MATHEMATICS 11 INTERMEDIATE
PART 1

Bilal Article

Chapter 5.

Partial Fraction

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Contents

Rational fraction:

The quotient of two polynomials $\frac{P(x)}{Q(x)}$ $where\ Q(x) \neq 0$ with no common factor is called rational fraction.

For example

$$\frac{x^2+1}{x^2-1}, \frac{x^4}{x^2+1}$$

Proper rational fraction:

A rational fraction $\frac{P(x)}{Q(x)}$ is called a proper rational fraction is the degree of polynomial $P(x)$ is less than the degree of polynomial $Q(x)$

For example

$$\frac{2x-5}{x^2+4}, \frac{3}{x+1}$$

Improper rational fraction:

A rational fraction $\frac{P(x)}{Q(x)}$ is called an improper ration fraction if the degree of the polynomial $P(x)$ is greater than or equal to the degree of polynomial $Q(x)$

$$\frac{3x^2+1}{x-1}, \frac{x^4}{x^2-1}$$

Partial fraction:

To express a single rational fraction as a sum of two or more single rational fractions is called partial fraction.

Partial fraction resolution:

Expressing a rational fraction as a sum of partial fraction is called partial fraction resolution.

Conditional equation:

It is an equation which is true for a particular valves of the variable

For example:

$$2x = 3 \text{ is true only } x = \frac{3}{2}$$

For simplicity, a conditional equation is called an equation.

Identity:

It as an equation which holds good for all valves of variable.

For example

$$(a+b)x = ax + bx$$

The symbol " $=$ " be used both for equation and identity.

Resolution of a rational fraction $\frac{P(x)}{Q(x)}$ in to partial fractions.

Following are the main points of resolving a rational fraction $\frac{P(x)}{Q(x)}$ in to partial fraction.

i The degree of $P(x)$ must be less than that of $Q(x)$. If not, divide and and work with the remainder theorem.

ii Clear the given equation of fractions.

iii Equate the coefficients of like term s (power of x)

iv Solve the resulting equations for the coefficients.

Case I

Resolution of $\frac{P(x)}{Q(x)}$ in to partial fractions when Q(x) has only repeated linear factors.

The polynomial $Q(x)$ may be written as

$Q(x) = (x-a_1)(x-a_2)\dots(x-a_n)$ where

$$a_1 \neq a_2 \neq \cdots \neq a_n$$

$$\because \frac{P(x)}{Q(x)} = \frac{A_1}{x-a_1} + \frac{A_2}{x-a_2} + \cdots + \frac{A_n}{x-a_n}$$

Are numbers to be found.

Note:

Which can be factorize, first of all can be factorized it

- **We uses partial fraction when the fraction $\frac{P(x)}{Q(x)}$ is proper rational fraction.**
- **If we are given improper fraction (division is possible) then first of all divide the fraction and make it proper fraction. After this sues partial fraction.**

Exercise 5.1

Resolve the following into partial fractions.

Question No.1

$$\frac{1}{x^2-1}$$

Solution:

$$\frac{1}{x^2-1} = \frac{1}{(x-1)(x+1)}$$

$\frac{1}{(x-1)(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x+1)}$ .... (Z)

Multiply both sides by (x-1)(x+1)

1=A(x+1)+B(x-1) ....(1)

Put x-1=0 $\Longrightarrow$ x=1 in equation (1)

1=A(1+1)+B(1-1)

1=A(2) $\Longrightarrow$ A=$\frac{1}{2}$

Now put x+1=0 $\Longrightarrow$ x= -1 in equation (1)

1=A(-1+1)+B(-1-1)

1=B(-2) $\Longrightarrow$B=$-\frac{1}{2}$

Now put A and B in equation (Z)

Hence $\frac{1}{(x-1)(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x+1)}$

$= \frac{1}{2(x-1)} - \frac{1}{2(x+1)}$

Question No.2

$$\frac{x^2+1}{(x+1)(x-1)}$$

Solution:- $\frac{x^2+1}{(x+1)(x-1)} = \frac{x^2+1}{x^2-1}$

$\frac{x^2+1}{x^2-1} = 1 + \frac{2}{x^2-1}$

Now consider $\frac{2}{(x+1)(x-1)} = \frac{A}{(x-1)} + \frac{B}{(x+1)}$ .... (Z)

Multiply both sides by (x-1)(x+1)

2=A(x+1)+B(x-1) ....(1)

Put x-1=0 $\Longrightarrow$ x=1 in equation (1)

2=A(1+1)+B(1-1)

2=A(2) $\Longrightarrow$ A=1

Now put x+1=0 $\Longrightarrow$ x= -1 in equation (1)

1=A(-1+1)+B(-1-1)

2=B(-2) $\Rightarrow$B=-1

Now put A and B in equation (Z)

$\frac{1}{(x-1)(x+1)}=\frac{A}{(x-1)}+\frac{B}{(x+1)}$

$=\frac{1}{(x-1)}-\frac{1}{(x+1)}$

Hence $\frac{x^2+1}{(x+1)(x-1)}=2+\frac{1}{(x-1)}-\frac{1}{(x+1)}$

Question No.3

$$\frac{2x+1}{(x-1)(x+2)(x+3)}$$

Solution:- $\frac{2x+1}{(x-1)(x+2)(x+3)}$

Now consider

$\frac{2x+1}{(x-1)(x+2)(x+3)}=\frac{A}{(x-1)}+\frac{B}{(x+2)}+\frac{c}{x+3}$ .... (Z)

Multiply both sides by (x-1)(x+2)(x+3)

2x+1= A(x+2)(x+3)+B(x-1)(x+3)+C(x-1)(x+2) ....(1)

Put x-1=0 $\Longrightarrow$ x=1 in equation (1)

2(1)+1=A(1+2)(1+3)

2(1)+1=A(12) $\Longrightarrow$ A=$\frac{1}{4}$

Now put x+2=0 $\Longrightarrow$ x= -2 in equation (1)

2(-2)+1=B(-2-1)(-2+3)

-3=B(-3) $\Rightarrow$B=1

Now put x+3=0 $\Longrightarrow$ x= -3 in equation (1)

2(-3)+1=C(-3-1)(-3+2)

-5=C(4) $\Rightarrow$C=$\frac{-5}{4}$

Now put A,B and C in equation (Z)

Hence $\frac{2x+1}{(x-1)(x+2)(x+3)}=\frac{\frac{1}{4}}{(x-1)}+\frac{1}{(x+2)}+\frac{\frac{-5}{4}}{x+3}$

$\Longrightarrow\frac{2x+1}{(x-1)(x+2)(x+3)}=\frac{1}{4(x-1)}+\frac{1}{(x+2)}-\frac{5}{4\,(x+3)}$

Question No.4

$$\frac{3x^2-4x-5}{(x-2)(x^2+7x+10)}$$

Solution:- AS $\frac{3x^2-4x-5}{(x-2)(x^2+7x+10)}=\frac{3x^2-4x-5}{(x-2)(x+5)(x+2)}$

$x^2+7x+10$=$x^2+5x+2x+10$

x(x+5)+2(x+5)=(x+5)(x+2)

Now consider

$\frac{3x^2-4x-5}{(x-2)(x+5)(x+2)}=\frac{A}{(x-2)}+\frac{B}{(x+5)}+\frac{C}{x+2}$ .... (Z)

Multiply both sides by (x-2)(x+5)(x+2)

$3x^2-4x-5$= A(x+5)(x+2)+B(x-2)(x+2)+C(x-2)(x+5) ....(1)

Put x-2=0 $\Longrightarrow$ x=2 in equation (1)

$3(2)^2-4(2)-5$=A(2+5)(2+2)

-1=A(28) $\Longrightarrow$ A=$\frac{-1}{28}$

Now put x+5=0 $\Longrightarrow$ x= -5 in equation (1)

$3(-5)^2-4(-5)-5$=B(-5-2)(-5+2)

90=B(21) $\Rightarrow$B=$\frac{90}{21}$

B=$\frac{30}{7}$

Now put x+2=0 $\Longrightarrow$ x= -2 in equation (1)

$3(-2)^2-4(-2)-5$=C(-2-2)(-2+5)

15=C(-12) $\Rightarrow$C=$\frac{-15}{12}$

C=$-\frac{5}{4}$

Now put A,B and C in equation (Z)

Hence $\frac{3x^2-4x-5}{(x-2)(x+5)(x+2)}=\frac{-1}{28(x-2)}+\frac{30}{7(x+5)}-\frac{5}{4(x+2)}$

Question No.5

$$\frac{1}{(x-1)(2x-1)(3x-1)}$$

Solution:- $\frac{1}{(x-1)(2x-1)(3x-1)}$

Now consider

$\frac{1}{(x-1)(2x-1)(3x-1)}=\frac{A}{(x-1)}+\frac{B}{(2x-1)}+\frac{c}{(3x-1)}$ .... (Z)

Multiply both sides by $(x-1)(2x-1)(3x-1)$

1= A$(2x-1)(3x-1)$+B$(x-1)(3x-1)$+C$(x-1)(2x-1)$...............(1)

Put x-1=0 $\Longrightarrow$ x=1 in equation (1)

1=A(2(1)-1)(3(1)-1)

1=A(12) $\Longrightarrow$ A=$\frac{1}{2}$

Now put 2x-1=0 $\Longrightarrow$ x=$\frac{1}{2}$ in equation (1)

1=B($\frac{1}{2}$-1)(3($\frac{1}{2}$)-1)

1=B(-$\frac{1}{4}$) $\Rightarrow$B=-4

Now put 3x-1=0 $\Longrightarrow$ x=$\frac{1}{3}$ in equation (1)

1=C$\left(-\frac{2}{3}\right)\left(-\frac{1}{3}\right)$

1=$\left(\frac{2}{9}\right)$C $\Rightarrow$C=$\frac{9}{2}$

Now put A,B and C in equation (Z)

Hence $\frac{1}{(x-1)(2x-1)(3x-1)}=\frac{1}{2(x-1)}-\frac{4}{(2x-1)}+\frac{9}{2(3x-1)}$

Question No.6

$$\frac{x}{(x-a)(x-b)(x-c)}$$

Solution:- $\frac{x}{(x-a)(x-b)(x-c)}$

Now consider

$\frac{x}{(x-a)(x-b)(x-c)}=\frac{A}{(x-a)}+\frac{B}{(x-b)}+\frac{c}{(x-c)}$ .... (Z)

Multiply both sides by $(x-a)(x-b)(x-c)$

x= A$(x-b)(x-c)$ +B$(x-a)(x-c)$+C$(x-b)(x-a)$...............(1)

Put x-a=0 $\Longrightarrow$ x=a in equation (1)

a=A(a-b)(a-c)

$\Longrightarrow$ A=$\frac{a}{(a-b)(a-c)}$

Now put x-b=0 $\Longrightarrow$ x= b in equation (1)

b=B(b-a)(b-c)

$\Rightarrow$B=$\frac{b}{(b-a)(b-c)}$

Now put x-c=0 $\Longrightarrow$ x= c in equation (1)

c=C$(c-b)(c-a)$

$\Longrightarrow$C=$\frac{c}{(c-a)(c-b)}$

Now put A,B and C in equation (Z)

Hence $\frac{x}{(x-a)(x-b)(x-c)}$= $\frac{a}{(a-b)(a-c)(x-a)}+$ $\frac{b}{(b-a)(b-c)(x-b)}+\frac{c}{(c-a)(c-b)(x-c)}$

Question No.7

$$\frac{6x^3+5x^2-7}{2x^2-x-1}$$

Solution:-Its is an improper fraction we first convert it into proper fraction by division

$\Longrightarrow$3x+4 +$\frac{7x-3}{2x^2-x-1}$

=3x+4 +$\frac{7x-3}{(x-1)(2x+1)}$

Now consider

$\frac{7x-3}{(x-1)(2x+1)}=\frac{A}{(x-1)}+\frac{B}{(2x+1)}$

Find value of A and B yourself you will get

A=$\frac{4}{3}$ and B=$\frac{13}{3}$

So $\frac{7x-3}{(x-1)(2x+1)}=\frac{4}{3(x-1)}+\frac{13}{3(2x+1)}$

Hence $\frac{6x^3+5x^2-7}{2x^2-x-1}$ = 3x+4 +$\frac{4}{3(x-1)}+\frac{13}{3(2x+1)}$

Question No.8

$$\frac{2x^3+x^2-5x+3}{2x^3+x^2-3x}$$

Solution:- Its is an improper fraction we first convert it into proper fraction by division

$\Longrightarrow$1 +$\frac{-2x+3}{2x^3+x^2-3x}$

=1 +$\frac{-2x+3}{x(2x^2+x-3)}$

=1+$\frac{-2x+3}{x(2x^2+3x-2x-3)}$

1+$\frac{-2x+3}{x(2x^2+3x-2x-3)}$

1+$\frac{-2x+3}{x(2x+3)(x-1)}$

Now consider

$\frac{-2x+3}{x(2x+3)(x-1)}=\frac{A}{x}+\frac{B}{(2x+3)}+\frac{C}{(X-1)}$ ....(Z)

Multiply both sides by$x(2x+3)(x-1)$

-2x+3= A$(2x+3)(x-1)$ +B$x(x-1)$+C$x(2x+3)$............... (1)

Put x=0 in equation (1)

-2(0)+3=A(3)(-1)

3=A(-3) $\Longrightarrow$ A=-1

Now put 2x+3=0 $\Longrightarrow$ x= $-\frac{3}{2}$ in equation (1)

-2($-\frac{3}{2}$)+3=B($-\frac{3}{2}$)$(-\frac{3}{2}-1)$

-2($-\frac{3}{2}$)+3=B(-$\frac{15}{4}$) $\Longrightarrow$B=$\frac{8}{5}$

Now put x-1=0 $\Longrightarrow$ x= 1 in equation (1)

-2(1)+3=C(1)(2(1)+3)

-2+3=5C $\Longrightarrow$C=$\frac{1}{5}$

Now put A,B and C in equation (Z)

$\Longrightarrow\frac{-2x+3}{x(2x+3)(x-1)}=\frac{-1}{x}+\frac{8}{5(2x+3)}+\frac{1}{5(x-1)}$

Hence 1 +$\frac{-2x+3}{2x^3+x^2-3x}=1-\frac{1}{x}+\frac{8}{5(2x+3)}+\frac{1}{5(x-1)}$

Question No.9

$$\frac{(x-1)(x-3)(x-5)}{(x-2)(x-4)(x-6)}$$

Solution:-

$\frac{(x-1)(x-3)(x-5)}{(x-2)(x-4)(x-6)}$=$\frac{(x-1)(x^2-3x-5x+15)}{(x-2)(x^2-4x-6x+24)}$

$\frac{(x-1)(x^2-3x-5x+15)}{(x-2)(x^2-4x-6x+24)}$=$\frac{(x-1)(x^2-8x+15)}{(x-2)(x^2-10x+24)}$

=$\frac{x^3-9x^2+23x-15}{x^3-12x^2+44x-48}$

Its is an improper fraction we first convert it into proper fraction by division

$\Longrightarrow$1 +$\frac{3x^2-21x+33}{x^3-12x^2+44x-48}$

=1 +$\frac{3x^2-21x+33}{(x-2)(x-4)(x-6)}$

Now consider $\frac{3x^2-21x+33}{(x-2)(x-4)(x-6)}=\frac{A}{(x-2)}+\frac{B}{(x-4)}+\frac{C}{(x-6)}$

Find value of A and B yourself you will get

A=$\frac{3}{8}$, B=$\frac{3}{4}$ and C=$\frac{15}{8}$

So $\frac{3x^2-21x+33}{(x-2)(x-4)(x-6)}=\frac{\frac{3}{8}}{(x-2)}+\frac{\frac{3}{4}}{(x-4)}+\frac{\frac{15}{8}}{(x-6)}$

Hence $\frac{(x-1)(x-3)(x-5)}{(x-2)(x-4)(x-6)}$ = 1 +$\frac{3}{8(x-2)}+\frac{3}{4(x-4)}+\frac{15}{8(x-6)}$

Question No.10

$$\frac{1}{(1-ax)(1-bx)(1-cx)}$$

Solution:- $\frac{1}{(1-ax)(1-bx)(1-cx)}$

Now consider

$\frac{1}{(1-ax)(1-bx)(1-cx)}$=$\frac{A}{(1-ax)}+\frac{B}{(1-bx)}+\frac{c}{(1-cx)}$ ...... (Z)

Multiply both sides by $(1-ax)(1-bx)(1-cx)$

1= A$(1-bx)(1-cx)$ +B$(1-ax)(1-cx)$+C$(1-ax)(1-bx)$...............(1)

Put 1-ax=0 $\Longrightarrow$ x=$\frac{1}{a}$ in equation (1)

1=A(1-b($\frac{1}{a}$))(1-c($\frac{1}{a}$))

1=A$\frac{(a-b)(a-c)}{a^2}$

$\Longrightarrow$ A=$\frac{a^2}{(a-b)(a-c)}$

Similarly we can find the value of B and C

$\Longrightarrow$B=$\frac{b^2}{(b-a)(b-c)}$

$\Longrightarrow$C=$\frac{c^2}{(c-a)(c-b)}$

Now put A,B and C in equation (Z)

Hence $\frac{1}{(1-ax)(1-bx)(1-cx)}$= $\frac{a^2}{(a-b)(a-c)(1-ax)}+$ $\frac{b^2}{(b-a)(b-c)(1-bx)}+\frac{c^2}{(c-a)(c-b)(1-cx)}$

Question No.11

$$\frac{x^2+a^2}{(x^2+b^2)(x^2+c^2)(x^2+d^2)}$$

Solution:

$$\frac{x^2+a^2}{(x^2+b^2)(x^2+c^2)(x^2+d^2)}$$

Replace x^2 by y

$$=\frac{\boldsymbol{y+a^2}}{\boldsymbol{(y+b^2)(y+c^2)(y+d^2)}}$$

Suppose

$$\frac{y+a^2}{(y+b^2)(y+c^2)(y+d^2)}$$

$$=\frac{A}{y+b^2}+\frac{B}{y+c^2}+\frac{C}{(y+d^2)}\rightarrow (i)$$

"x"by $(y+b^2)(y+c^2)(y+d^2)+c(y+a^2)(y+c^2)\rightarrow (ii)$

Put $y+b^2=0\Rightarrow y=-b^2\ in\ (ii)$

$$-b^2+a^2=A(-b^2+c^2)(-b^2+d^2)$$

$$\Rightarrow\quad A=\frac{a^2-b^2}{(c^2-b^2)(d^2-b^2)}$$

Put $y+d^2=0\Rightarrow y=-d^2in\ (ii)$

$$-d^2+a^2=c(-d^2+a^2)(-d^2+c^2)$$

$$\Rightarrow\quad c=\frac{a^2-d^2}{(a^2-d^2)(c^2-d^2)}$$

Put valves in (i)

$$\frac{y+a^2}{(y+b^2)(y+c^2)(y+d^2)}$$

$$=\frac{a^2-b^2}{(c^2-b^2)(d^2-b^2)(y-b^2)}$$

$$+\frac{a^{-2}-c^2}{(b^2-c^2)(d^2-c^2)(y+c^2)}+\frac{a^2-d^2}{(b^2-d^2)(c^2-d^2)}$$

Thus

$$\frac{x^2+a^2}{(x^2+b^2)(x^2+c^2)(x^2+d^2)}$$

$$=\frac{a^2-b^2}{(c^2-b^2)(d^2-b^2)(y-b^2)}$$

$$+\frac{a^{-2}-c^2}{(b^2-c^2)(d^2-c^2)(y+c^2)}+\frac{a^2-d^2}{(b^2-d^2)(c^2-d^2)}$$

Case II

When $Q(x)$ has repeated linear factors.

$if\ Q(x)$ has a factor $(x-a)^n$,$n\geq 2$ and n is +ve integer, then $\frac{P(x)}{Q(x)}$ may be written as the following identity

$$\because\frac{P(x)}{Q(x)}=\frac{A_1}{(x-a_1)}+\frac{A_2}{(x-a_2)}+\cdots+\frac{A_n}{(x-a_n)}$$

Where $A_1,A_2,\ldots.A_n\ are$ numbers to found.

Exercise 5.2

Resolve the following into partial fractions:

Question No.1

$$\frac{2x^2-3x+4}{(x-1)^3}$$

Solution:

$$\frac{2x^2-3x+4}{(x-1)^3}$$

Resolve into partial fraction

Now consider $\frac{2x^2-3x+4}{(x-1)^3}$= $\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{c}{(x-1)^3}$ …………… (Z)

Multiply both sides by $(x-1)^3$

$2x^2-3x+4$= A$(x-1)^2$+B$(x-1)$+C……………(1)

Put x-1=0 ⟹ x=1 in equation (1)

$2(1)^2-3(1)+4$= C

3=C ⟹ C=3

Now equation (1) implies

$2x^2-3x+4$= A(x^2-2x+1)+B$(x-1)$+C

comoaring the coefficients of $x^2,x\ and\ x^0$

2=A

-3=-2(2)+B

-3=-4+B ⟹ B=1

Now put A,B and C in equation (Z)

Hence $\frac{2x^2-3x+4}{(x-1)^3}$= $\frac{2}{(x-1)}+\frac{1}{(x-1)^2}+\frac{3}{(x-1)^3}$

Question No.2

$$\frac{5x^2-2x+3}{(x+2)^3}$$

Solution:

$$\frac{5x^2-2x+3}{(x+2)^3}$$

Resolve into partial fraction

Now consider

$\frac{5x^2-2x+3}{(x+2)^3}=\frac{A}{(x+2)}+\frac{B}{(x+2)^2}+\frac{c}{(x+2)^3}$ …………… (Z)

Multiply both sides by $(x+2)^3$

$5x^2-2x+3$= A$(x+2)^2$+B$(x+2)$+C……………(1)

Put x+2=0 ⟹ x=-2 in equation (1)

$5(-2)^2-2(-2)+3$= C

27=C ⟹ C=27

Now equation (1) implies

$5x^2-2x+3$= A(x^2+4x+4)+B$(x+2)$+C

comparing the coefficients of $x^2,x\ and\ x^0$

5=A

B=-22

Now put A,B and C in equation (Z)

Hence $\frac{5x^2-2x+3}{(x+2)^3}=\frac{5}{(x+2)}-\frac{22}{(x+2)^2}+\frac{27}{(x+2)^3}$

Question No.3

$$\frac{4x}{(x+1)^2(x-1)}$$

Solution:

$$\frac{4x}{(x+1)^2(x-1)}$$

Resolve into partial fraction

Now consider

$\frac{4x}{(x+1)^2(x-1)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x-1)}$ …………… (Z)

Multiply both sides by $(x+1)^2(x-1)$

$4x$= A$(x+1)(x-1)$+B$(x-1)$+C$(x+1)^2$……………(1)

Put x-1=0 ⟹ x=1 in equation (1)

4(1)= C$(1+1)^2$

4=4C ⟹ C=1

Put x+1=0 ⟹ x=-1 in equation (1)

$4(-1)$= B$(-1-1)$

⟹B=2

Now equation (1) implies

$4x$= A(x^2-1)+B$(x-1)$+C(x^2-2x+1)

comoaring the coefficients of $x^2, x\ and\ x^0$

0=A +C

0=1+A

⟹A=-1

Now put A,B and C in equation (Z)

Hence $\frac{4x}{(x+1)^2(x-1)} = \frac{-1}{(x+1)} + \frac{2}{(x+1)^2} + \frac{1}{(x-1)}$

Question No.4

$$\frac{9}{(x+2)^2(x-1)}$$

Solution:

$$\frac{9}{(x+2)^2(x-1)} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{x-1} \to (i)$$

$'x' by\ (x+2)^2(x-1) we\ get$

$9 = A(x+2)(x-1) + 8(x-1) + C(x+2)^2 \to (ii)$

Put $x-1=0 \Rightarrow x=1\ in\ (ii)$

$$9 = C(1+2)^2 \Rightarrow 9 = C(3)^2$$

⇨ $9 = 9C \Rightarrow C = 1$

Put $x+2=0$

⇨ $x = -2\ \ in\ (ii)$

$$9 = B(-2-1) \Rightarrow 9 = -3B$$

$$B = -3$$

From (ii)

$$9 = A(x^2 - x + 2x - 2) + B(x) - B + C(x^2 + 4 + 4x)$$

$$9 = Ax^2 + Ax - 2A + Bx - B + C(x^2 + 4 + 4x)$$

$$Equating\ coefficients$$

$$x^2,\quad 0 = A + C \Rightarrow A + 1$$

$$= A = -1$$

(i) becomes as

$$\frac{9}{(x+2)^2(x+1)} = -\frac{1}{(x+2)} - \frac{3}{(x+2)^2} + \frac{1}{(x-1)}$$

Question No.5

$$\boldsymbol{\frac{1}{(x-3)(x+1)}}$$

Solution:

$$\frac{1}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{c}{x+1} \to (i)$$

$$'x' by\ (x-3)^2(x+1), we\ get$$

$1 = A(x-3)(x+1) + B(x+1) + C(x-3)^2 \to (ii)$

Put $x-3=0 \Rightarrow x=3\ in\ (ii)$

$$1 = B(3+1)$$

⇨ $1 = 4B$

⇨ $B = \frac{1}{4}$

Put $x+1=0 \Rightarrow x=-1\ in\ (ii)$

$$1 = C(-1-3)^2$$

⇨ $1 = C(-4)^2$

$$1 = 16C$$

$$C = \frac{1}{16}$$

$$1 = A(x^2 + x - 3x - 3) + Bx + B + C(x^2 + 9 - 9x)$$

$$1 = Ax^2 - 2Ax - 3A + Bx + B + Cx^2 + 9C - 9Cx$$

Equating coefficients

$$x^2;\quad 0 = A + C$$

⇨ $0 = A + \frac{1}{16}$

⇨ $A = -\frac{1}{16}$

(i) becomes as

$$\frac{1}{(x-3)^2(x+1)} = -\frac{1}{16(x-3)} + \frac{1}{4(x-3)^2} + \frac{1}{16(x+1)}$$

Question No.6

$$\frac{x^2}{(x-2)(x-1)^2}$$

Solution: Suppose

$$\frac{x^2}{(x-2)(x-1)^2} = \frac{A}{x-2} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \to (i)$$

$$'x' by\ (x-2(x-1)^2, we\ get$$

$$x^2 = A(x-1)^2 + B(x-2)(x-1) + C(x-2)$$

$$\to (ii)$$

Put $x-2=0$

$$\Rightarrow x^2 = A(x-1)^2 + B(x-2)(x-1) + C(x-2)$$

$$\to (ii)$$

$$put\ x-2=0 \Rightarrow x=2\ \ in\ (ii)$$

$$(2)^2 = A(2-1)^2$$

$$\Rightarrow 4 = A$$

Put $x-1=0 \Rightarrow x=1\ in\ (ii)$

$$(1)^2 = C(1-2)$$

⇨ $1 = -C$

⇨ $C = -1$

From (ii)

$$x^2 = A(x^2 + 1 - 2x) + B(x^2 - x - 2x + 2) + Cx - 2C$$

$$x^2 = Ax^2 + A - 2Ax + Bx^2 - 3Bx + 2B + Cx - 2C$$

Equating coefficients

$$x^2;\quad 1 = A + B$$

$$1 = 4 + B$$

$$B = 1 - 4$$

$$B = -3$$

$$(i)\ becomes\ as$$

$$\frac{x^2}{(x-2)(x-1)^2} = \frac{4}{x-2} - \frac{3}{x-1} - \frac{1}{(x-1)^2}$$

Question No.7

$$\boldsymbol{\frac{1}{(x-1)^2(x+1)}}$$

Solution: Suppose

$$\frac{1}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} \rightarrow (i)$$

$$'x' \ by \ (x-1)^2(x+1), we \ get$$

$$1 = A(x-1)(x+1) + B(x+1) + C(x-1)^2 \rightarrow (ii)$$

Put $x - 1 = 0 \Rightarrow x = 1 \ in \ (ii)$

$$1 = B(1+1) \Rightarrow 1 = 2B \Rightarrow B = \frac{1}{2}$$

Put $x + 1 = 0 \ \Rightarrow x = -1 \ in \ (ii)$

$$1 = C(-1-1)^2$$
$$1 = C(-2)^2$$
$$1 = 4C$$
$$C = \frac{1}{4}$$

From (ii)

$$1 = B(1+1) \Rightarrow 1 = 2B \Rightarrow B = \frac{1}{2}$$

Put $x + 1 = 0 \Rightarrow x = -1 \ in \ (ii)$

$$1 = C(-1-1)^2$$
$$1 = C(-2)^2$$
$$1 = 4C$$
$$C = \frac{1}{4}$$

From (ii)

$$1 = A(x^2-1) + Bx + B + C(x^2 - 2x + 1)$$
$$1 = Ax^2 - A + Bx + Cx^2 - 2Cx + C$$

Equating coefficients

$$x^2; \quad 0 = A + C$$
$$\Rightarrow 0 = A + \frac{1}{4}$$
$$A = -\frac{1}{4}$$
$$(i) becomes \ as$$

$$\frac{1}{(x-1)^2(x+1)}$$
$$= -\frac{1}{4(x-1)} + \frac{1}{2(x-1)^2} + \frac{1}{4(x+1)}$$

Question No.8

$$\frac{x^2}{(x-1)^3(x+1)}$$

Solution:

$$\frac{x^2}{(x-1)^3(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{D}{x+1}$$

$$'x' by \ (x-1)^3 \ (x+1) we \ get$$

$$x^2 = A(x-1)^2(x+1) + B(x-1)(x+1)$$
$$+ C(x+1) + D(x-1)^3 \rightarrow (ii)$$

$$put \ x - 1 = 0 \Rightarrow x = 1 \ in \ (ii)$$

$$(1)^2 = C(1+1) \Rightarrow 1 = 2C \Rightarrow C = \frac{1}{2}$$

Put $x + 1 = 0 \Rightarrow x = -1 \ in \ (ii)$

$$(-1)^2 = D = (-1-1)^3$$
$$\Rightarrow 1 = D(-2)^3$$
$$1 = -8D$$

⇨ $d = -\frac{1}{8}$

From (ii)

$$x^2 = A(x^2 + 1 - 2x)(x+1) + B(x^2 - 1) + Cx + C$$
$$+D(x^3 - 1 - 3x^2 + 3x)$$

$$x^2 = A(x^3 + x^2 + x + 1 - 2x^2 - 2x) + Bx^2 - B$$
$$+Cx + C + Dx^3 - D - 3Dx^2 + 3Dx$$

$$x^2 = Ax^3 - Ax^2 - Ax + A + Bx^2 - B + Cx + C$$
$$+ Dx^3 - d - 3Dx^2 + 3Dx$$

$$x^2 = Ax^3 - Ax^2 - Ax + A + Bx^2 - B + Cx + C$$
$$+ Dx^3 - D - 3Dx^2 + 3Dx^2 + 3Dx$$

Equating coefficients

$$x^3; \quad 0 = A + D \Rightarrow 0 = A - \frac{1}{8}$$
$$A = \frac{1}{8}$$
$$= x^2; \quad 1 = -A + B - 3D$$
$$1 = -\frac{1}{8} + B - 3\left(-\frac{1}{8}\right)$$

⇨ $1 = -\frac{1}{8} + B + \frac{3}{B}$

$$8 = 1 + \frac{1}{8} - \frac{3}{8}$$
$$8 = \frac{8 + 1 - 3}{8} = \frac{6}{8} = \frac{3}{4}$$

⇨ $B = \frac{3}{4}$

(i) becomes as

$$\frac{x^2}{(x-1)^3(x+1)}$$
$$= \frac{1}{8(x-1)} + \frac{3}{4(x-1)^2} + \frac{1}{2(x-1)^3}$$
$$- \frac{1}{8(x+1)}$$

Question No.9

$$\frac{x-1}{(x-2)(x+1)^2}$$

Solution:

Suppose

$$\frac{x-1}{(x-2)(x+3)^2} = \frac{A}{x-2} + \frac{B}{x+1} + \frac{c}{(x+1)^2} + \frac{d}{(x+1)^3}$$
$$\rightarrow (i)$$

$$'x' by \ (x-2)(x-3)^3, \quad we \ get \ (i)$$
$$'x' by \ (x-2)(x+1)^3, \quad we \ get$$

$$x - 1 = A(x+1)^3 + B(x-2)(x+1)^2 + C(x-2)(x+1)$$
$$+d(x-2) \rightarrow (ii)$$

Put $x - 2 = 0 \Rightarrow x = 2 \ in \ (ii)$

$$2 - 1 = A(2+1)^3 \Rightarrow 1 = A(3)^3 \Rightarrow 1 = 27A$$
$$A = \frac{1}{27}$$

Put $x + 1 = 0 \Rightarrow x = -1 \ in \ (ii)$

$$-1 - 1 = D(-1-2)$$

⇨ $-2 = -3D$

⇨ $D = \frac{2}{3}$

From (ii) $x - 1 = A(x^3 + 1 + 3x^2 + 3x) + B(x-2)(x^2 + +2x) + C(x^2 + x - 2x - 2) + Dx - 2D$

$$x - 1 = Ax^3 + A + 3Ax^2 + 3Ax$$
$$+ B(x^3 + x + 2x^2 - 2x^2 - 2 - 4x)$$
$$+Cx^2 - Cx - 2C + Dx - 2D$$

$$x - 1 = Ax^3 + A + 3Ax^2 + 3Ax + Bx^3 - 3Bx - 2B + Cx^2$$
$$- Cx - 2C + Bx - 2D$$

$$x - 1 = Ax^3 + A + 3Ax^2 + 3Ax^2 + 3Ax + Bx^3 - 3Bx$$
$$- 2B + Cx^2 - Cx - 2C + Dx - 2D$$

Equating coefficients

$$x^3; \quad 0 = A + B \Rightarrow 0 = \frac{1}{27} + B$$

$$B = -\frac{1}{27}$$

$$x^2; 0 = 3A + C \Rightarrow\Rightarrow 0 = 3\left(\frac{1}{27}\right) + C$$

$$C = -\frac{1}{9}$$

So (i) becomes as

$$\frac{x-1}{(x-2)(x+1)^3} = \frac{1}{27(x-2)} - \frac{1}{27(x+1)} - \frac{1}{9(x+1)^2} + \frac{2}{3(x+1)^3}$$

Question No.10

$$\frac{4x^3}{(x^2-1)(x-1)^2}$$

Solution:

$$\frac{4x^3}{(x^2-1)(x-1)^2} = \frac{4x^3}{(x-1)(x+1)(x+1)^2}$$

$$= \frac{4x^3}{(x-1)(x+1)^3}$$

Suppose

$$\frac{4x^3}{(x-1)(x+1)^3} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3} \to (i)$$

$$'x' \ by \ (x-1)(x+1)^3, we \ get$$

$$4x^3 = A(x+1)^3 + B(x-1)(x+1)^2 + C(x-1)(x+1) + D(x-1) \to (ii)$$

$$put \ x - 1 = 0 \Rightarrow x = 1 \ in \ (i)$$

$$4(1)^3 = A(1+1)^3$$

$$4 = A(2)^3 \Rightarrow 8A = 4 \Rightarrow A = \frac{1}{2}$$

Put $x + 1 = 0 \Rightarrow x = -1 \ in \ (ii)$

$$4(-1)^3 = D(-1-1) \Rightarrow -4 = -2D$$

$$D = 2$$

From (ii)

$$4x^3 = A(x^3 + 1 + 3x^2 + 3x) + 3ax + B(x^3 + x + 2x^2 - x^2 - 1 - 2x) + Cx^2 - C + Dx - D$$

$$4x^3 = Ax^3 + A + 3Ax^2 + 3Ax + Bx^3 + Bx^3 + Bx^2 - Bx - B + Cx^2 - C + Dx - D$$

Equating coefficients

$$x^3; 4 = A + B \Rightarrow 4 = \frac{1}{2} + B$$

$$B = 4 - \frac{1}{2} = \frac{8-1}{2} = \frac{7}{2} = B = \frac{7}{2}$$

$$x^2; 0 = 3A + B + C$$

$$0 = 3\left(\frac{1}{2}\right) + \frac{7}{2} + C$$

$$C = -\frac{3}{2} - \frac{7}{2} = -\frac{10}{2} = -5 = C = -5$$

So (i) becomes

$$\frac{4x^3}{(x-1)(x+1)^3} = \frac{1}{2(x-1)} + \frac{7}{2(x+1)} + \frac{-5}{2(x+1)^2} + \frac{2}{(x+1)^3}$$

Or

$$\frac{4x^3}{(x-1)(x+1)^3} = \frac{1}{2(x-1)} + \frac{7}{2(x+1)} - \frac{5}{2(x+1)^2} + \frac{2}{(x+1)^3}$$

Question No.11

$$\frac{2x+1}{(x+3)(x-1)(x+2)^2}$$

Solution:

Suppose

$$\frac{2x+1}{(x+3)(x-1)(x+2)^2} = \frac{A}{x+3} + \frac{B}{x-1} + \frac{C}{x+2} + \frac{D}{(x+2)^2} \to (i)$$

$$'x' \ by \ (x+3)(x-1)(x+2)^2 \ \ we \ get$$

$$\Rightarrow 2x + 1 = A(x-1)(x+2)^2 + B(x+3)(x+2)^2 + +C(x+3)(x-1)(x+2) + D(x+3)(x-1) \to (ii)$$

$$put \ \ x + 3 = 0 \Rightarrow x = -3 \ in \ (ii)$$

$$2(-3) + 1 = A(-3-1)(-3+2)^2$$

$$-6 + 1 = A(-3-1)(-3+2)^2$$

$$-6 + 1 = A(-4)(-1)^2 \Rightarrow -5 = A(-4)(1)$$

$$-5 = -4A \Rightarrow A = \frac{5}{4}$$

Put $x - 1 = 0 \Rightarrow x = 1 \ in \ (ii)$

$$2(1) + 1 = B(! + 3)(1 + 2)^2$$

$$3 = B(4)(9) \Rightarrow 3 = 36B$$

$$B = \frac{3}{36} = B = \frac{1}{12}$$

Put $x + 2 = 0 \Rightarrow x = -2 \ in \ (ii)$

$$2(-2) + 1 = D(-2+3)(-2-1)$$

$$-4 + 1 = D(1)(-3)$$

⇨ $-3 = -3D$

⇨ **D=1**

From (ii)

$$2x + 1 = A(x-1)(x^2 + 4 + 4x) + B(x+3)(x^2 + 4 + 4x) + C(x+3)(x^2 + 2x - x - 2) + D(x^2 - x + 3x - 3)$$

$$2x + 1 = A(x^3 + 3x^2 - 4) + B(x^3 + 7x^2 + 16x + 12) + C(x^3 + x^2 - 2x + 3x^2 + 3x - 6) + Dx^2 + 2Dx - 3D$$

$$2x + 1 = Ax^3 + 3Ax^2 - 4A + Bx^3 + 7Bx^2 + 16B + 12B + Cx^3 + 4Cx^2 + Cx - 6x - 6C + Dx^2 + 2Dx - 3D$$

Equating coefficients

$$x^3; \quad 0 = A + B + C$$

$$0 = \frac{5}{4} + \frac{1}{12} + C$$

⇨ $C = -\frac{5}{4} - \frac{1}{12} = \frac{-15-1}{12} = -\frac{16}{12}$

$$C = -\frac{4}{3}$$

So (i) becomes as

$$\frac{2x+1}{(x+3)(x-1)(x+2)^2} = \frac{5}{4(x+3)^2} + \frac{1}{12(x-1)} - \frac{4}{3(x-1)} + \frac{1}{(x+2)^2}$$

Question No.12

$$\frac{2x^4}{(x-3)(x+2)^2}$$

Solution:

$$\frac{2x^4}{(x-3)(x+2)^2}(improper)$$

$$= \frac{2x^4}{(x-3)(x^2+4x+4)}$$

$$= \frac{2x^4}{x^3+4x^2+4x-3x^2-12x-12}$$

$$= \frac{2x^4}{x^3+x^2-18x-12}$$

$$\begin{array}{r|l} & 2x-2 \\ \hline x^3+x^2-8x-12 & 2x^4 \\ & \underline{\pm 2x^4 \pm 2x^3 \mp 16x^2 \mp 24x} \\ & -2x^3+16x^2+24x \\ & \underline{\mp 2x^3 \mp 2x^2 \pm 16x \pm 24} \\ & 18x^2+8x-24 \end{array}$$

So

$$\frac{2x^4}{x^3+x^2-18x-12} = 2x-2+\frac{18x^2+8x-24}{x^3+x^2-8x-12}(proper)$$

Now suppose

$$\frac{18x^2+8x-24}{(x-3)(x+2)^2} = \frac{A}{x-3}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$$

$$'x' \ by \ (x-3)(x+2)^2 \ \ we \ get$$

$$18x^2+8x-24 = A(x+2)^2+B(x-3)(x+2) + C(x-3) \rightarrow (ii)$$

Put $x-3=0 \Rightarrow x=3 \ \ in \ (ii)$

$$18(3)^2+8(3)-24 = A(3+2)^2$$

$$162+24-24 = 25A \Rightarrow A = \frac{162}{25}$$

Put $x-3=0 \Rightarrow x=3 \ in \ (ii)$

$$18(-2)^2+8(-2)-24 = C(-2-3)$$

$$72-16-24 = -5C$$

$$32 = -5C$$

$$C = -\frac{32}{5}$$

$$18x^2+8x-24 = A(x^2+4+4x)+B(x^2+2x-3x-6) + Cx-3C$$

$$18x^2+8x-24 = Ax^2+4A+4Ax+Bx^2-Bx-6B + Cx-3C$$

Equating coefficients

$$x^2; \quad 18 = A+B = 18 = \frac{162}{25}+B$$

$$B = A+B \Rightarrow 18 = \frac{162}{25}+B$$

$$B = 18-\frac{162}{25} = \frac{450-162}{25}$$

$$B = \frac{288}{25}$$

So (i) becomes

$$\frac{18x^2+8x-24}{(x-3)(x+2)^2} = \frac{162}{25(x-3)}+\frac{288}{25(x+2)}-\frac{32}{5(x+2)^2}$$

Hence

$$\frac{2x^4}{(x-3)(x+2)^2} = 2x-2+\frac{162}{25(x-3)}+\frac{288}{25(x+2)} - \frac{32}{5(x+2)^2}$$

Case III

When $Q(x)$ contains non represented irreducible quadratic factors:

If $Q(x)$ contains non-repeated irreducible quadratic factor than $\frac{P(x)}{Q(x)}$ may be written as the identity having partial fractions of the form $\frac{Ax+B}{ax^2+bx+c}$ where $A \ and \ B$ are numbers to be found

Irreducible Quadratic:

A quadratic factor is irreducible if it cannot be written as the product of two linear factors with real coefficient

$e.g \ x^2+x+1 \ \ and \ x^2+3$

Exercise 5.3

Question No.1

$$\frac{9x-7}{(x^2+1)(x+3)}$$

Solution:

Suppose $\frac{9x-7}{(x^2+1)(x+3)} = \frac{Ax+B}{x^2+1}+\frac{C}{x+3} \rightarrow (i)$

$$'\times' \ by \ (x^2+1)(x+3)+C(x^2+1) \rightarrow (ii)$$

$$put \ (x+3)=0 \Rightarrow x=-3 \ \ in(ii)$$

$$9(-3)-7 = C((-3)^2+1)$$

$$-27-7 = C(9+1) \Rightarrow C = -\frac{34}{10} = -\frac{17}{5}$$

$$C = -\frac{17}{5}$$

From (ii)

$$9x-7 = Ax^2+3Ax+Bx+3B+Cx^2+C$$

Equating coefficients

$$x^2; \quad 0 = A+C \Rightarrow 0 = A-\frac{17}{5}$$

$$\Rightarrow A = \frac{17}{5}$$

$$x; \quad 9 = 2A+B \Rightarrow 9 = 3\left(\frac{17}{5}\right)+B$$

$$\Rightarrow 9 = \frac{51}{5}+B \Rightarrow B = 9-\frac{51}{5}$$

$$B = \frac{45-51}{5} = -\frac{6}{5} \Rightarrow B = -\frac{6}{5}$$

So (i) becomes as 17x-6

$$\frac{9x-7}{(x^2+1)(x+3)} = \frac{\frac{17}{5}x-\frac{6}{5}}{(x^2+1)}+\frac{-\frac{17}{5}}{(x+3)}$$

Or

$$\frac{9x-7}{(x^2+1)(x+3)} = \frac{17x-6}{5(x^2+1)}-\frac{17}{5(x+3)}$$

Question No.2

$$\frac{1}{(x^2+1)(x+1)}$$

Solution: Suppose

$$\frac{1}{(x^2+1)(x+1)} = \frac{Ax+B}{x^2+1}+\frac{C}{x+1}$$

$$\times \ by(\ x^2+1)(x+1) \ we \ get$$

$$1 = (Ax+B)(x+1)+C(x^2+1) \rightarrow (ii)$$

Put $x+1=0 \ \ \Rightarrow x=-1 \ \ in \ (ii)$

$$1 = C((-1)^2+1) \Rightarrow 1 = C(1+1)$$

$$C = \frac{1}{2}$$

From (ii)

$$1 = Ax^2 + Ax + Bx + B + Cx^2 + C$$

Equating confidents

$$x^2, \quad A + C = 0 \Rightarrow A + \frac{1}{2} = 0$$

$$A = -\frac{1}{2}$$

$$x; \quad o = A + B \Rightarrow 0 = -\frac{1}{2} + B$$

$$B = -\frac{1}{2}$$

So (i) becomes

$$\frac{1}{(x^2+1)(x+1)} = \frac{-\frac{1}{2}x + \frac{1}{2}}{x^2+1} + \frac{\frac{1}{2}}{x+1}$$

Or $\frac{1}{(x^2+1)(x+3)} = \frac{-x+1}{2(x^2+1)} + \frac{1}{2(x+1)}$

Question No.3

$$\frac{3x+7}{(x^2+4)(x+3)}$$

Solution:

$$\frac{3x+7}{(x^2+4)(x+3)} = \frac{Ax+B}{x^2+4} + \frac{C}{x+3} \rightarrow (i)$$

$$'\times' \ by \ (x^2+4)(x+3) \ we \ get$$

$$3x + 7 = (Ax+B)(x+3) + C(x^2+4) \rightarrow (ii)$$

Put $x + 3 = 0 \Rightarrow x = -3 \quad in \ (ii)$

$$3(-3) + 7 = (Ax+B)(x+3) + C(x^2+4) \rightarrow (ii)$$

Put $x + 3 = 0 \Rightarrow x = -3 + C \quad in \ (ii)$

$$3(-3) + 7 = C(-3)^2 + 4$$

$$-9 + 7 = C(9+4)$$

$$\Rightarrow -2 = 13C$$

$$or \quad C = -\frac{2}{13}$$

From (ii)

$$3x + 7 = Ax^2 + 3Ax + Bx + 3B + Cx^2 + 4C$$

Equating coefficients

$$x^2; \quad 0 = A + C \Rightarrow A = -\frac{2}{13}$$

$$A = \frac{2}{13}$$

$$x; \quad 3 = 3A + B$$

$$\Rightarrow 3 = 3\left(\frac{2}{12}\right) + B = 3 - \frac{6}{13}$$

⇨ $B = \frac{39-6}{13} \Rightarrow B = \frac{33}{13}$

So (i) becomes as

$$\frac{3x+7}{(x^2+4)(x+3)} = \frac{\frac{2}{13}x + \frac{33}{13}}{x^2+4} + \frac{-\frac{12}{3}}{x+3}$$

$$\frac{3x+7}{(x^2+4)(x+3)} = \frac{2x+33}{13(x^2+4)} - \frac{12}{13(x+3)}$$

Question No.4

$$\frac{x^2+15}{(x^2+2x+5)(x-1)}$$

Solution:

Suppose

$$\frac{x^2+15}{(x^2+2x+5)(x-1)} = \frac{Ax+B}{x^2+2x+5} + \frac{C}{x-1} \rightarrow (i)$$

$$' \times by \ (x^2+2x+5)(x-1) \ \ we \ get$$

$$x^2 + 15 = (Ax+B)(x-1) + C(x^2+2x+6) \rightarrow (ii)$$

Put $x - 1 = 0$

$$x = 1 \ in \ (ii)$$

$$(1)^2 + 15 = c((1)^2 + 2(1) + 5)$$

$$16 = C(1+2+5) \Rightarrow 16 - 8C$$

$$C = 2$$

From (ii)

$$x^2 + 15 = Ax^2 - Ax + Bx - B + Cx^2 + 2Cx + 5C$$

Equation coefficients

$$x^2; \quad 1 = A + C \Rightarrow 1 = A + 2$$

$$A = 1 - 2 \Rightarrow A = -1$$

$$x; \quad 0 = -A + B + 2C$$

$$0 = -(-1) + B + 2(2)$$

⇨ $1 + B + 4 \Rightarrow 0 = B + 5$

$$or \quad B = -5$$

So (i)becomes as

$$\frac{x^2+15}{(x^2+2x+5)(x-1)} = \frac{(-1)x + (-5)}{x^2+2x+5} + \frac{2}{x-1}$$

$$\frac{x^2+15}{(x^2+2x+5)(x-1)} = \frac{-x-5}{x^2+2x+5} + \frac{2}{x-1}$$

Question No.5 $\frac{x^2}{(x^2+4)(x+2)}$

Solution: suppose

$$\frac{x^2}{(x^2+4)(x+2)} = \frac{Ax+B}{x^2+4} + \frac{C}{x+2} \rightarrow (i)$$

$$'\times' \ \ by \ (x^2+4)(x+2), \ \ we \ get$$

$$x^2 = (Ax+B)(x+2) + C(x2+4) \rightarrow (ii)$$

Put $x + 2 = 0 \Rightarrow x = -2 \ \ in \ (ii)$

$$(-2)^2 = C((-2)^2 + 4) \Rightarrow 4 = C(4+4)$$

$$4 = 8C \Rightarrow C = \frac{1}{2}$$

From (ii)

$$x^2; \quad 1 = A + C \Rightarrow 1 = A + \frac{1}{2}$$

$$1 - \frac{1}{2} = A \Rightarrow A = \frac{1}{2}$$

$$0 = 2A + B$$

$$0 = 2\left(\frac{1}{2}\right) + B \Rightarrow B = -1$$

So (i) becomes as

$$\frac{x^2}{(x^2+4)(x+2)} = \frac{\frac{1}{2}x + (-1)}{x^2+4} + \frac{\frac{1}{2}}{x+2}$$

$$\frac{x^2}{(x^2+4)(x+2)} = \frac{\frac{1}{2}x - 1}{x^2+4} + \frac{\frac{1}{2}}{x+2}$$

$$\frac{x^2}{(x^2+4)(x+2)} = \frac{x-2}{2(x^2+4)} + \frac{1}{2(x+2)}$$

Question No.6

$$\frac{x^2+1}{x^3+1}$$

Solution:

$$\frac{x^2+1}{x^3+1} = \frac{x^2+1}{(x+1)(x^2-x+1)}$$

Suppose

$$\frac{x^2+1}{(x+1)(x^2-x+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2-x+1}\to(i)$$

$$'\times'\ (x+1)(x^2-x+1)\quad we\ get$$

⇨ $x^2+1=A(x^2-x+1)+(Bx+C)(x+1)\to(ii)$

Put $x+1=0\ \Rightarrow x=-1\ in\ (ii)$

$$(-1)^2+1=A[(-1)^2-(-1)+1]$$

$$\Rightarrow 1+1=A[1+1+1]$$

$$2=3A$$

$$A=\frac{2}{3}$$

From (ii)

$$x^2+1=Ax^2-Ax+A+Bx^2+Bx+Cx+C$$

Equating coefficients

$$x^2;\quad 1=A+B\Rightarrow 1=\frac{2}{3}+B$$

$$\Rightarrow 1-\frac{2}{3}=B\Rightarrow B=\frac{1}{3}$$

$$x;\quad 0=-A+B+C$$

$$0=-\frac{2}{3}+\frac{1}{3}+C$$

⇨ $0=-\frac{1}{3}+C$

⇨ $C=\frac{1}{3}$

So $(i)\quad c=becomes\ as$

$$\frac{x^2+1}{(x+1)(x^2-x+1)}=\frac{2}{3(x+1)}+\frac{\frac{1}{3}x+\frac{1}{3}}{x^2-x+1}$$

$$\frac{x^2+1}{x^3+1}=\frac{2}{3(x+1)}+\frac{x+!}{3(x^2-x+1)}$$

Question No.7 $\boldsymbol{\frac{x^2+2x+2}{(x^2+3)(x+1)(x-1)}}$

Solution:

$$\frac{x^2+2x+2}{(x^2+3)(x+1)(x-1)}=\frac{Ax+B}{x^2+3}+\frac{C}{x+1}+\frac{D}{x-1}\to\boldsymbol{(i)}$$

$$'\times'\ \boldsymbol{by}\ (x^2+3)(x+1)(x-1)\ we\ get$$

$$x^2+2x+2=(Ax+B)(x+1)(x-1)+C(x^2+3)(x-1)$$
$$+D(x^2+3)(x+1)\to(ii)$$

Put $x+1=0\quad\Rightarrow x=-1\ \ in\ (ii)$

$$(-1)^2+2(-1)+2=C[(-1)^2+3](-1-1)$$

$$1-2+2=C(4)(-2)\Rightarrow 1=-8C$$

$$C=-\frac{1}{8}$$

Put$x-1=0\quad\Rightarrow x=1\ \ in\ (ii)$

$$(1)^2+2(1)+2=D((1)^2+3)(1+1)$$

$$1+2+2=D(1+3)(2)$$

⇨ $5=8D\Rightarrow D=\frac{5}{8}$

From (ii)

$$x^2+2x+2=(Ax+B)(x^2-1)+C(x^3-x^2+3x-3)$$
$$+D(x^3+x^2+3x+3)$$

$$x^2+2x+2=Ax^3-Ax+Bx^2-B+Cx^3-Cx^2+3Cx$$
$$-3C+Dx^3+Dx^2+3Dx+3D$$

Equating coefficients

$$x^3;\qquad 0=A+C+D$$

$$0=A-\frac{1}{8}+\frac{5}{8}\Rightarrow 0=A+\frac{4}{8}$$

⇨ $0=A+\frac{1}{2}\quad or\quad A=-\frac{1}{2}$

$$x^2;\qquad 1=B-C+D$$

$$1=B-\left(-\frac{1}{8}\right)+\frac{5}{8}$$

$$1=B+\frac{1}{8}+\frac{5}{8}$$

$$1=B+\frac{1}{8}+\frac{5}{8}\Rightarrow 1=B+\frac{6}{8}$$

$$\Rightarrow 1=B+\frac{3}{4}\Rightarrow B=1-\frac{3}{4}$$

$$B=\frac{1}{4}$$

$$so\ (i)becomes\ as$$

$$\frac{x^2+2x+2}{(x^2+3)(x+1)(x-1)}=\frac{-\frac{1}{2}x+\frac{1}{4}}{x^2+3}+\frac{-\frac{1}{8}}{x+1}+\frac{\frac{5}{8}}{x-1}$$

$$\frac{x^2+2x+2}{(x^2+3)(x+1)(x-1)}$$
$$=\frac{\frac{1}{4}(-2x+1)}{x^2+3}-\frac{1}{8(x+1)}+\frac{5}{8(x-1)}$$

$$\frac{x^2+2x+2}{(x^2+3)(x+1)(x-1)}$$
$$=\frac{(-2x+1)}{4(x^2+3)}-\frac{1}{8(x+1)}+\frac{5}{8(x-1)}$$

Question No.8

$$\frac{1}{(x-1)^2(x^2+2)}$$

Solution:

$$\frac{1}{(x-1)^2(x^2+2)}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{Cx+D}{x^2+2}\to(i)$$

$$'\times by(x-1)^2(x^2+2)\quad we\ get$$

$$1=A(x-1)(x^2+2)+B(x^2+2)+Cx+D(x-1)^2\to(ii)$$

$$put\ x-1=0\Rightarrow x=1\ in(ii)$$

$$1=B(1^2+2)\Rightarrow 1-3B\Rightarrow B=\frac{1}{3}$$

From (ii)

$$1=A(x^3+2x-x^2-2)+Bx^2+2B+(Cx+D)(x^2+1-2x)$$

Equating coefficients

$$x^3;\qquad 0=A+C\to(iii)$$

$$x^2;\quad 0=B-A-2C+D$$

⇨ $B+D-A-2C=0\to(iv)$

$$x;\qquad 0=2A+C-2D$$

$$\Rightarrow 2A+C-2D=0\to(v)$$

Put $\quad B=\frac{1}{3}\ \ in\ (iv)$

$\frac{1}{3}+D-A-2C=0$

$$D-A-2C=-\frac{1}{3}$$

⇨ $2D-2A-4C=-\frac{2}{3}\to(vi)$

$$(\times'\ by\ 2)$$

By $(v)+(vi)$

$$-2D+2A+C=0$$

$$2D-2A-4c=-\frac{2}{3}$$

$$-3C=-\frac{2}{3}\Rightarrow C=\frac{2}{9}$$

So (iii)

$$0=A+\frac{2}{9}\Rightarrow A=-\frac{2}{9}$$

Now (v)

$$2\left(-\frac{2}{9}\right)+\frac{2}{9}-2D=0$$

$$(\div\ \ by\ 2)\Rightarrow -\frac{2}{9}+\frac{1}{9}-D=0$$

$$-\frac{1}{9} = D \;\; or \; D = -\frac{1}{9}$$

So (i) becomes

$$\frac{1}{(x-1)^2(x^2+2)} = -\frac{2}{9(x-1)} + \frac{1}{3(x-1)^2} + \frac{2x-1}{9(x^2+2)}$$

Question No.9

$$\boldsymbol{\frac{x^4}{1-x^4}(improper)}$$

Solution:

$$\begin{array}{r|l} & \mathbf{-1} \\ \hline \mathbf{1-x^4} & \mathbf{x^4} \\ & \mathbf{\pm x^4 \mp 1} \\ \hline & \mathbf{1} \end{array}$$

$$\frac{x^4}{1-x^4} = -1 + \frac{1}{1-x^4} \quad (proper)$$

$$= -1 + \frac{1}{(1-x^2)(1+x^2)}$$

$$\frac{x^4}{1-x^4} = -1 + \frac{1}{(1-x)(1+x)(1+x^2)} \rightarrow (i)$$

$$'\times' \;\; by\;(1-x)(1+x)(1+x^2)\; we\; get$$

$$1 = A(1+x)(1+x^2) + B(1-x)(1+x^2)$$
$$+ (Cx+D)(1-x)(1+x) \rightarrow (ii)$$

Put $1-x=0 \Rightarrow x = 1 \;\; in\;(ii)$

$$1 = A(1+1)(1+(1)^2)$$

⇨ $1 = a(2)(2)$

$$A = \frac{1}{4}$$

Put $1+x=0 \quad \Rightarrow x = -1 \;\; in\;(ii)$

$$1 = B\big((1-)-1\big)(1+(-1)^2)$$

$$1 = B(2)(2) \Rightarrow B = \frac{1}{4}$$

From (ii)

$$1 = A(1+x^2+x+x^3) + B(1+x^2-x-x^3) +$$
$$Cx - Cx^3 + D - Dx^2$$

Equating coefficients

$$x^3; \quad 0 = A - B - C$$

$$0 = \frac{1}{4} - \frac{1}{4} - C \Rightarrow 0 = -C$$

$$C = 0$$

$$x^2 \quad ; \quad 0 = A + B - D \Rightarrow 0 = \frac{1}{4} + \frac{1}{4} - D$$

So (i) $0 = \frac{2}{4} - D \Rightarrow D = \frac{1}{2}$

$$so \quad (i)\; becomes$$

$$\frac{1}{(1-x)(1+x)(1+x^2)}$$

$$= \frac{1}{4(1-x)} + \frac{1}{4(1+x)} + \frac{\frac{1}{2}}{4(1+x^2)}$$

Hence

$$\frac{x^4}{1-x^4} == \frac{1}{4(1-x)} + \frac{1}{4(1+x)} + \frac{1}{2(1+x^2)}$$

Question No.10

$$\frac{x^2-2x+3}{x^4+x^2+1}$$

Solution :

$$\frac{x^2-2x+3}{x^4+x^2+1} = \frac{x^2-2x+3}{x^4+2x^2+1-x^2}$$

$$= \frac{x^2-2x+3}{(x^2)^2+2x^2+(1)-(x)^2} = \frac{x^2-2x+3}{(x^2+1)^2-(x)^2}$$

$$= \frac{x^2-2x+3}{(x^2+1+x)(x^2+1-x)}$$

Suppose

$$\frac{x^2-2x+3}{(x^2+1+x)(x^2+1-x)} = \frac{Ax+B}{x^2+x+1} + \frac{Cx+D}{x^2-x+1}$$
$$\rightarrow (i)$$

$$'\times'\; by(x^2+1+x)(x^2+1-x) \;\; we\; get$$

$$x^2-2x+3 = (Ax+B)(x^2-x+1)(Cx+D)(x^2$$
$$+x+1)$$

$$x^2-2x+3 = Ax^3 - Ax^2 + Ax + Bx^2 + Bx + B$$
$$+ Cx^3 + Cx^2 + Cx + Dx^2 + Dx + D$$

$$Equating\; coefficients$$

$$x^3; \quad o = A + C \rightarrow (ii)$$

$$x^2; \quad 1 = -A + B + C + D \rightarrow (iii)$$

$$x; -2 = A - B + C + D \rightarrow (iv)$$

$$constant\; term \quad 3 = B + D \rightarrow (v)$$

$$put \;\; A + C = 0 \;\; in\;(iv)$$

$$-B + D = -2 \rightarrow (vi)$$

$$by\;(v)+(vi) \Rightarrow 2D = 1 \Rightarrow D = \frac{1}{2}$$

$$NOw\;(v) \Rightarrow 3 = B + \frac{1}{2} \Rightarrow B = 3 - \frac{1}{2}$$

$$B = \frac{5}{2}$$

Put $B + D = 3 \; in\;(iii)\; 3 + C - A = 1$

⇨ $C - A = -2 \rightarrow (vii)$

By $(ii) + (vii) \Rightarrow 2C = -2 \Rightarrow C = -1$

So $(ii) \Rightarrow o = A - 1 \Rightarrow A = 1$

$$\frac{x^2-2x+3}{(x^2+1+x)(x^2+1-x)} = \frac{(1)x+5/2}{x^2+x+1} + \frac{(-1)x+1/2}{x^2-x+1}$$

$$\frac{x^2-2x+3}{(x^2+1+x)(x^2+1-x)} = \frac{x+5/2}{x^2+x+1} + \frac{-x+1/2}{x^2-x+1}$$

$$= \frac{2x+5}{2(x^2+x+1)} + \frac{-2x+1}{x^2-x+1}$$

$$= \frac{2x+5}{2(x^2+x+1)} - \frac{2x-1}{x^2-x+1}$$

Case IV

When $Q(x)$ has repeated irreducible quadratic factors. $if\ \ Q(x)$ contains a repeated irreducible quadratic factors.

$(ax^2+bx+c)^n,\ \ n\geq 2$ and n is $a\ \ +ve$ integer, then

$\frac{P(x)}{Q(x)}$ $may\ be\ written\ as\ the\ following\ identity$

$$\frac{P(x)}{Q(x)}=\frac{A_1x+B_1}{ax^2+bx+c}+\frac{A_2x+B_2}{(ax^2+bx+c)^2}+\cdots$$
$$+\frac{A_nx+B_n}{(ax^2+bx+c)^n}$$

Where $A_1, A_2, B_1, B_2, \dots, A_n, B_n$ are $no.s$ to be found.

Exercise 5.4

Resolve into partial fractions:

Question No.1

$$\frac{x^3+2x+2}{(x^2+x+1)^2}$$

Solution: Suppose

$$\frac{x^3+2x+2}{(x^2+x+1)^2}=\frac{Ax+B}{x^2+x+1}+\frac{Cx+D}{(x^2+x+1)^2}\rightarrow(i)$$

$xby\ (x^2+x+1)^2\ \ we\ get$

$$x^3+2x+2=(Ax+B)(x^2+x+1)+Cx+D$$
$$x^3+2x+2=Ax^3+Ax^2+Ax+Bx^2+Bx+B$$
$$+Cx+D$$

Equating coefficients

$$x^3; 1=A, \qquad x^2; 0=A+B$$
$$0=1+B, B=-1$$

$x;\quad 2=A+B+C$

$$2=1+(-1)+C$$

⇨ $C=2$

Const. term $2=B+D\Longrightarrow 2=-1+D$

$$D=3$$

So (i) becomes as

$$\frac{x^3+2x+2}{(x^2+x+1)^2}=\frac{x-1}{x^2+x+1}+\frac{2x+3}{(x^2+x+1)^2}$$

Question No.2

$$\boldsymbol{\frac{x^2}{(x^2+1)^2(x-1)}}$$

Solution: Suppose

$$\frac{x^2}{(x^2+1)^2(x-1)}=\frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2}+\frac{E}{x-1}\rightarrow(i)$$

$'\times'$ **by** $(x^2+1)^2(x-1)$ we get

$$x^2=(Ax+B)(x^2+1)(x-1)+(Cx+D)(x-1)$$
$$+E(x^2+1)^2\rightarrow(ii)$$
$$x^2=Ax^4-Ax^3+Ax^2-Ax+Bx^3-Bx^2+Bx-B$$
$$+Cx^2-Cx+Dx-D+Ex^4+E+2Ex^2\rightarrow(iii)$$

Put $x-1=0\Rightarrow x=1\ in\ (ii)$

$$(1)^2=E((1)^2+1)^2\Rightarrow 1=E(2)^2\Rightarrow E=\frac{1}{4}$$

Comparing coefficients of eq. (iii)

$$x^4;\quad 0=A+E\Rightarrow 0=A+\frac{1}{4}$$

Comparing coefficients of $eq.\ (iii)$

$$x^4;\quad 0=A+E\Rightarrow 0=A+\frac{1}{4}$$
$$A=-\frac{1}{4}$$
$$x^3;\quad 0=B-A\Rightarrow 0=B-\left(-\frac{1}{4}\right)$$

$$\frac{1}{4}+B=0\Rightarrow B=-\frac{1}{4}$$
$$x^2;\quad 1=A-B+C+2E$$
$$1=-\frac{1}{4}-\left(-\frac{1}{4}\right)+C+2\left(\frac{1}{4}\right)$$
$$1=-\frac{1}{4}+\frac{1}{4}+C+\frac{1}{2}$$
$$C=1-\frac{1}{2}\Rightarrow C=\frac{1}{2}$$
$$x;\quad 0=B-A-C-D$$
$$0=-\frac{1}{4}-\left(-\frac{1}{4}\right)-\frac{1}{2}+D$$
$$0=-\frac{1}{4}+\frac{1}{4}-\frac{1}{2}+D$$

⇨ $o=-\frac{1}{2}+D\Rightarrow D=\frac{1}{2}$

So (i) becomes

$$\frac{x^2}{(x^2+1)^2(x-1)}=\frac{-\frac{1}{4}x-\frac{1}{4}}{x^2+1}+\frac{\frac{1}{2}x+\frac{1}{2}}{(x^2+1)^2}+\frac{\frac{1}{4}}{x-1}$$
$$\frac{x^2}{(x^2+1)^2(x-1)}$$
$$=\frac{-(x+1)}{4(x^2+1)}+\frac{x+1}{2(x^2+1)^2}+\frac{1}{4(x-1)}$$

Question No.3

$$\boldsymbol{\frac{2x-5}{(x^2+2)^2(x-2)}}$$

Solution: Suppose

$$\frac{2x-5}{(x^2+1)^2(x-2)}=\frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2}+\frac{E}{x-2}\rightarrow(i)$$

$'\times'$ **by** $(x^2+1)^2(x-2)$ we get

$$2x-5=(Ax+B)(x^2+1)(x-2)+(Cx+D)(x-2)$$
$$+E(x^2+1)^2\rightarrow(ii)$$

From (ii)

$$2x-5=(Ax+B)(x^3+2x-2x^2-4)+Cx^2$$
$$-2Cx$$
$$+Dx-2D+E(x^4+4+4x^2)$$
$$2x-5=Ax^4+2Ax^2-2x^3A-4Ax+Bx^3-Bx^2+2Bx$$
$$-2Bx^2-4B$$
$$+Cx^2-2Cx+Dx-2D+Ex^4+4E+4Ex^2\rightarrow(iii)$$

Put $x-2=0\Rightarrow x=2\ in\ (ii)$

$$2(2)-5=E((2)^2+2)^2\Rightarrow 4-5=E(4+2)^2$$
$$-1=E(36)\Rightarrow E=-\frac{1}{36}$$

Equating the coefficients

$$x^4;\quad 0=A+E\Rightarrow 0=A-\frac{1}{36}$$
$$A=\frac{1}{36}$$
$$x^3;\quad 0=-2A+B\Rightarrow 0=-2\left(\frac{1}{36}\right)+B$$

$$0 = -\frac{1}{18} + B \Rightarrow B = \frac{1}{18}$$

$$x^2; \quad 0 = 2A - 2B + C + 4E$$

$$0 = 2\left(\frac{1}{36}\right) - 2\left(\frac{1}{18}\right) + C + 4\left(-\frac{1}{36}\right)$$

$$0 = \frac{1}{18} - \frac{1}{9} + C - \frac{1}{9}$$

⇨ $\frac{1}{9} + \frac{1}{9} - \frac{1}{18} = C$

$$\frac{2+2-1}{18} = C$$

$$C = \frac{3}{18} = \frac{1}{6}$$

$x;$

$2 = 2B - 4A - 2C + D$

$$2 = 2\left(\frac{1}{18}\right) - 4\left(\frac{1}{36}\right) - 2\left(\frac{1}{6}\right) + D$$

$$2 = \frac{1}{9} - \frac{1}{9} + \frac{1}{3} + D$$

$$D = 2 + \frac{1}{3}$$

$so\ (i) becomes$ as

$$\frac{2x-5}{(x^2+1)^2(x-2)} = \frac{\frac{1}{36}x + \frac{1}{18}}{x^2+1} + \frac{\frac{1}{6}x + \frac{7}{3}}{(x^2+1)^2} + \frac{-\frac{1}{36}}{x-2}$$

$$\frac{2x-5}{(x^2+1)^2(x-2)} = \frac{x+2}{36(x^2+1)} + \frac{x+14}{6(x^2+1)^2} - \frac{1}{36(x-2)}$$

Question No.4

$$\mathbf{\frac{8x^2}{(x^2+1)^2(1-x^2)}}$$

Solution:

$$\frac{8x^2}{(x^2+1)^2(1-x^2)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2}$$

$$+\frac{E}{1-x} + \frac{F}{1+x} \rightarrow (i)$$

$$'\times'\ by\ (x^2+1)^2(-1x)(1+x), \qquad we\ get$$

$$8x^2 = (Ax+B)(x^2+1)(1-x)(1+x)$$

$$+(Cx+D)(1-x)(1+x) + E(x^2+1)^2(1+x)$$

$$+F(x^2+1)^2(1-x) \rightarrow (ii)$$

Put $1 - x = 0 \Rightarrow x = 1\ \ in(ii)$

$$8(1)^2 = E(1^2+1)^2(1+1) \Rightarrow 8 = E(2)^2(2)$$

$$8 = E(8) \Rightarrow E = 1$$

Put $1 + x = 0 \quad \Rightarrow x = -1\ \ in(ii)$

$$8(-1)^2 = F((-1)^2+1)^2\big(1-(-1)\big)$$

$$8 = F(2)^2(2) \Rightarrow 8 = 8F \Rightarrow F = 1$$

From (ii)

$$8x^2 = (Ax^3 + Ax + Bx^2 + B)(1-x^2) + (Cx+D)$$

$$(1-x^2) + E(x^4+1+2x^2)(1+x) +$$

$$F(x^4+1+2x^2)(1-x)$$

$$8x^2 = Ax^3 + Ax + Bx^2 + B - Ax^5 - Ax^3 - Bx^4$$

$$-Bx^2 + Cx - Cx^3 + D - Dx^2 + (Ex^4 + E + 2Ex^2)(1+x)$$

$$+(Fx^4 + F + 2Fx^2)(1-x)$$

$$8x^2 = Ax^3 + Ax + Bx^2 + B - Ax^5 - Ax^3 - Bx^4 - Bx^2$$

$$+Cx - Cx^3 + D - Dx^2 + Ex^4 + E + 2Ex^2 + Ex^5 + Ex + 2Ex^3$$

$$+$$

$$Fx^4 + F + 2fx^2 - Fx^5 - Fx - 2Fx^3$$

Equating coefficients

$$x^5; \quad 0 = -A + E - F$$

$$0 = -A + 1 - 1 \Rightarrow 0 = -A$$

$$A = 0$$

$$x^4; \quad 0 = -B + E + F$$

$$0 = -B + E + F$$

$$0 = -B + 1 + 1 \Rightarrow B = 2$$

$$x^3; \quad o = -C + 2E - 2F$$

$$0 = -C + 2(1) - 2(1)$$

$$0 = -C + 2 - 2 \Rightarrow c = 0$$

$$x^2; \qquad 8 = 2E + 2F \Rightarrow 4 = E + F$$

$$x; \quad 8 = 2E + 2F \Rightarrow 4 = E + F$$

$$x; \quad 0 = A + C + E - F$$

Constant term; $0 = B + D + E - F$

$$0 = 2 + D + 1 + 1 \Rightarrow 0 = 4 + D$$

$$D = -4$$

$$\frac{8x^2}{(x^2+1)^2(1-x^2)} = \frac{(0)x+2}{x^2+1} + \frac{(0)x-4}{(x^2+1)^2} + \frac{1}{1-x} + \frac{1}{1+x}$$

Or

$$\frac{8x^2}{(x^2+1)^2(1-x^2)} = \frac{2}{x^2+1} + \frac{-4}{(x^2+1)^2} + \frac{1}{1-x} + \frac{1}{1+x}$$

Question No.5

$$\mathbf{\frac{4x^4 + 3x^3 + 6x^2 + 5x}{(x-1)(x^2+x+1)^2}}$$

Solution: Suppose

$$\frac{4x^4 + 3x^3 + 6x^2 + 5x}{(x-1)(x^2+x+1)^2} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1} + \frac{Dx+E}{(x^2+x+1)^2}$$

$$\rightarrow (i)$$

$$'\times'\ by\ (x-1)(x^2+x+!)^2 \quad we\ get$$

$$4x^4 + 3x^3 + 6x^2 + 5x = A(x^2+x+1)^2$$

$$+(Bx+C)(x-1)(x^2+x+1) + (Dx+E)(x-1) \rightarrow (ii)$$

Put $x - 1 = 0 \Rightarrow x = 1\ in\ (ii)$

$$4(1)^4 + 3(1)^3 + 6(1)^2 + 5(1) = A[(1)^2 + 1 + 1]^2$$

$$4 + 3 + 6 + 5 = A(3)^2 \Rightarrow 18 = 9A$$

$$\Rightarrow A = 2$$

From (ii)

$$4x^4 + 3x^3 + 6x^2 + 5x = A(x^4 + 2x^3 + 3x^2 + 2x + 1) +$$

$$(Bx+C)(x^3-1) + Dx^2 - Dx + Ex - E$$

$4x^4 + 3x^3 + 6x^2 + 6x$

$= Ax^4 + 2Ax^3 + 3Ax^2 + 2Ax + A + Bx^4 - Bx + Cx^3 - C + 2Ax$

$$+A + Bx^4 - Bx + Cx^3 - C + Dx^2 - Dx + Ex - E$$

$$4x^4 + 3x^3 + 6x^2 + 6x = Ax^4 + 2Ax^3 + 3Ax^2 + 2Ax + A + Bx^4$$

$$-Bx + Cx^3 - C + Dx^2 - Dx + Ex - E$$

Equating the coefficients

$$x^4; \quad 4 = A + B \Rightarrow 4 = 2 + B \Rightarrow B = 2$$

$$x^3; \quad 3 = 2A + C \Rightarrow 3 = 2(2) + C$$

$$3 = 4 + C \Rightarrow 3 - 4 = C \Rightarrow C = -1$$

$$x^2; \qquad 6 = 3A + D \ \Rightarrow 6 = 3(2) + D \Rightarrow d = 0$$

$$x; \qquad 5 = 2A - B - D + E$$

$$5 = 2(2) - 2 - 0 + E \Rightarrow 5 = 4 - 2 + E$$

$$5 = 2 + E \Rightarrow 5 - 2 = E = 3$$

So (i) becomes as

$$\frac{4x^4 + 3x^3 + 6x^2 + 5x}{(x-1)(x^2+x+1)^2}$$

$$= \frac{2}{x-1} + \frac{2x-1}{x^2+x+1} + \frac{3}{(x^2+x+1)^2}$$

Question No.6

$$\mathbf{\frac{2x^4 - 3x^3 - 4x}{(x^2+2)^2(x+1)^2}}$$

Solution: Suppose

$$\frac{2x^4-3x^3-4x}{(x^2+2)^2(x+1)^2}=\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{Cx+D}{x^2+2}+\frac{Ex+F}{(x^2+2)^2}\rightarrow (i)$$

$$'\times'\ by(x^2+2)^2(x+1)^2\quad we\ get$$

$$2x^4-3x^3-4x=A(x+1)(x^2+2)^2+B(x^2+2)^2+(Cx+D)$$

$$(x^2+2)(x^2+2)^2(x+1)^2+(Ex+F)(x+1)^2\rightarrow (ii)$$

Put $x+1=0 \Rightarrow x=-1\ \ in(ii)$

$$2(-1)^4-3(-1)^3-4(-1)=B((-1)^2+2)^2$$

$$2+3+4=B(1+2)^2 \Rightarrow 9=9B$$

$$B=1$$

From (ii)

$$2x^4-3x^3-4x=A(x+1)(x^4+4x^2+4)+B(x^4+4+4x^2)+$$

$$(Cx+D)(x^2+2)(x^2+1+2x)+(Ex+F)(x^2+1+2x)$$

$$2x^4-3x^3-4x=A(x^5+4x^3+4x+x^4+4x^2+4)$$

$$B(x^4+4+4x^2)+(Cx+D)(x^4+2x^3+x^2+2x^2+4x+2)$$

$$+Ex^3+Ex+2Ex^2+Fx^2F+2Fx$$

$$2x^4-3x^3-4x=Ax^5+4Ax^3+4Ax+Ax^4+4Ax^2+4A$$

$$+Bx^4+4B+4Bx^2+Cx^5+2Cx^4+3Cx^3+4Cx^2+2Cx$$

$$Dx^4+2Dx^3+3Dx^2+4Dx+2D+Ex^3+Ex+2Ex^2+$$

$$Fx^2+F+2Fx$$

$x^5;$ $\qquad 0=A+C\rightarrow (iii)$

$x^4;$ $\qquad 2=A+B+2C+D\rightarrow (iv)$

$x^3;$ $\qquad -3=4A+3C+2D+E\rightarrow (v)$

x^2 $\qquad 0=4A+4B+4C+3D+2E+F\rightarrow (vi)$

$x;$ $\qquad -4=4A+2C+4D+E+2F\rightarrow (vii)$

Constant term,

$$0=4A+4B+2D+F\rightarrow (viii)$$

Now

$$(iii)\Rightarrow C=-A\ put\ valves\ of\ B\ and\ C\ in\ (iv)$$

$$(iv)\Rightarrow 2=A+1+2(-A)+D\Rightarrow 2=A+1-2A+D$$

$$2-1=-A+D\Rightarrow 1=-A+D$$

$$or\quad D=1+A\rightarrow (ix)$$

Put valves of $C\ and\ D\ in\ (v)$

$$(v)\Rightarrow -3=4A+3)-A)+2(1+A)+E$$

$$-3=4A-3A+2+2A+E\Rightarrow -3=3A+2+E$$

$$or\ \ E=-5-3A\rightarrow (x)$$

Put valves of $B, C, D\ and\ E\ in\ (vi)$

$$(vi)\Rightarrow 0=4A+4(1)+4(-A)+3(1+A)+2(-5-3A)+F$$

$$0=4A+4-4A+3+3A-10-6A+F$$

$$0=-3-3A+F\Rightarrow F=3+3A\rightarrow (xi)$$

Put valves of $C, D\ \ and\ E\ \ in\ (vii)$

$$(vii)\Rightarrow -4=4A+2(-A)+4(1+A+(-5-3A)+2F$$

$$-4=4A-2(A)+4+4A-5-3A+2F$$

$$-4=3A-1+2F\Rightarrow -4+1=3A+2F$$

$$3a+2F=-3\rightarrow (xii)$$

By $(xi)+(xii)\Rightarrow$

$$\begin{array}{c} -3=-3A+F \\ -3=3A+2F \\ \hline 0=3F\Rightarrow F=0 \end{array}$$

So

$$(xi)\Rightarrow\quad 3=-3A+0A=-1$$

$$(iii)\Rightarrow 0=-1+C\Rightarrow C=1$$

$$(ix)\Rightarrow 1=-(-1)+D$$

$$1=1+D\Rightarrow D=0$$

$$(x)\Rightarrow -5=3(-1)+E\Rightarrow -5=-3+E\Rightarrow E=-2$$

So (i) becomes as

$$\frac{2x^4-3x^3-4x}{(x^2+2)^2(x+1)^2}=\frac{-1}{x+1}+\frac{1}{(x+1)^2}+\frac{x+0}{x^2+2}+\frac{-2x+0}{(x^2+2)^2}$$

$$\frac{2x^4-3x^3-4x}{(x^2+2)^2(x+1)^2}=\frac{-1}{x+1}+\frac{1}{(x+1)^2}+\frac{x}{x^2+2}-\frac{2x}{(x^2+2)^2}$$