SHORT QUESTIONS

1.1 Name several repetitive phenomenon occurring in nature which could serve as reasonable time standard.

Ans. Any natural phenomenon that repeats itself after exactly same time interval can be used as a measure of time. The repetitive phenomenon could serve as reasonable time standard, occurring in nature are as follows:

1. **Sun:** Sun served as reasonable time standard because sunset and sunrises gives the information of time.
2. **Moon:** Moon is also reasonable time standard because it gives the information of time.
3. **Weather:** Changing of weather can also give information about time.
4. Rotation of Earth on its axis.
5. Rotation of Earth around the sun.
6. Oscillation of a simple pendulum.

1.2 Give draw backs to use the period of a pendulum as a time standard.

Ans. As we know that the time period of a simple pendulum depends upon the length and value of g at any place. Since

$$T = 2\pi\sqrt{\frac{l}{g}}$$

(i) It is clear that time period of a simple pendulum depends upon the value of g which is different at different places. So a pendulum of same length may have different time period at difference places. So period of pendulum cannot be taken as standard for measuring time.

(ii) **Friction:** Time period of a simple pendulum changes due to air resistance.

(iii) **Temperature:** In summer due to increase in temperature, length of simple pendulum changes so time period changes.

1.3 Why do we find it useful to have two units for the amount of substance, the kilogram and the mole?

Ans. It is very useful to have two units for the amount of substance i.e., kilogram and mole. If we want to consider a specific amounts of mass without considering number of microscopic atoms present in it, it is useful to use **kilogram**. Because one kilogram of different substances contains different number of molecules. While if we want to consider a fixed number of atoms present in it then it is useful to use **mole**. Because one mole of any substance contains the same number of atoms or molecules.

1.4 Three student's measured the length of a needle with a scale on which minimum division is 1 mm and recorded as (i) 0.2145m (ii) 0.21m (iii) 0.214m. Which record is correct and why?

Ans. In these records (iii) 0.214 m is more correct than the other records because the least count of a scale is 1 mm which can be written as 0.001 m. So according to this figure, the student measure that type of record is correct.

1.5 An old saying is that "A chain is only as strong as its weakest link". What analogue statement can you make regarding experimental data used in computation?

Ans. The analogous statement regarding experimental data used in computation is "A result obtained from an experimental data used in computation is only as accurate as its least accurate reading".

1.6 The period of simple pendulum is measured by a stopwatch. What type of errors are possible in the time period?

Ans. When the period of a simple pendulum is measured by a stopwatch, the following types of errors are possible:

1. **Systematic Error:** The error due to the fault in the measuring instrument is called systematic error i.e., zero error.
2. **Personal Error:** The error due to the faulty procedure of an observer is called personal error.

1.7 Does dimensional analysis give any information on constant of proportionality that may appear in an algebraic expression? Explain.

Ans. Dimensional analysis does not give any information about the constant of proportionality or dimensionless constant. For example

$$v = \text{Constant} \times \sqrt{\frac{E}{\rho}}$$

The numerical value of this constant cannot determined by dimensional analysis.

1.8 Write the dimension of:

(i) Pressure (ii) Density

Ans. (i) Dimensions of Pressure:

$$\text{As} \quad P = \frac{F}{A} = \frac{ma}{A}$$

$$\text{Unit of P} = \frac{\text{kg ms}^{-2}}{\text{m}^2} = \text{kg m}^{-1}\text{s}^{-2}$$

$$\Rightarrow \quad [P] = [ML^{-1}T^{-2}]$$

(ii) Dimensions of Density:

$$\text{As} \quad \text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Unit of density} = \frac{\text{kg}}{\text{m}^3} = \text{kg m}^{-3}$$

$$[\text{Density}] = [ML^{-3}]$$

1.9 The wavelength λ of a wave depends on the speed v of the wave and its frequency f. Knowing that:

$[\lambda] = [L], [v] = [LT^{-1}]$ and $[f] = [T^{-1}]$

Decide which of the following is correct, $f = v\lambda$ or $f = \frac{v}{\lambda}$.

Ans. In 1st case if $f = v\lambda$ where f is frequency. Its dimension is $[T^{-1}]$, v is speed, its dimensions are $[LT^{-1}]$.

λ is the wavelength, its dimension is [L].

So, $$[T^{-1}] = [LT^{-1}][L]$$

$$[T^{-1}] = [L^2T^{-1}]$$

Hence the equation $f = v\lambda$ is not dimensionally correct because left hand side dimension is not equal to right hand side dimension.

In second case

$$f = \frac{v}{\lambda}$$

So $$[T^{-1}] = \frac{[LT^{-1}]}{[L]}$$

$$[T^{-1}] = [T^{-1}]$$

Hence the equation $f = \frac{v}{\lambda}$ is dimensionally correct because left hand side dimensions is equal to right hand side dimension.

PROBLEMS WITH SOLUTIONS

PROBLEMS 1.1

A light year is the distance light travels in one year. How many metres are there in one light year? (Speed of light = 3.0×10^8 ms^{-1})

Data

Time = t = 1 year

= 365 days

= $365 \times 24 \times 3600$

= 31536000 sec.

Speed of light = C = 3×10^8 m/s

To Find

Distance covered by light = d = ?

SOLUTION

As we know that

$$v = \frac{d}{t}$$

$$d = v \times t$$

$$= C \times t$$

$$= 3 \times 10^8 \times 31536000$$

$$= 94608000 \times 10^8$$

$$= 9.46 \times 10^{15} \text{ m}$$

Result

Distance covered by light = d = 9.46×10^{15} m

PROBLEM 1.2

(a) How many seconds are there in 1 year?

(b) How many nanoseconds in 1 year?

(c) How many years in 1 second?

Data

One year = 365 days

= $365 \times 24 \times 3600$

= 3.1536000

= 3.15×10^7 sec.

To Find

(a) Seconds in one year = ?

(b) Nanosecond in one year = ?

(c) Years in one second = ?

SOLUTION

(a) As we know that

$$1 \text{ year} = 365 \text{ days}$$
$$= 365 \times 24 \times 3600$$
$$= 3.15 \times 10^7 \text{ sec.}$$

Seconds in one year $= 3.15 \times 10^7$ sec.

(b) As 1 year $= 3.15 \times 10^7 \times 10^9$ n sec. Since 1 ns $= 10^{-9}$ s

$= 3.15 \times 10^{16}$ nanosecond $\therefore$ 1 s $= 10^9$ ns

(c) As 1 year $= 3.15 \times 10^7$ sec.

$$\frac{1}{3.15 \times 10^7} \text{ year} = 1 \text{ second}$$

$$1 \text{ second} = 0.317 \times 10^{-7} \text{ years}$$
$$= 3.17 \times 10^{-8} \text{ years}$$

Result

(a) Number of seconds in one year $= 3.15 \times 10^7$ seconds

(b) Number of nanoseconds in one year $= 3.15 \times 10^{16}$ nanosecond

(c) Number of years in one second $= 3.17 \times 10^{-8}$ years

PROBLEM 1.3

The length and width of a rectangular plate are measured to be 15.3 cm and 12.80 cm, respectively. Find the area of the plate.

Data

Length of rectangular plate = L = 15.3 cm

Width of rectangular plate = W = 12.80 cm

To Find

Area of the plate = A = ?

SOLUTION

As we know that

$$\text{Area} = \text{Length} \times \text{Width}$$
$$= 15.3 \times 12.80$$
$$= 195.84 \text{ cm}^2$$

Result

Area of rectangular plate = A = 196 cm^2

PROBLEM 1.4

Add the following masses given in kg upto appropriate precision. 2.189, 0.089, 11.8 and 5.32.

Data

The given masses are 2.189, 0.089, 11.8 and 5.32

To Find

Sum of masses upto appropriate precision = ?

SOLUTION

Sum of masses = 2.189 + 0.089 11.8 + 5.32

= 19.398

= 19.4 kg

Result

Sum of masses upto appropriate precision = 19.4 kg

PROBLEM 1.5

Find the value of 'g' and its uncertainty using $T = 2\pi\sqrt{\frac{l}{g}}$ from the following measurements made during an experiment.

Length of simple pendulum l = 100 cm

Time for 20 vibrations = 40.2 s

Length was measured by a metre scale of accuracy upto 1 mm and time by stop watch of accuracy upto 0.1 s.

Data

Length of simple pendulum = l = 100 cm

= 1 m

Time for 20 vibration = t = 40.2 s

Time period = $T = \frac{t}{20} = \frac{40.2}{20} = 2.01$ sec.

Least count of metre scale = 1 mm = 0.1 cm

To Find

Acceleration due to gravity = g = ?

SOLUTION

As we know that

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Squaring

$$T^2 = 4\pi^2 \times \frac{l}{g}$$

$$\boxed{g = \frac{4\pi^2 l}{T^2}}$$

Putting the values

$$g = \frac{4(3.14)^2 \times 1}{(2.01)^2}$$

$$= \frac{39.4384}{4.04}$$

$$g = 9.76 \text{ m/s}^2$$

Since Uncertainty in length = 0.1 cm

% uncertainty in length = 0.1%

% uncertainty in time $= \frac{0.005}{2.01} \times 100$ Since $\frac{0.1}{20} = 0.005$ sec.

= 0.25%

Thus Total uncertainty in "g" = % uncertainty in time + 2(% uncertainty in time)

= 0.1 + 2(0.25)

= 0.1 + 0.5

= 0.6%

Thus Uncertainty in calculated value of g $= \frac{0.6}{100} \times 0.97$

$= 0.06 \text{ m/s}^2$

Hence $g = (9.76 \pm 0.06) \text{ m/s}^2$

Result

Acceleration due to gravity $= g = (9.76 \pm 0.06) \text{ m/s}^2$

PROBLEM 1.6

What are the dimensions and units of gravitational constant G in the formula?

$$\mathbf{F = G\frac{m_1 m_2}{r^2}}$$

Data

The given formula is

$$F = G\frac{m_1 m_2}{r^2}$$

To Find

Dimensions of G = ?

Unit of G = ?

SOLUTION

Now for dimensions

$$F = G\frac{m_1 m_2}{r^2}$$

$$G = \frac{F \times r^2}{m_1 m_2} \qquad \text{Since} \quad F = ma$$

$$= \frac{ma \times r^2}{m_1 \times m_2}$$

$$\text{Unit of G} = \frac{kg\ .\ m/s^2 \times m^2}{kg\ .\ kg}$$

$$= \frac{m^3}{kg\ s^2}$$

$$\text{Dimensions of G} = [M^{-1}L^3T^{-2}]$$

For unit of G

$$G = \frac{F \times r_2}{m_1 \times m_1} = \frac{N\text{-}m^2}{kg\ .\ kg}$$

$$G = Nm^2/kg^2$$

Result

$$\text{Dimensions of G} = [M^{-1}L^3T^{-2}]$$

$$\text{Unit of G} = N\text{-}m^2/kg^2$$

PROBLEM 1.7

Show that the expression $v_f = v_i + at$ dimensionally correct, where v_1 is the velocity at t = 0, a is acceleration and v_f is the velocity at time t.

Data

The given equation is

$$v_f = v_i + at$$

To Find

Is the equation dimensionally correct = ?

SOLUTION

Now $v_f = v_i + at$

In unit form

$$\frac{m}{s} = \frac{m}{s} + \frac{m}{s^2} \times s$$

$$\frac{m}{s} = \frac{m}{s} + \frac{m}{s}$$

$$\frac{L}{T} = \frac{L}{T} + \frac{L}{T}$$

Where 2 is constant so it is dimensionless

$$\frac{L}{T} = \frac{L}{T}$$

$$[LT^{-1}] = [LT^{-1}]$$

$$[LT^{-1}] = [LT^{-1}]$$

Result

Hence the equation $v_f = v_i + at$ is dimensionally correct.

PROBLEM 1.8

The speed v of sound waves through a medium may be assumed to depend on (a) the density ρ of the medium and (b) its modulus of elasticity E which is the ratio of stress to strain. Deduce by the method of dimensions, the formula for the speed of sound.

SOLUTION

As we know that the speed of sound depends upon the following two factors

(i) Density ρ^a and (ii) Elasticity E^b

Since $v \propto \rho^a E^b$

$$v = \text{Constant} \times \rho^a E^b \qquad \ldots\ldots \text{(i)}$$

Writing dimensions of quantities on both the sides.

Dimensions of velocity $v = \left[\frac{S}{T}\right] = [LT^{-1}]$

The dimensions of density $\rho = \frac{\text{Mass}}{\text{Volume}}$

$$\text{Unit of } \rho = \frac{kg}{m^3}$$

$$[\,\rho\,] = [ML^{-3}]$$

and Dimensions of elasticity $E = \frac{\text{Stress}}{\text{Strain}} = \frac{F}{A}$

Where strain has no dimensions

$$E = \frac{ma}{A}$$

$$\text{Unit of E} = \frac{kg\ m/s^2}{m^2}$$

$$[\,E\,] = \frac{kg}{m.s^2} = [ML^{-1}T^{-2}]$$

Putting in equation (i)

$$[LT^{-1}] = \text{Constant } [ML^{-3}]^a\ [ML^{-1}T^{-2}]^b$$

$$[LT^{-1}] = \text{Constant} \times [M^aL^{-3a}][M^bL^{-b}T^{-2b}]$$

$$= \text{Constant} \times [M^{a+b}\ L^{-3a-b}\ T^{-2b}]$$

Comparing the exponents

For L $\quad -3a - b = 1$

For T $\quad -2b = -1$

For M $\quad a + b = 0$

As $\quad -2b = -1$

$$b = \frac{1}{2}$$

and $\quad a + b = 0$

$$a + \frac{1}{2} = 0$$

$$\boxed{a = -\frac{1}{2}}$$

Putting the values in eq. (i)

$$v = \text{Constant} \times \rho^{-1/2} E^{1/2}$$

$$v = \text{Constant} \times \frac{E^{1/2}}{\rho^{1/2}}$$

$$v = \text{Constant} \times \sqrt{\frac{E}{\rho}}$$

Result

The formula for the speed of sound is

$$v = \text{Constant} \times \sqrt{\frac{E}{\rho}}$$

PROBLEM 1.9

Show that the famous "Einstein equation" $E = mc^2$ is dimensionally consistent.

SOLUTION

The given equation is

$$E = mc^2$$

Writing the dimension of both sides

Dimension of energy (E) = Work = F . d

$$= ma \cdot d$$

$$\text{Unit of work} = kg \frac{m}{s^2} \cdot m$$

$$= kg \frac{m^2}{s^2}$$

$$[W] = [ML^2T^{-2}] \quad \ldots\ldots \text{(i)}$$

$$\text{Unit of } mc^2 = kg \left(\frac{m}{s}\right)^2$$

$$= kg \frac{m^2}{s^2}$$

$$[mc^2] = [ML^2T^{-2}] \quad \ldots\ldots \text{(ii)}$$

From eq. (i) and (ii)

$$[ML^2T^{-2}] = [ML^2T^{-2}]$$

Result

Hence the Einstein's equation $E = mc^2$ is dimensionally consistent.

PROBLEM 1.10

Suppose, we are told that the acceleration of a particle moving in a circle of radius r with uniform speed v is proportional to some power of r, say r^n, and some power of v, say v^m, determine the powers of r and v?

SOLUTION

According to statement, the acceleration of particle moving in a circle can be written as

$$a \propto r^n v^m$$

$$a = \text{Constant} \times r^n v^m \quad \ldots\ldots \text{(i)}$$

Writing the dimension of both sides

Dimensions of acceleration $= [\,a\,] = [LT^{-2}]$

Dimensions of radius $= [\,r\,] = m = [\,L\,]$

Dimensions of velocity $= [\,v\,] = [LT^{-1}]$

Putting in eq. (i)

$$[LT^{-2}] = \text{Constant} \times [\,L\,]^n [LT^{-1}]^m$$

$$[LT^{-2}] = \text{Constant} \times [\,L^n\,] [L^m T^{-m}]$$

$$[LT^{-2}] = \text{Constant} \times [L^{n+m}\, T^{-m}]$$

Comparing the exponents

$$n + m = 1$$

$$-m = -2$$

$$\boxed{m = 2}$$

Putting in above

$$n + 2 = 1$$

$$n = 1 - 2$$

$$\boxed{n = -1}$$

Putting in eq. (i)

$$a = \text{Constant} \times r^{-1} v^2$$

$$a = \text{Constant} \times \frac{v^2}{r}$$

Result

The acceleration of a particle moves with velocity in a circle of radius r is

$$a = \text{Constant} \times \frac{v^2}{r}$$