SHORT QUESTIONS

2.1 Define the terms (i) unit vector (ii) Position vector (iii) Components of a vector.

Ans. (i) Unit Vector: A vector whose magnitude is one called unit vector. It is used to find the direction of a vector. The formula for the unit vector is

$$\hat{A} = \frac{\vec{A}}{|A|}$$

(ii) Position Vector: It is a vector that describe the location of a particle with respect to origin. The position vector $\vec{r}$ of point P(a, b) in x-y plane is given by

$$\vec{a} = a\hat{i} + b\hat{j} + c\hat{k}$$

where $\hat{k}$ is the unit vector along z-axis. In three dimension, the position and $\vec{r}$ from origin will

$$\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$$

where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors along x, y and z-axis respectively.

(iii) Component of a Vector: The part of a vector effective in a particular direction is called the components of a vector. Usually a vector has two or more components, one along x-axis is called horizontal component and other along y-axis is called vertical component.

2.2 The vector sum of three vectors gives a zero resultant. What can be the orientation of the vectors?

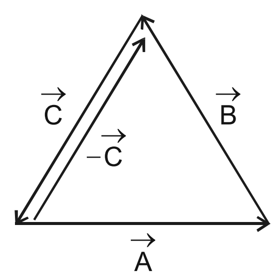
Ans. The resultant of three vectors of equal magnitudes is equal to zero if they are represented by the three adjacent sides of a triangle as shown. If we have three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$. By using head to tail rule where $-\vec{C}$ is the resultant of $\vec{A}$ and $\vec{B}$.

Hence

$$-\vec{C} = \vec{A} + \vec{B}$$

$$\vec{A} + \vec{B} + \vec{C} = 0$$

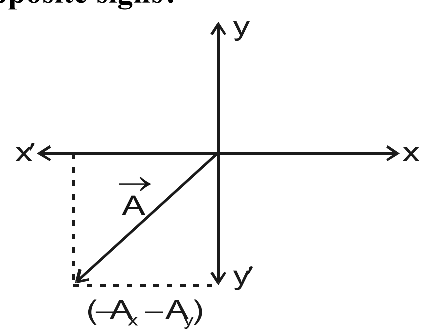
Thus the vector sum of three vectors is zero.

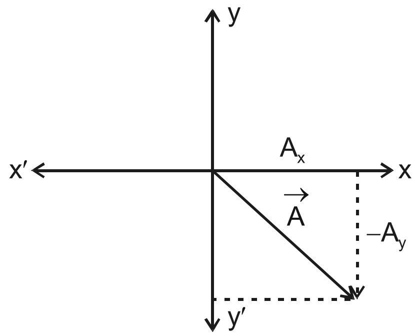


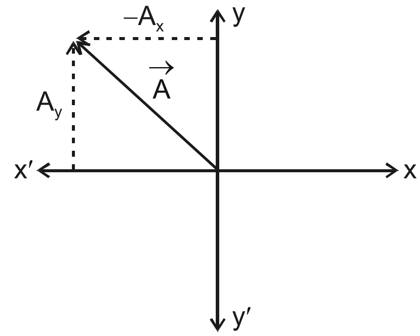
2.3 Vector A lies in the xy-plane. For what orientation will both of its rectangular components be negative. For what orientation will its components have opposite signs?

Ans. ***Case-I:*** If a vector $\vec{A}$ lies in third quadrant then both of its rectangular components A_x and A_y will be negative as shown.

Case-II: If a vector $\vec{A}$ lies in second and fourth quadrant then both of it, rectangular components A_x and A_y have in opposite sign as shown.







2.4 If one of the rectangular components of a vector is not zero, can its magnitude be zero? Explains.

Ans. No, its magnitude cannot be zero because, the magnitude of vector contains the sum of square of its components. So if one of the components of a vector is not zero and even if they have the opposite signs then the magnitude of a vector cannot be zero. According to formula

$$A = \sqrt{A_x^2 + A_y^2}$$

If $A_y = 0$

Then $A = \sqrt{A_x^2 + 0^2}$

$$A = \sqrt{A_x^2}$$

$$A = A_x$$

$\therefore \quad A \neq 0$

So if one of the rectangular components of a vector is not zero then its magnitude cannot be zero.

2.5 Can a vector have a component greater than the vector's magnitude?

Ans. (i) No, the magnitude of a vector cannot have a component greater than its magnitude because the components of a vector is always less in magnitude of resultant vector. Only in case of equilateral triangle, they are equal.

(ii) Yes, the statement is correct if we do not take the case of rectangular component. So a vector has a component greater than vector magnitude.

2.6 Can the magnitude of a vector have a negative value?

Ans. No, the magnitude of a vector cannot have a negative value. The magnitude of a vector always has a positive value. For example, if we have a vector $-3\vec{A}$, where 3 is the magnitude of a vector and the negative sign shows its direction.

(OR)

As magnitude of $\vec{A}$ is

$$A = \sqrt{A_x^2 + A_y^2}$$

Hence magnitude of a vector cannot have a negative value. e.g.,

If $A_x = -5$ and $A_y = 2$

then $A = \sqrt{(-5)^2 + (2)^2}$

$$= \sqrt{25 + 4}$$

$$= \sqrt{29}$$

2.7 If $\vec{A} + \vec{B} = \vec{0}$, what can you say about the components of the two vectors?

Ans. If

$$\vec{A} + \vec{B} = \vec{0}$$

$$\vec{A} = -\vec{B}$$

In case of rectangular components

$$A_x\hat{i} + A_y\hat{j} + A_z\hat{k} = -(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$A_x\hat{i} + A_y\hat{j} + A_z\hat{k} = -B_x\hat{i} - B_y\hat{j} - B_z\hat{k}$$

Comparing the coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$

$$A_x = -B_x$$

$$A_y = -B_y$$

$$A_z = -B_z$$

So it means that if the sum of the two vectors is zero then their rectangular components will be of the same magnitude but in opposite direction.

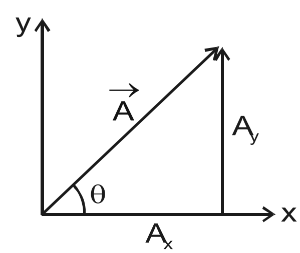
2.8 Under what circumstances would a vector have components that are equal in magnitude?

Ans. If θ be the angle which vector $\vec{A}$ makes with horizontal line having components A_x and A_y then

$$\tan\theta = \frac{A_y}{A_x}$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

According to question



$$A_x = A_y$$

then

$$\theta = \tan^{-1}\left(\frac{A_y}{A_y}\right)$$

$$\theta = \tan^{-1}(1)$$

$$\theta = 45°$$

So, if $\vec{A}$ makes an angle of 45° with x-axis then its both components will be equal in magnitude.

2.9 Is it possible to add a vector quantity to a scalar quantity? Explain.

Ans. No, a vector quantity cannot be added to a scalar quantity because scalar has only magnitude while vector has both the magnitude and direction. So they cannot be added to each other.

2.10 Can you add zero to a null vector?

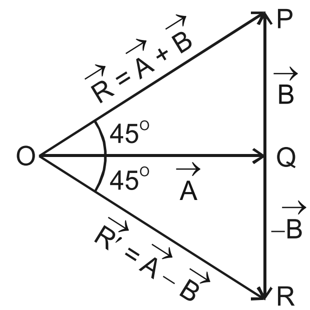
Ans. No, zero is not added to a null vector because zero is a scalar and null vector is a vector quantity.

2.11 Two vectors have unequal magnitudes. Can their sum be zero? Explain.

Ans. No, the sum of two vectors of unequal magnitude cannot be zero. It is only possible when two vectors have same magnitude and in opposite direction.

2.12 Show that the sum and difference of two perpendicular vectors of equal lengths are also perpendicular and of the same length.

Ans. Consider two vectors $\vec{A}$ and $\vec{B}$ having equal magnitudes as shown. From head to tail rule, $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$ are the resultant having same magnitude because they are the hypotenuse of right angled triangle.



Since in ΔOPQ

$$\angle POQ = \angle QPO = 45°$$

and magnitude of $\vec{R}$ will be

$$|\vec{R}| = \sqrt{A^2 + B^2}$$

and in ΔOQR

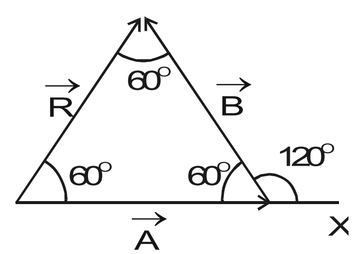
$$\angle ROQ = \angle ORQ = 45°$$

So $\angle POQ + \angle ROQ = 45° + 45° = 90°$

Hence the resultants $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$ are perpendicular to each other and they are in equal length.

2.13 How would the two vectors of the same magnitude have to be oriented, if they were to be combined to give a resultant equal to a vector of the same magnitude.

Ans. If two vectors $\vec{A}$ and $\vec{B}$ make an angle of 120°. Then their resultant would have the same magnitude as that of $\vec{A}$ and $\vec{B}$.



We know that:

$$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB + \cos\theta}$$

According to given condition A = B = R.

$$\because \quad R = \sqrt{R^2 + R^2 + 2R^2\cos\theta}$$

$$R^2 = 2R^2 + 2R^2\cos\theta$$

$$R^2 - 2R^2 = 2R^2\cos\theta$$

$$\frac{-R^2}{2R^2} = \cos\theta \quad \Rightarrow \quad \cos\theta = -\frac{1}{2}$$

$$\theta = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$\theta = 120°$$

2.14 The two vectors to be combined have magnitudes 60 N and 35 N. Pick the correct answer from those given below and tell why is it the only one of the three that is correct.

(i) 100N (ii) 70N (iii) 20N

Ans. When 60 N and 35 N forces are in same direction then maximum resultant is

$$= 60 + 35 = 95 \text{ N}$$

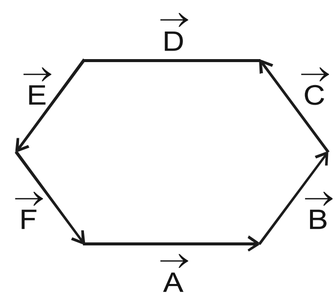
When 60 N and 35 N forces are in opposite direction then minimum resultant is

$$= 60 - 35 = 25 \text{ N}$$

Hence the range of the force is 25 N – 95 N then (ii) 70 N is correct.

2.15 Suppose the sides of a closed polygon represent vector arranged head to tail? What is the sum of these vectors?

Ans. We know that if the vectors are arranged by head to tail rule, which makes a closed polygon then its resultant is zero because there is no place to draw resultant. Consider $\vec{A}, \vec{B}, \vec{C}, \vec{D}, \vec{E}$ and $\vec{F}$ are the vectors which are arranged by head to tail rule then their resultant is zero i.e.,



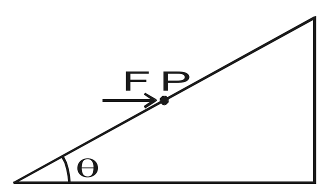
$$\vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E} + \vec{F} = 0$$

2.16 Identify the correct answer:

(i) Two ships X and Y are traveling in different directions at equal speeds. The actual direction of motion of X is due north but to an observer on Y, the apparent direction of motion of X is north-east. The actual direction of motion of Y as observed from the shore will be.

(a) East (b) West (c) South-east (d) South-west

(ii) A horizontal force F is applied to a small object P of mass m at rest on a smooth plane inclined at an angle θ to the horizontal as shown in figure. The magnitude of the resultant force acting up and along the surface of the plane, on the object is:



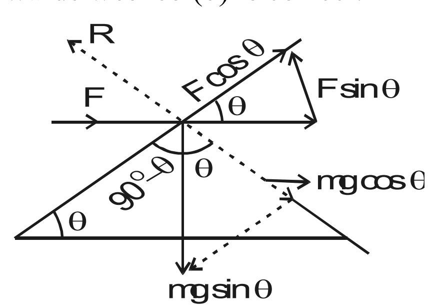
(a) $F \cos\theta - mg \sin\theta$ (b) $F \sin\theta - mg \cos\theta$

(c) $F \cos\theta + mg \cos\theta$ (d) $F \sin\theta + mg \sin\theta$

(e) $mg \tan\theta$

Ans. (i) We know that the ship x is moving towards North from shore and according to observer on ship y, the ship x is moving towards north-east direction so ship y is approaching towards the line of motion of ship x. Thus the motion of ship y is towards west so (b) is correct.

(ii) Now the horizontal force F and weight mg of the body can be resolved into its rectangular components as shown. The force acting up along the plane of the surface is



$$= F \cos\theta - mg \sin\theta$$

So (a) is correct.

2.17 If all the components of the vectors, $\vec{A_1}$ and $\vec{A_2}$ were reversed, how would this alter $\vec{A_1} \times \vec{A_2}$?

Ans. The vectors $\vec{A_1}$ and $\vec{A_2}$ can be resolved into its rectangular components i.e.,

$$\vec{A_1} = A_{1x}\hat{i} + A_{1y}\hat{j} + A_{1z}\hat{k}$$

$$\vec{A_2} = A_{2x}\hat{i} + A_{2y}\hat{j} + A_{2z}\hat{k}$$

$$\vec{A_1} \times \vec{A_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_{1x} & A_{1y} & A_{1z} \\ A_{2x} & A_{2y} & A_{2z} \end{vmatrix}$$

On reversing the components

$$\vec{A_1} = -A_{1x}\hat{i} - A_{1y}\hat{j} - A_{1z}\hat{k}$$

$$\vec{A_2} = -A_{2x}\hat{i} - A_{2y}\hat{j} - A_{2z}\hat{k}$$

So $$\vec{A_1} \times \vec{A_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -A_{1x} & -A_{1y} & -A_{1z} \\ -A_{2x} & -A_{2y} & -A_{2z} \end{vmatrix}$$

Taking (−1) as common from R_1 and R_2

$$\vec{A_1} \times \vec{A_2} = (-)(-)\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_{1x} & A_{1y} & A_{1z} \\ A_{2x} & A_{2y} & A_{2z} \end{vmatrix}$$

$$\vec{A_1} \times \vec{A_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_{1x} & A_{1y} & A_{1z} \\ A_{2x} & A_{2y} & A_{2z} \end{vmatrix}$$

Hence if the components of the vectors are reversed then there is no effect on $\vec{A_1} \times \vec{A_2}$.

2.18 Name the three different conditions that could make $\vec{A_1} \times \vec{A_2} = 0$.

Ans. We know that

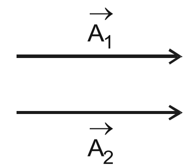
$$\vec{A_1} \times \vec{A_2} = A_1A_2 \sin\theta\, \hat{n}$$

Therefore the required three conditions are

(i) Either $\vec{A_1}$ or $\vec{A_2}$ is null vector i.e., $|\vec{A_1}| = |\vec{A_2}| = 0$.

$$\vec{A_1} \times \vec{A_2} = 0$$

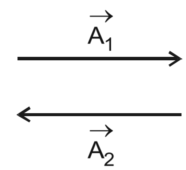
(ii) $\vec{A_1}$ and $\vec{A_2}$ are parallel i.e., $\theta = 0°$.



$$\vec{A_1} \times \vec{A_2} = A_1A_2 \sin 0° \hat{n} \qquad \because \sin 0° = 0$$

$$\vec{A_1} \times \vec{A_2} = 0$$

(iii) $\vec{A_1}$ and $\vec{A_2}$ are antiparallel i.e., $\theta = 180°$



$$\vec{A_1} \times \vec{A_2} = A_1A_2 \sin 180° \hat{n}$$

$$\Rightarrow \quad \vec{A_1} \times \vec{A_2} = 0 \qquad \because \sin 180° = 0$$

2.19 Identify true or false statements and explain the reason:

(a) A body in equilibrium implies that it is not moving nor rotating.

(b) If coplanar forces acting on a body form a closed polygon, then the body is said to be in equilibrium.

Ans. (a) This statement is false because the body is said to be in equilibrium if it is moving with constant velocity or rotating with constant angular velocity.

(OR)

This statement is true because when a body is not moving nor rotating then it is in static equilibrium.

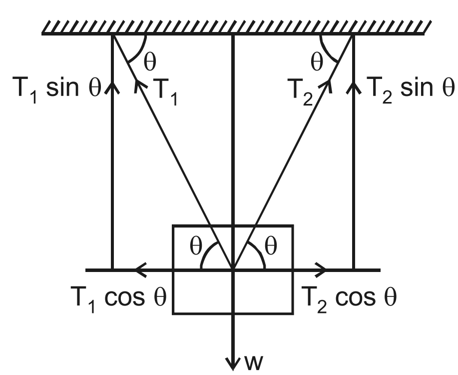
(b) This statement is true because when coplanar forces (vectors) acting on a body in the form of a closed polygon then $\sum \vec{F} = \vec{0}$ i.e., 1^{st} condition of equilibrium, is satisfied so the body is in translational equilibrium.

(OR)

This statement is false because there may be any torque due to these forces i.e., 2^{nd} condition of equilibrium is not satisfied so the body is not in complete equilibrium.

2.20 A picture is suspended from a wall by two strings. Show by diagram the configuration of the strings for which the tension in the settings will be minimum.

Ans. Let we suspend the picture from the wall by two strings as shown in figure.



Let T_1 and T_2 be the tension in string from figure.

Along x-axis

$$T_1 \cos\theta = T_2 \cos\theta$$

$$\Rightarrow \quad T_1 = T_2 = T$$

Along y-axis

$$T_1 \sin\theta + T_2 \sin\theta = w$$

$$T \sin\theta + T \sin\theta = w \qquad \because T_1 = T_2 = T_3$$

$$T = \frac{w}{2 \sin \theta}$$

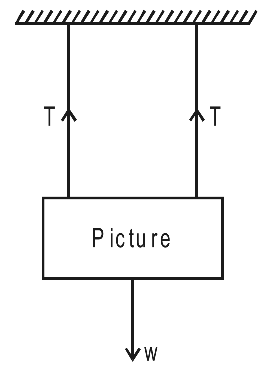
For minimum tension sin θ should have max. value and max. value of sin θ = 1.

$$\Rightarrow \quad \sin \theta = 1$$

$$\sin \theta = \sin 90°$$

$$\theta = 90°$$

So for minimum tension θ should be 90° as shown in figure.



From figure

$$2T = w$$

$$T = \frac{w}{2}$$

So only in this case, the tension in both the string will be minimum.

2.21 Can a body rotate about its centre of gravity under the action of its weight?

Ans. No, a body cannot rotate about its centre of gravity under the action of weight because the line of action of force (weight) passes through its center of gravity (pivot) i.e., movement arm r = 0.

So $\tau = rF$

$$\tau = (0)\,F$$

$$\tau = 0$$

So the torque is zero.

PROBLEMS WITH SOLUTIONS

PROBLEM 2.1

Suppose, in a rectangular coordinates system, a vector $\vec{A}$ has its tail at the point P(–2, –3) and its tip at Q(3, 9). Determine the distance between these two points?

Data

The given points are

P(–2, –3) and Q(3, 9)

To Find

Distance between P and Q = r = ?

SOLUTION

$$\text{Position vector of a} = \vec{r}_1 = -2\hat{i} - 3\hat{j}$$

$$\text{Position vector of b} = \vec{r}_2 = 3\hat{i} + 9\hat{j}$$

$$\Rightarrow \quad \vec{A} = \vec{r}_2 - \vec{r}_1$$

$$= 3\hat{i} + 9\hat{j} - (-2\hat{i} - 3\hat{j})$$

$$= 3\hat{i} + 9\hat{j} + 2\hat{i} + 3\hat{j}$$

$$\vec{A} = 5\hat{i} + 12\hat{j} \qquad \ldots\ldots \text{(i)}$$

The magnitude of $\vec{A}$ is:

$$A = \sqrt{(5)^2 + (12)^2}$$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169}$$

$$= 13 \text{ units}$$

Result

Distance between P and Q = r = 13 units

PROBLEM 2.2

A certain corner of a room is selected as the origin of the rectangular coordinate system. If an insect is crawling on an adjacent wall at a point having coordinates (2, 1), where the units are in meters? What is the distance of the insect from this corner of the room?

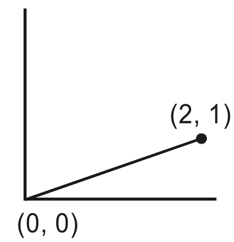
Data

In a coordinate system, the given point is (2, 1)

To Find

Distance of insect from the corner of room = r = ?

SOLUTION



Let the corner of the room whose coordinate are (0, 0)

The position vector is

$$\vec{r} = 2\hat{i} + 1\hat{j}$$

The magnitude of position vector $\vec{r}$ is

$$|\vec{r}| = \sqrt{(2)^2 + (1)^2}$$

$$= \sqrt{4 + 1}$$

$$= \sqrt{5}$$

$$r = 2.24 \text{ m}$$

Result

Distance of insect = r = 2.24 m

PROBLEM 2.3

What is the unit vector in the vector $\vec{A} = 4\hat{i} + 3\hat{j}$?

Data

The given vector is

$$\vec{A} = 4\hat{i} + 3\hat{j}$$

To Find

Unit vector = $\hat{A}$ = ?

SOLUTION

By formula

$$\hat{A} = \frac{\vec{A}}{|A|}$$

$$= \frac{4\hat{i} + 3\hat{j}}{\sqrt{4^2 + 3^2}}$$

$$= \frac{4\hat{i} + 3\hat{j}}{\sqrt{16 + 9}}$$

$$\hat{A} = \frac{4\hat{i} + 3\hat{j}}{5}$$

Result

$$\text{Unit vector} = \hat{A} = \frac{4\hat{i} + 3\hat{j}}{5}$$

PROBLEM 2.4

Two particles are located at $\vec{r}_1 = 3\hat{i} + 7\hat{j}$ and $\vec{r}_2 = -2\hat{i} + 3\hat{j}$ respectively. Find both the magnitude of vector $(\vec{r}_2 - \vec{r}_1)$ and its orientation with respect to the x-axis.

Data

The position vectors are

$$\vec{r}_1 = 3\hat{i} + 7\hat{j}$$

$$\vec{r}_2 = -2\hat{i} + 3\hat{j}$$

To Find

Magnitude of vector $= \vec{r}_2 - \vec{r}_1 = ?$

Direction $= \theta = ?$

SOLUTION

As we know that

$$\vec{r}_2 - \vec{r}_1 = -2\hat{i} + 3\hat{j} - (3\hat{i} + 7\hat{j})$$

$$= -2\hat{i} + 3\hat{j} - 3\hat{i} - 7\hat{j}$$

$$= -5\hat{i} - 4\hat{j}$$

$$|\vec{r}_2 - \vec{r}_1| = \sqrt{(-5)^2 + (-4)^2}$$

$$= \sqrt{25 + 16} = \sqrt{41}$$

$$|\vec{r}_2 - \vec{r}_1| = 6.4$$

Orientation w.r.t. x-axis:

$$\phi = \tan^{-1}\left(\frac{\text{y-component}}{\text{x-component}}\right)$$

$$= \tan^{-1}\left(\frac{4}{5}\right)$$

$$\phi = 39°$$

As both components are –ive so $(\vec{r}_2 - \vec{r}_1)$ lies in 3rd quadrant.

$$\Rightarrow \quad \theta = 180 + \phi$$
$$= 180 + 39$$
$$\theta = 219°$$

Result

Magnitude of vector $\vec{r}_2 - \vec{r}_1 = 6.4$

Orientation $= \theta = 219°$

PROBLEM 2.5

If a $\vec{B}$ is added to vector $\vec{A}$, the result is $6\hat{i} + \hat{j}$. If $\vec{B}$ is subtracted from $\vec{A}$, the result is $-4\hat{i} + 7\hat{j}$. What is the magnitude of vector $\vec{A}$?

Data

If $\vec{B}$ is added to $\vec{A}$

$$\vec{A} + \vec{B} = 6\hat{i} + \hat{j}$$

If $\vec{B}$ is subtracted from $\vec{A}$

$$\vec{A} - \vec{B} = -4\hat{i} + 7\hat{j}$$

To Find

Magnitude of vector $\vec{A} = ?$

SOLUTION

Then

$$\vec{A} + \vec{B} = 6\hat{i} + \hat{j}$$
$$\vec{A} - \vec{B} = -4\hat{i} + 7\hat{j}$$
$$2\vec{A} = 2\hat{i} + 8\hat{j}$$
$$2\vec{A} = 2(\hat{i} + 4\hat{j})$$
$$\vec{A} = \hat{i} + 4\hat{j}$$

Magnitude of vector $\vec{A} = \sqrt{(1)^2 + (4)^2}$
$$= \sqrt{1 + 16}$$
$$= \sqrt{17}$$
$$= 4.12$$

Result

Magnitude of vector $\vec{A} = 4.12$

PROBLEM 2.6

Given that $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = 3\hat{i} - 4\hat{j}$, find the magnitude and direction of

(a) $\vec{C} = \vec{A} + \vec{B}$ (b) $\vec{D} = 3\vec{A} - 2\vec{B}$

Data

The given vectors are

$$\vec{A} = 2\hat{i} + 3\hat{j}$$

$$\vec{B} = 3\hat{i} - 4\hat{j}$$

To Find

(a) Magnitude of $\vec{C} = \vec{A} + \vec{B} = ?$

Direction of $\vec{C} = ?$

(b) Magnitude of $\vec{D} = 3\vec{A} - 2\vec{B} = ?$

Direction of $\vec{D} = ?$

SOLUTION

(a) Now $\vec{C} = \vec{A} + \vec{B}$

Putting the values

$$\vec{C} = 2\hat{i} + 3\hat{j} + 3\hat{j} - 4\hat{j}$$

$$\vec{C} = 5\hat{i} - \hat{j}$$

Magnitude of $\vec{C} = \sqrt{(5)^2 + (-1)^2}$

$$= \sqrt{25 + 1}$$

$$= \sqrt{26}$$

$$= 5.1$$

For direction, $\tan\phi = \dfrac{C_y}{C_x}$

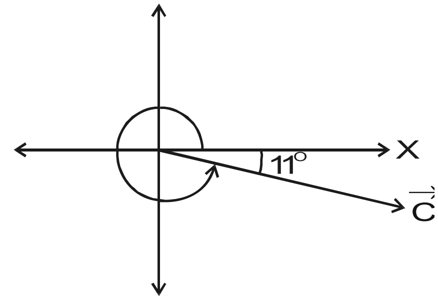
$$\phi = \tan^{-1}\left(\frac{C_y}{C_x}\right)$$ where $C_y = -1$, $C_x = 5$

So $\phi = \tan^{-1}\left(\dfrac{1}{5}\right)$

$$\phi = 11°$$

Since C_x is positive and C_y is negative.

So $\vec{C}$ lies in 4th quadrant.



$$\theta = 360° - 11°$$
$$\theta = 349°$$

(b) Now $\vec{D} = 3\vec{A} - 2\vec{B}$

$$= 3(2\hat{i} + 3\hat{j}) - 2(3\hat{i} - 4\hat{j})$$
$$= 5\hat{i} + 9\hat{j} - 6\hat{i} + 8\hat{j}$$
$$= 17\hat{j}$$

So the magnitude of $\vec{D} = \sqrt{(17)^2}$

$$= 17$$

And the direction of $\vec{D}$ is

$$\theta = \tan^{-1}\left(\frac{D_y}{D_x}\right) \quad \text{where } D_y = 17, \; D_x = 0$$
$$\theta = \tan^{-1}\left(\frac{17}{0}\right)$$
$$= \tan^{-1}(\infty)$$
$$\theta = 90°$$

Result

(a) Magnitude of vector $\vec{C}$ = 5.0

Direction of vector $\vec{C}$ = θ = 349°

(b) Magnitude of vector $\vec{D}$ = 17

Direction of vector $\vec{D}$ = θ = 90°

PROBLEM 2.7

Find the angle between two vectors $\vec{A} = 5\hat{i} + \hat{j}$ and $\vec{B} = 2\hat{i} + 4\hat{j}$.

Data

The given vectors are

$$\vec{A} = 5\hat{i} + \hat{j}$$
$$\vec{B} = 2\hat{i} + 4\hat{j}$$

To Find

Angle between the vectors = θ = ?

SOLUTION

As we know that

$$\vec{A} \cdot \vec{B} = AB \cos\theta$$

$$\boxed{\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB}} \qquad \ldots\ldots \text{(i)}$$

where $\vec{A} \cdot \vec{B} = (5\hat{i} + \hat{j}) \cdot (2\hat{i} + 4\hat{j})$

$$= 10(\hat{i} \cdot \hat{i}) + 20(\hat{i} \cdot \hat{j}) + 2(\hat{j} \cdot \hat{i}) + 4(\hat{j} \cdot \hat{j})$$

Since $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

and $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$

So $\vec{A} \cdot \vec{B} = 10 + 4 = 14$

The magnitude of vector $\vec{A} = \sqrt{5^2 + 1^2}$

$$= \sqrt{25 + 1}$$

$$= \sqrt{26}$$

$$A = 5.1$$

The magnitude of vector $\vec{B} = \sqrt{2^2 + 4^2}$

$$= \sqrt{4 + 16}$$

$$= \sqrt{20}$$

$$B = 4.5$$

Putting the value in eq. (i)

$$\cos\theta = \frac{14}{(5.1)(4.5)}$$

$$\cos\theta = 0.61$$

$$\theta = \cos^{-1}(0.61)$$

$$\theta = 52^\circ$$

Result

Angle between vectors $= \theta = 52^\circ$

PROBLEM 2.8

Find the work done when the point of application of the force $3\hat{i} + 2\hat{j}$ moves in a straight line from the point (2, –1) to the point (6, 4).

Data

Force $= \vec{F} = 3\hat{i} + 2\hat{j}$

and the given points are A(2, –1), B(6, 4).

To Find

Work done = W = ?

SOLUTION

By formula

$$W = \vec{F} \cdot \vec{d}$$

Where $\vec{d}$ is distance between two given points

So Coordinates $\vec{d}$ = (6, 4) – (2, –1)
= (6 – 2, 4 + 1)
= (4, 5)

Now $\vec{d} = 4\hat{i} + 5\hat{j}$

Therefore

$$W = (3\hat{i} + 2\hat{j}) \cdot (4\hat{i} + 5\hat{j})$$
$$= 12(\hat{i} \cdot \hat{i}) + 10(\hat{j} \cdot \hat{j})$$
$$= 12 + 10$$
$$= 22 \text{ Joule}$$

Result

Work done = W = 22 J

PROBLEM 2.9

Show that the three vectors $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} - 3\hat{j} + \hat{k}$ and $4\hat{i} + \hat{j} - 5\hat{k}$ are mutually perpendicular.

Data

The given vectors are

$$\vec{A} = \hat{i} + \hat{j} + \hat{k}$$
$$\vec{B} = 2\hat{i} - 3\hat{j} + \hat{k}$$
$$\vec{C} = 4\hat{i} + \hat{j} - 5\hat{k}$$

To Find

Are the vectors mutually perpendicular = ?

SOLUTION

If $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{C} = \vec{C} \cdot \vec{A} = 0$ so the vectors are mutual perpendicular.

Thus
$$\vec{A} \cdot \vec{B} = (\hat{i} + \hat{j} + \hat{k}) \cdot (2\hat{i} - 3\hat{j} + \hat{k})$$
$$= 2(\hat{i} \cdot \hat{i}) - 3(\hat{j} \cdot \hat{j}) + 1(\hat{k} \cdot \hat{k})$$
$$= 2 - 3 + 1$$
$$\vec{A} \cdot \vec{B} = 0$$

and
$$\vec{B} \cdot \vec{C} = (2\hat{i} - 3\hat{j} + \hat{k}) \cdot (4\hat{i} + \hat{j} - 5\hat{k})$$
$$= 8(\hat{i} \cdot \hat{i}) - 3(\hat{j} \cdot \hat{j}) - 5(\hat{k} \cdot \hat{k})$$
$$= 8 - 3 - 5$$
$$\vec{B} \cdot \vec{C} = 0$$
$$\vec{C} \cdot \vec{A} = (4\hat{i} + \hat{j} - 5\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k})$$
$$= 4(\hat{i} \cdot \hat{i}) + 1(\hat{j} \cdot \hat{j}) - 5(\hat{k} \cdot \hat{k})$$
$$= 4 + 1 - 5$$
$$\vec{C} \cdot \vec{A} = 0$$

Hence
$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} = \vec{C} \cdot \vec{A} = 0$$

Result

So the given three vectors are mutually perpendicular.

PROBLEM 2.10

Given that $\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{B} = 3\hat{i} - 4\hat{k}$, find the length of the projection of $\vec{A}$ on $\vec{B}$.

Data

The given vectors are

$$\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}$$
$$\vec{B} = 3\hat{i} - 4\hat{k}$$

To Find

Projection of $\vec{A}$ on $\vec{B}$ = $A \cos\theta$ = ?

SOLUTION

By formula

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

$$\boxed{A\cos\theta = \frac{\vec{A} \cdot \vec{B}}{B}}$$

Now $\vec{A} \cdot \vec{B} = (\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (3\hat{i} - 4\hat{k})$

$$= 3(\hat{i} \cdot \hat{i}) - 12(\hat{k} \cdot \hat{k})$$

$$= 3 - 12$$

$$= -9$$

And magnitude of vector $\vec{B}$

$$B = \sqrt{(3)^2 + (-4)^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25}$$

$$= 5$$

Putting in above equation

$$A\cos\theta = \frac{-9}{5}$$

Result

Projection of $\vec{A}$ on $\vec{B}$ $= A\cos\theta = -\frac{9}{5}$

PROBLEM 2.11

Vector $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 4 unit north, 3 unit west and 8 unit east, respectively. Describe carefully (a) $\vec{A} \times \vec{B}$ (b) $\vec{A} \times \vec{C}$ (c) $\vec{B} \times \vec{C}$.

Data

The given vectors are

$$\vec{A} = 4 \text{ units north}$$

$$\vec{B} = 3 \text{ units west}$$

$$\vec{C} = 8 \text{ units east}$$

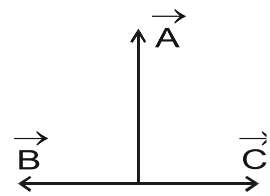
To Find

(a) $\vec{A} \times \vec{B} = ?$

(b) $\vec{A} \times \vec{C} = ?$

(c) $\vec{B} \times \vec{C} = ?$

SOLUTION



(a) As $\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n}$

$\vec{A} \times \vec{B} = (4)(3) \sin 90°\, \hat{n}$

= 12 units vertically upward according to right hand rule

(b) $\vec{A} \times \vec{C} = AC \sin\theta\, \hat{n}$

$= (4)(8) \sin 90°\, \hat{n}$

= 32 units vertically downward

(c) $\vec{B} \times \vec{C} = BC \sin\theta\, \hat{n}$

$= (3)(8) \sin 180°\, \hat{n}$

$= 24(0)\, \hat{n}$

$\vec{B} \times \vec{C} = \vec{0}$

Result

(a) $\vec{A} \times \vec{B}$ = 12 units vertically upward

(b) $\vec{A} \times \vec{C}$ = 32 units vertically downward

(c) $\vec{B} \times \vec{C} = \vec{0}$ (Null vector)

PROBLEM 2.12

The torque or turning effect of force about a given point is given by $\vec{r} \times \vec{F}$ where $\vec{r}$ is the vector from the given point to the point of application of $\vec{F}$. Consider a force $\vec{F} = -3\hat{i} + \hat{j} + \hat{k}$ acting on the point $7\hat{i} + 3\hat{j} + \hat{k}$ (m). What is the torque in Nm about the origin?

Data

Force $= \vec{F} = -3\hat{i} + \hat{j} + 5\hat{k}$

Position vector $= \vec{r} = 7\hat{i} + 3\hat{j} + \hat{k}$

To Find

Torque acting $= \vec{\tau} = \vec{r} \times \vec{F} = ?$

SOLUTION

As $\vec{\tau} = \vec{r} \times \vec{F}$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 3 & 1 \\ -3 & 1 & 5 \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix} - \hat{j}\begin{vmatrix} 7 & 1 \\ -3 & 5 \end{vmatrix} + \hat{k}\begin{vmatrix} 7 & 3 \\ -3 & 1 \end{vmatrix}$$

$$= \hat{i}\,(15 - 1) - \hat{j}\,(35 + 3) + \hat{k}\,(7 + 9)$$

$$= 14\hat{i} - 38\hat{j} + 16\hat{k}$$

Result

Torque acting $= \vec{\tau} = 14\hat{i} - 38\hat{j} + 16\hat{k}$ (Nm)

PROBLEM 2.13

The line of action of force $\vec{F} = \hat{i} - 2\hat{j}$, passes through the point whose position vector is $(-\hat{j} + \hat{k})$. Find (a) the moment of F about the origin (b) the moment of $\vec{F}$ about the point of which the position vector is $\hat{i} + \hat{k}$.

Data

Force $= \vec{F} = \hat{i} - 2\hat{j}$

Position vector $= \vec{r} = -\hat{j} + \hat{k}$

To Find

(a) Moment of force about origin $= \vec{\tau} = ?$

(b) Moment of force about $\hat{i} + \hat{k} = \vec{\tau'} = ?$

SOLUTION

(a) First for the moment of force about origin is

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -1 & 1 \\ 1 & -2 & 0 \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} -1 & 1 \\ -2 & 0 \end{vmatrix} - \hat{j}\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} 0 & -1 \\ 1 & -2 \end{vmatrix}$$

$$= \hat{i}(0+2) - \hat{j}(0-1) + \hat{k}(0+1)$$

$$\vec{\tau} = 2\hat{i} + \hat{j} + \hat{k}$$

(b) The moment of force about $\hat{i} + \hat{k}$ is

$$\vec{\tau} = \vec{r} \times \vec{F}$$

So $$\vec{r}' = (-\hat{j} + \hat{k}) - (\hat{i} + \hat{k})$$

$$= -\hat{j} + \hat{k} - \hat{i} - \hat{k}$$

$$= -\hat{i} - \hat{j}$$

So $$\vec{\tau}' = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 0 \\ 1 & -2 & 0 \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} -1 & 0 \\ -2 & 0 \end{vmatrix} - \hat{j}\begin{vmatrix} -1 & 0 \\ 1 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} -1 & -1 \\ 1 & -2 \end{vmatrix}$$

$$= \hat{i}(0) - \hat{j}(0) + \hat{k}(2+1)$$

$$= 0\hat{i} - 0\hat{j} + 3\hat{k}$$

$$\vec{\tau}' = 3\hat{k}$$

Result

(a) Moment of force about origin $= \vec{\tau} = 2\hat{i} + \hat{j} + \hat{k}$

(b) Moment of force about $\hat{i} + \hat{k} = \vec{\tau}' = 3\hat{k}$

PROBLEM 2.14

The magnitude of dot and cross products of two vectors are $6\sqrt{3}$ and 6 respectively. Find the angle between the vector.

Data

Magnitude of dot product $= \vec{A}\cdot\vec{B} = 6\sqrt{3}$

Magnitude of vector product $= |\vec{A}\times\vec{B}| = 6$

To Find

Angle between the vectors $= \theta = ?$

SOLUTION

As $\vec{A}\cdot\vec{B} = AB\cos\theta$ …… (i)

$|\vec{A}\times\vec{B}| = AB\sin\theta$ …… (ii)

Divide eq. (ii) by (i)

$$\frac{|\vec{A}\times\vec{B}|}{\vec{A}\cdot\vec{B}} = \frac{AB\sin\theta}{AB\cos\theta}$$

$$\frac{6}{6\sqrt{3}} = \tan\theta$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\theta = 30^\circ$$

Result

Angle between vectors $= \theta = 30^\circ$

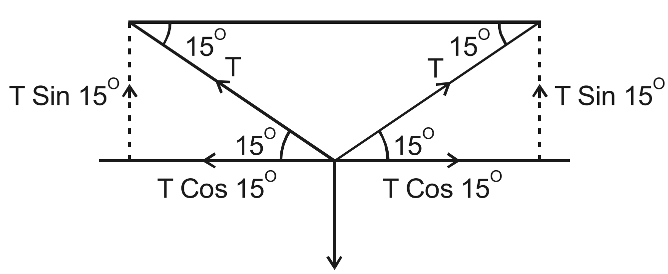
PROBLEM 2.15

A load of 10N is suspended from a clothes line. This distorts the line so that it makes an angle of 15° with the horizontal at each end. Find the tension in the clothes line.

Data

Load $= W = 10\text{ N}$

Angle with horizontal $= \theta = 15^\circ$



To Find

Tension in the string $= T = ?$

SOLUTION

According to 1st condition of equilibrium.

Now using $\sum F_y = 0$ and $\sum F_x = 0$

So $T_1 \sin\theta + T_2 \sin\theta - \text{Load} = 0$ $\qquad T_2 \cos 15° - T_1 \cos 15° = 0$

$2T \sin\theta = \text{Load}$ $\qquad T_2 \cos 15° = T_1 \cos 15°$

$T = \dfrac{\text{Load}}{2 \sin\theta}$ $\qquad T_1 = T_2$

$= \dfrac{5}{0.2588}$ $\qquad \therefore \quad T_1 = T_2 = T$

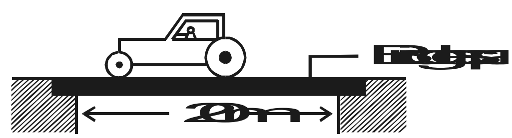
$T = 19.3 \text{ N}$

Result

Tension in string $= T = 19.3$ N

PROBLEM 2.16

A tractor of weight 15000N crosses a single span bridge of weight 8000 N and of length 21m. The span bridge is supported half a metre from either end. The bridge is supported half a metre from either end. The tractors front wheels take 1/3 of the total weight of the tractor, and the rear wheels are 3m behind the front wheels. Calculate the force on the bridge supports when the rear wheels are at the middle of the bridge span.



Data

Weight of tractor $= W_1 = 15000$ N

Weight of bridge $= W_2 = 8000$ N

Length of bridge $= l_1 = 21$ m

Length of tractor $= l_2 = 3.0$ m

Weight of front wheels $= W_3 = \dfrac{1}{3} \times 15000$

$= 5000$ N

Weight of rear wheels $= W_4 = 15000 - 5000$

$= 10000$ N

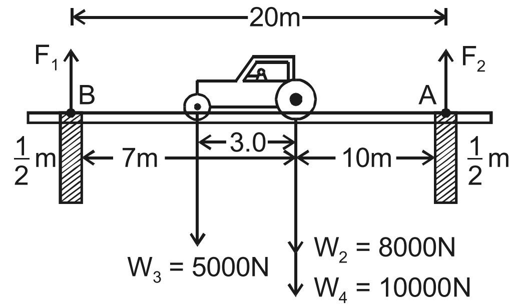
To Find

Forces on the bridge supports

$F_1 = ?$

and $F_2 = ?$

SOLUTION



By using 1st condition of equilibrium

$$\sum F_y = 0$$

$$F_1 + F_2 - 5000 - 8000 - 10000 = 0$$

$$F_1 + F_2 = 23000 \text{ N} \quad \ldots\ldots \text{(i)}$$

According to 2nd condition of equilibrium

$$\sum \tau = 0$$

Suppose the point B is the point of rotation therefore

Torque of Force $F_1 = \tau_1 = F_1 \times 0$

$= 0$

Torque of Force $F_2 = \tau_2 = F_2 \times 20$

Torque of Force W_2 and $W_4 = \tau_3 = -18000 \times 10$

Torque of Force $W_3 = \tau_4 = -5000 \times 7$

Then $\tau_1 + \tau_2 + \tau_3 + \tau_4 = 0$

$$0 + F_2 \times 20 - 180000 - 35000 = 0$$

$$20F_2 = 215000$$

$$F_2 = \frac{215000}{20}$$

$$F_2 = 10750 \text{ N}$$

$$F_2 = 10.750 \times 10^3 \text{ N}$$

$$F_2 = 10.75 \text{ KN}$$

Putting in eq. (i)

$$F_1 + 10750 = 23000$$

$$F_1 = 23000 - 10750$$

$$= 12250 \text{ N}$$

$$F_1 = 12.25 \times 10^3 \text{ N}$$

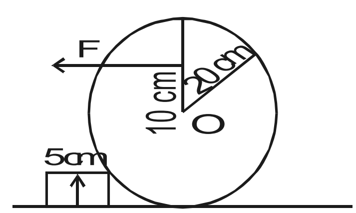
$$= 12.25 \text{ KN}$$

Result

Force on the bridge supports

$F_1 = 12.25$ KN and $F_2 = 10.75$ KN

PROBLEM 2.17



A spherical ball of weight 50N is to be lifted over the step as shown in the figure.

Calculate the minimum force needed just to lift it above the floor.

Data

Weight of the spherical ball = W = 50 N

Radius of the spherical ball = r = 20 cm

= 0.2 m

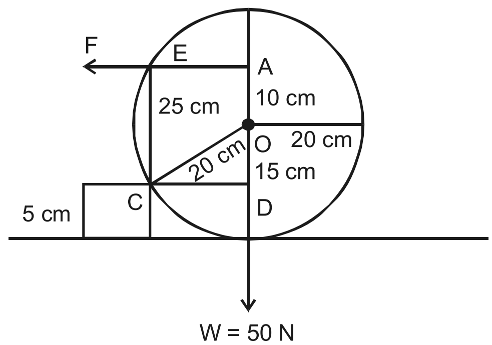
Height of the step = h = 5 cm

= 0.05 m

To Find

Minimum force required to lift the ball = F = ?

SOLUTION



Now consider the triangle ΔOCD is right angled triangle, by Pythagorean theorem

$$(OC)^2 = (OD)^2 + (CD)^2$$

$$(CD)^2 = (OC)^2 - (OD)^2$$

$$= (20)^2 - (15)^2$$

$$\sqrt{(CD)^2} = \sqrt{1750}$$

$$= 13.2 \text{ cm}$$

$$CD = 0.13 \text{ m}$$

By using 2nd condition of equilibrium

$$\sum\tau = 0$$

So $$-W \times CD + F \times CB = 0$$

$$-50 \times 0.13 + F \times 0.25 = 0$$

$$0.25\ F = 6.5$$

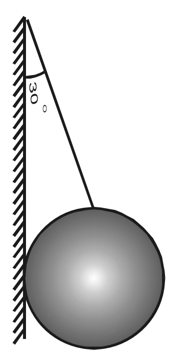
$$F = \frac{6.5}{0.25}$$

$$F = 26 \text{ N}$$

Result

Minimum force required to left ball = F = 26 N

PROBLEM 2.18



A uniform sphere of weight 10 N is held by a string attached to a frictionless wall so that the string makes an angle of 30° with the wall. Find the tension in the string and the force exerted on the sphere by the wall.

Data

Weight of the sphere = W = 10.0 N

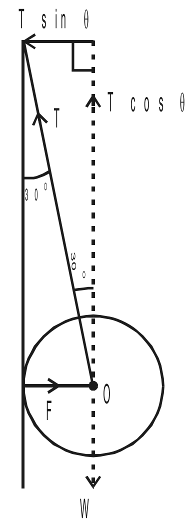
Angle of string with wall = θ = 30°

To Find

(a) Tension in the string = T = ?

(b) Force exerted by the wall = F = ?

SOLUTION



(a) Let F be the force exerted on sphere by wall, T is the tension in string and W is the weight of the sphere.

By using 1st condition of equilibrium

$$\sum F_y = 0 \quad \text{and} \quad \sum F_x = 0$$

So $F - T\sin\theta = 0$

$$F = T\sin\theta$$

and $T\cos\theta - W = 0$

$$T\cos\theta = W$$

$$T = \frac{W}{\cos\theta}$$

$$= \frac{10.0}{\cos 30} = \frac{10.0}{0.866}$$

$$T = 11.55\text{ N}$$

and $F = T\sin\theta$

$$= 11.55 \sin 30°$$

$$F = 5.8\text{ N}$$

Result

(a) Tension in the string = T = 11.55 N

(b) Force exerted by the wall = F = 5.8 N