLUTION

By using the 2nd equation of motion

$$S = v_i t + \frac{1}{2} g t^2$$

$$156.8 = -19.6\,t + \frac{1}{2} \times 9.8\,t^2$$

$$156.8 = -19.6\,t + 4.9\,t^2$$

Divide by 4.9

$$\frac{156.8}{4.9} = \frac{-19.6}{4.9}t + \frac{4.9}{4.9}t^2$$

$$32 = -4t + t^2$$

$$t^2 - 4t - 32 = 0$$

$$t^2 - 8t + 4t - 32 = 0$$

$$t(t-8) + 4(t-8) = 0$$

$$(t+4)(t-8) = 0$$

$$t + 4 = 0 \quad , \quad t - 8 = 0$$

$$t = -4 \text{ sec. (neglected)}, \quad t = 8 \text{ sec.}$$

Result

Time taken by stone to reach the ground = t = 8 sec.

PROBLEM 3.2

Using the following data, draw a velocity-time graph for a short journey on a straight road of a motorbike.

Velocity (ms^{-1})	**0**	**10**	**20**	**20**	**20**	**20**	**0**
Time (s)	**0**	**30**	**60**	**90**	**120**	**150**	**180**

Use the graph to calculate

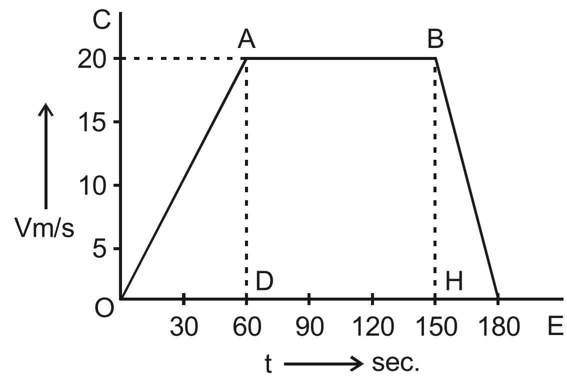
(a) The initial acceleration

(b) The final acceleration

(c) The total distance travelled by the motorcyclist.

SOLUTION

The velocity time graph is as shown.



(a) For initial acceleration

Initial acceleration

$$a_i = \frac{\text{Change in velocity}}{\text{Time}}$$

$$a_i = \frac{\Delta v}{\Delta t}$$

Since $\Delta v = 20$ m/s

$\Delta t = 60$ sec.

So, $a_i = \frac{20}{60}$

$= \frac{1}{3}$ m/s^2

$a_i = 0.33$ m/s^2

(b) For final acceleration

$$a_f = \frac{\text{Change in velocity}}{\text{Time}}$$

Since $\Delta v = v_f - v_i$

$= 0 - 20$

$= -20$ m/s

and $\Delta t = 30$ sec.

$a_f = \frac{-20}{30}$

$a_f = -0.67$ m/s^2

(c) For total distance travelled by motorcyclist

Total distance = Area of ΔOAD + Area of rectangle ABHD + Area of ΔBHE

Thus;

Area of ΔOAD $= \frac{1}{2}$ Base × Height

$= \frac{1}{2} \times 60 \times 20$

$= 600$ m

Area of rectangle ABHD = Length × Breadth

$= 90 \times 20$

$= 1800$ m/s

$$\text{Area of } \Delta BHE = \frac{1}{2} \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times 30 \times 20$$

$$= 300 \text{ m}$$

Putting in above equation

$$\text{Total distance travelled} = 600 + 1800 + 300$$

$$= 2700 \text{ m}$$

$$= 2.7 \text{ km}$$

Result

(a) Initial acceleration = a_i = 0.33 m/s^2

(b) Final acceleration = a_f = -0.67 m/s^2

(c) Total distance travelled by motorcyclist = 2.7 km

PROBLEM 3.3

A proton moving with speed of 1.0×10^7 ms^{-1} passes through a 0.02 cm thick sheet of paper and emerges with a speed of 2.0×10^6 ms^{-1}. Assuming uniform deceleration, find retardation and time taken to pass through the paper.

Data

Initial speed of proton = v_i = 1.0×10^7 m/s

Distance covered = S = 0.02 cm

= 2×10^{-4} m

Final speed of proton = v_f = 2.0×10^6 m/s

To Find

Retardation (negative acceleration) = a = ?

Time taken = t = ?

SOLUTION

For the retardation by using 3[rd] equation of motion

$$2as = v_f^2 - v_i^2$$

$$2a \times 2 \times 10^{-4} = (2 \times 10^6)^2 - (1.0 \times 10^7)^2$$

$$4 \times 10^{-4} a = 4 \times 10^{12} - 1.0 \times 10^{14}$$

$$= 10^{12} (4 - 1.0 \times 10^2)$$

$$= 10^{12} (4 - 100)$$

$$4 \times 10^{-4} a = -96 \times 10^{12}$$

$$a = \frac{-96 \times 10^{12}}{4 \times 10^{-4}}$$

$$a = -24 \times 10^{16}$$

$$a = -2.4 \times 10^{17} \text{ m/s}^2$$

Negative sign shows retardation.

For the time taken, by using 1st equation of motion

$$v_f = v_i + at$$

$$t = \frac{v_f - v_i}{a}$$

$$= \frac{2.0 \times 10^6 - 1.0 \times 10^7}{-2.4 \times 10^{17}}$$

$$= \frac{10^6 (2 - 1.0 \times 10)}{-2.4 \times 10^{17}}$$

$$= \frac{10^6 (2 - 10)}{-2.4 \times 10^{17}}$$

$$= \frac{-8 \times 10^6}{-2.4 \times 10^{17}}$$

$$= 3.3 \times 10^{6-17}$$

$$= 3.3 \times 10^{-11} \text{ sec.}$$

Result

Retardation (negative acceleration) $= a = -2.4 \times 10^7 \text{ m/s}^2$

Time taken $= t = 3.3 \times 10^{-11}$ sec.

PROBLEM 3.4

Two masses m_1 and m_2 are initially at rest with a spring compressed between them. What is the magnitude of ratio of their velocities after the spring has been released?

Data

1st mass $= m_1$

2nd mass $= m_2$

Initial velocity of mass m_1 $= v_i = 0$

Initial velocity of mass m_2 $= v_i = 0$

To Find

Ratio of their velocities $= \frac{v_1}{v_2} = ?$

SOLUTION

According to law of conservation momentum

$$m_1v_i + m_2v_i = m_1v_1 + m_2v_2$$

Therefore;

$$m_1v_1 + m_2v_2 = 0$$

$$m_1v_1 = -m_2v_2$$

$$\frac{v_1}{v_2} = -\frac{m_2}{m_1}$$

Result

Hence after releasing the spring, the ratio of magnitude of their velocities is equal to the inverse ratio of their masses.

PROBLEM 3.5

An amoeba of mass 1.0×10^{-12} kg propels itself through water by blowing a jet of water through a tiny orifice. The amoeba ejects water with a speed of 1.0×10^{-4} ms^{-1} and at a rate of 1.0×10^{-13} kgs^{-1}. Assume that the water is being continuously replenished so that the mass of the amoeba remains the same.

(a) If there were no force on amoeba other than the reaction force caused by the emerging jet, what would be the acceleration of the amoeba?

(b) If amoeba moves with constant velocity through water, what is force of surrounding water (exclusively of jet) on the amoeba?

Data

Mass of amoeba $= m = 1.0 \times 10^{-12}$ kg

Speed of ejected water $= v = 1.0 \times 10^{-4}$ m/s

Rate of water $= \frac{m}{t} = 1.0 \times 10^{-13}$ kg/s

To Find

(a) Acceleration of amoeba $= a = ?$

(b) Force of surrounding water $= F = ?$

SOLUTION

(a) By formula

$$\boxed{F = \frac{m}{t} \times v}$$

$$F = 1.0 \times 10^{-13} \times 1.0 \times 10^{-4}$$

$$= 1.0 \times 10^{-17}\ N$$

So by second law of motion

$$F = ma$$

$$a = \frac{F}{m}$$

$$a = \frac{1.0 \times 10^{-17}}{1.0 \times 10^{-12}}$$

$$= 1.0 \times 10^{-17+12}$$

$$= 1.0 \times 10^{-5} \text{ m/s}^2$$

(b) The force of surrounding water is

$$F = \frac{m}{t} \times v$$

$$= 1.0 \times 10^{-13} \times 1.0 \times 10^{-4}$$

$$= 1.0 \times 10^{-17} \text{ N}$$

Result

(a) Acceleration of amoeba = a = 1.0×10^{-5} m/s^2

(b) Force of surrounding water = F = 1.0×10^{-17} N

PROBLEM 3.6

A boy places a fire cracker of negligible mass in an empty can of 40 g mass. He plugs the end with a wooden block of mass 200 g. After igniting the firecracker, he throws the can straight up. It explodes at the top of its path. If the block shoots out with a speed of 3 ms^{-1}, how fast will the can be going?

Data

Mass of can = m_1 = 40 g = 0.04 kg

Mass of wooden block = m_2 = 200 g
= 0.2 kg

Speed of wooden block = v_2' = 3 m/s

To Find

Speed of can = v_1' = ?

SOLUTION

According to law of conservation of momentum

Momentum before explosion = Momentum after explosion

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

$$0 = m_1v_1' + m_2v_2'$$

$$m_1v_1' = -m_2v_2'$$

$$\boxed{v_1' = -\frac{m_2 v_2}{m_1}}$$

$$= -\frac{0.2 \times 3}{0.04}$$

$$= -15 \text{ m/s}$$

Result

Speed of can $= v_1' = 15$ m/s

–ve sign shows that the can and wooden block moves in opposite direction.

PROBLEM 3.7

An electron ($m = 9.1 \times 10^{-31}$ kg) traveling at 2.0×10^{7} ms^{-1} undergoes a head on collision with a hydrogen atom ($m = 1.67 \times 10^{-27}$ kg) which is initially at rest. Assuming the collision to be perfectly elastic and a motion to be along a straight line, find the velocity of hydrogen atom?

Data

Mass of electron $= m_1 = 9.1 \times 10^{-31}$ kg

Velocity of electron $= v_1 = 2.0 \times 10^{7}$ m/s

Mass of hydrogen atom $= m_2 = 1.67 \times 10^{-27}$ kg

Velocity of hydrogen atom $= v_2 = 0$

To Find

Velocity of hydrogen atom $= v_2' = ?$

SOLUTION

$$\boxed{v_2' = \frac{2m_1 v_1}{m_1 + m_2}}$$

$$v_2' = \frac{2 \times 9.1 \times 10^{-31} \times 2.0 \times 10^{7}}{9.1 \times 10^{-31} + 1.67 \times 10^{-27}}$$

$$= \frac{36.4 \times 10^{-31+7}}{10^{-27}(9.1 \times 10^{-4} + 1.67)}$$

$$= \frac{36.4 \times 10^{-24+27}}{0.00091 + 1.67}$$

$$= \frac{36.4 \times 10^{3}}{1.67091}$$

$$= 21.78 \times 10^{3}$$

$$= 2.18 \times 10^{4} \text{ m/s}$$

Result

Velocity of hydrogen atom $= v_2' = 2.18 \times 10^{4}$ m/s

PROBLEM 3.8

A truck weighing 2500 kg and moving with a velocity of 21 ms^{-1} collides with a stationary car weighing 1000 kg. The truck and the car move together after the impact. Calculate their common velocity.

Data

Mass of truck = m_1 = 2500 kg

Velocity of truck = v_1 = 21 m/s

Mass of car = m_2 = 1000 kg

Velocity of car = v_2 = 0

To Find

Common velocity after collision = v = ?

SOLUTION

According to law of conservation of momentum

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

Since $v_1' = v_2' = v$

$$m_1v_1 + m_2v_2 = m_1v + m_2v$$

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v$$

$$\boxed{v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}}$$

Putting values

$$v = \frac{2500 \times 21 + 1000 \times 0}{2500 + 1000}$$

$$= \frac{52500}{3500}$$

$$v = 15 \text{ m/s}$$

Result

Common velocity of truck and car after collision = v = 15 m/s

PROBLEM 3.9

Two blocks of masses 2.0 kg and 0.5 kg are attached at the two ends of a compressed spring. The elastic potential energy stored in the spring in 10 J. Find the velocities of the blocks if the spring delivers its energy to the blocks when released.

Data

Mass of 1st block = m_1 = 2.0 kg

Mass of 2nd block = m_2 = 0.5 kg

Elastic potential energy = P.E = 10 J

To Find

Velocity of mass m_1 = v'_1 = ?

Velocity of mass m_2 = v'_2 = ?

SOLUTION

According to law of conservation of energy

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v'^2_1 + \frac{1}{2}m_2v'^2_2$$

$$\frac{1}{2}m_1v'^2_1 + \frac{1}{2}m_2v'^2_2 = 10$$

$$m_1v'^2_1 + m_2v'^2_2 = 20$$

$$2v'^2_1 + 0.5v'^2_2 = 20 \qquad \ldots\ldots \text{(i)}$$

And according to law of conservation of momentum

$$m_1v_1 + m_2v_2 = m_1v'_1 + m_2v'_2$$

$$m_1v'_1 + m_2v'_2 = 0$$

$$2v'_1 + 0.5v'_2 = 0$$

$$0.5v'_2 = -2v'_1$$

$$v'_2 = -\frac{2v'_1}{0.5} = -4v'_1$$

$$\boxed{v'_2 = -4v'_1}$$

Putting eq. (i)

$$2v'^2_1 + 0.5(-4v'_1)^2 = 20$$

$$2v'^2_1 + 0.5(16v'^2_1) = 20$$

$$2v'^2_1 + 8v'^2_1 = 20$$

$$10v'^2_1 = 20$$

$$v'^2_1 = \frac{20}{10}$$

$$v'_1 = 1.41 \text{ m/s}$$

and

$$v'_2 = -4v_1$$

$$= -4(1.41)$$

$$v'_2 = -5.65 \text{ m/s}$$

Result

Velocity of mass m_1 = v'_1 = 1.41 m/s

Velocity of mass m_2 = v'_2 = –5.65 m/s

PROBLEM 3.10

A football is thrown upward with an angle of 30^o with respect to the horizontal. To throw a 40 m pass what must be the initial speed of the ball?

Data

Angle with horizontal = θ = $30°$

Horizontal distance = R = 40 m

The value of g = $9.8\ m/s^2$

To Find

Initial speed of ball = v_i = ?

SOLUTION

By formula

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

$$v_i^2 = \frac{R \times g}{\sin 2\theta}$$

$$= \frac{40 \times 9.8}{\sin 2(30)}$$

$$= \frac{392}{0.866}$$

$$\sqrt{v_i^2} = \sqrt{452.64}$$

$$v_i = 21.27$$

$$= 21.3\ m/s$$

Result

Initial velocity of ball = v_i = 21.3 m/s

PROBLEM 3.11

A ball is thrown horizontally from a height of 10 m with velocity of 21 ms^{-1}. How far off it hit the ground and with what velocity?

Data

Initial horizontal velocity = v_{ix} = 21 m/s

Initial vertical velocity = v_{iy} = 0

Vertical distance = y = 10 m

To Find

Horizontal distance = R = x = ?

Velocity to hit the ground = v = ?

SOLUTION

Formula for horizontal distance

$$\boxed{x = v_{ix} \times t} \quad \ldots\ldots \text{(i)}$$

For time

$$y = v_{iy}t + \frac{1}{2}gt^2$$

$$10 = 0 \times t + \frac{1}{2} \times 9.8\, t^2$$

$$10 = 4.9\, t^2$$

$$t^2 = \frac{10}{4.9}$$

$$t^2 = 2.04$$

$$t = 1.42 \text{ sec.}$$

Therefore; putting in eq. (i)

$$x = R = v_{ix} \times t$$

$$= 21 \times 1.42$$

$$= 29.8$$

$$= 30 \text{ m}$$

For velocity

$$v = \sqrt{v_{fx}^2 + v_{fy}^2}$$

As $v_{fx} = v_{ix} = 21$ m/s

and $v_{fy} = v_{iy} + gt$

$$= 0 + 9.8 \times 1.42$$

$$v_{fy} = 13.91 \text{ m/s}$$

So $v = \sqrt{(21)^2 + (13.91)^2}$

$$= \sqrt{441 + 193.48}$$

$$= \sqrt{634.48}$$

$$v = 25.1 \text{ m/s}$$

Result

Horizontal distance = x = R = 30 m

Velocity to hit the ground = v = 25 m/s

PROBLEM 3.12

A bomber dropped a bomb at a height of 490 m when its velocity along the horizontal was 300 kmh^{-1}.

(a) At what distance from the point vertically below the bomber at the instant the bomb was dropped, did it strike the ground?

(b) How long was it in air?

Data

Height of bomber $= y = 490$ m

Horizontal velocity $= v_{ix} = 300$ km/hr

$$= \frac{300 \times 1000}{3600}$$

$$= 83.3 \text{ m/s}$$

To Find

(a) Horizontal distance $= x = R = ?$

(b) Time in air $= t = ?$

SOLUTION

(a) For horizontal distance

$$\boxed{x = v_{ix} \times t} \quad \ldots\ldots \text{(i)}$$

For time

$$y = v_{iy}t + \frac{1}{2}gt^2 \quad \text{Since} \quad v_{iy} = 0$$

$$490 = 0 \times t + \frac{1}{2} \times 9.8\, t^2$$

$$490 = 4.9\, t^2$$

$$t^2 = \frac{490}{4.9}$$

$$t^2 = 100$$

$$t = 10 \text{ sec.}$$

So putting in eq. (i), we get

$$x = v_{ix} \times t$$

$$x = 10 \times 83.3$$

$$= 833 \text{ m}$$

(b) Time in air $= t = 10$ sec.

Result

(a) Horizontal distance $= x = 833$ m

(b) Time in air $= t = 10$ sec.

PROBLEM 3.13

Find the angle of projection of a projectile for which its maximum height and horizontal range are equal.

Data

The given that

Horizontal range = Maximum height

To Find

Angle of projection = θ = ?

SOLUTION

As we know that the range of projection is

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

and maximum height is

$$h = \frac{v_i^2 \sin^2 \theta}{2g}$$

Therefore; $R = h$

$$\frac{v_i^2 \sin 2\theta}{g} = \frac{v_i^2 \sin^2 \theta}{2g}$$

$$\sin 2\theta = \frac{\sin^2 \theta}{2}$$

Since $\sin 2\theta = 2 \sin \theta \cos \theta$

$$2 \sin \theta \cos \theta = \frac{\sin^2 \theta}{2}$$

$$2 \cos \theta = \frac{\sin \theta}{2}$$

$$2 \times 2 = \frac{\sin \theta}{\cos \theta}$$

$$\tan \theta = 4$$

$$\theta = \tan^{-1} (4)$$

$$\theta = 75.9$$

$$= 76^\circ$$

Result

Angle of projection = θ = 76°

PROBLEM 3.14

Prove that for angles of projection, which exceed or fall short of 45° by equal amounts, the ranges are equal.

SOLUTION

Let the two angles 30° and 60° which exceed or fall short of 45° by equal of 15°.

Now we have to find, the ranges at these two angles so the range of projectile is

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

When θ = 30°

$$R_1 = \frac{v_i^2 \sin 2(30°)}{g}$$

$$R_1 = \frac{v_i^2 \sin 60°}{g}$$

$$R_1 = \frac{0.866\, v_i^2}{g}$$

And when θ = 60°

$$R_2 = \frac{v_i^2 \sin 2(60°)}{g}$$

$$R_2 = \frac{v_i^2 \times 0.866}{g}$$

$$R_2 = \frac{0.866\, v_i^2}{g}$$

Thus $R_1 = R_2$

Result

Hence for angle of projection which exceed or fall short of 45° by equal amounts, the ranges are equal.

PROBLEM 3.15

A SLBM (submarine launched ballistic missile) is fired from a distance of 3000 km. If the Earth were flat and the angle of launch is 45° with horizontal, find the time taken by SLBM to hit the target and the velocity with which the missile is fired.

Data

Horizontal distance $= x = 3000$ km

$= 3000 \times 1000$

$= 3 \times 10^6$ m

Angle of launch $= \theta = 45°$

To Find

Time taken by SLBM to hit the ground = t = ?

Velocity of the missile = v_i = ?

SOLUTION

For velocity of the missile

$$R = x = \frac{v_i^2 \sin 2\theta}{g}$$

$$\boxed{v_i^2 = \frac{x \times g}{\sin 2\theta}}$$

$$v_i^2 = \frac{3 \times 10^6 \times 9.8}{\sin 2(45°)}$$

$$v_i^2 = \frac{29.4 \times 10^6}{\sin 90°} \qquad \text{Since} \quad \sin 90° = 1$$

$$v_i^2 = 29.4 \times 10^6$$

$$v_i = 5.42 \times 10^3 \text{ m/s}$$

$$= 5.42 \text{ km/s}$$

For time

$$\boxed{t = \frac{2v_i \sin\theta}{g}}$$

$$t = \frac{2(5.42 \times 10^3) \sin 45°}{9.8}$$

$$= \frac{10.84 \times 10^3 \times 0.707}{9.8}$$

$$t = 0.982 \times 10^3$$

$$t = 782 \text{ sec.}$$

$$t = 13 \text{ min.}$$

Result

Time taken by SLMB to hit the ground = t = 13 min.

Velocity of the missile = v_i = 5.42 km/s