MATHEMATICS 11 INTERMEDIATE PART 1

Bilal Article

Chapter 4.

# QUADRATIC EQUATION

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# Contents

**Quadratic Equation:**

An equation containing one or more terms in which the variable is raised to maximum positive power two. In general $ax^2 - 7x + 10 = 0$;

$a = 1, b = -7, c = 10$

**Solution of Quadratic Equation:**

**We solve Quadratic Equation by following three method.**

i) By factorization

ii) By completing square

iii) By quadratic formula

**Note:**

- **The solution of an equation are also called its roots.**

**By Quadratic formula Derivation of the quadratic Formula:**

Quadratic equation is standard form is

$$ax^2 + bx + c = 0$$

Dividing both sides by a

⇨ $\frac{ax^2}{a} + \frac{b}{a}x + \frac{c}{a} = 0$

⇨ $x^2 + \frac{b}{a}x = -\frac{c}{a}$

$$Adding\ \left(\frac{1}{2}.\frac{b}{a}\right)^2\ on\ both\ sides$$

$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$

⇨ $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$

⇨ $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$

⇨ $x + \frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$

⇨ $x + \frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{2a}$

⇨ $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$

⇨ $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

⇨ $this\ is\ called\ Quadratic\ formula$"
Quadratic equation is also known as second degree polynomial."

In quadratic equation $ax^2 + bx + c = 0\ \forall\ a, b, c \in R$

# Exercise 4.1

**Solve the following equations by factorization:**

**Question#1**

$\mathbf{3x^2 + 4x + 1 = 0}$

**Solution:**

$3x^2 + 3x + x + 1 = 0$

$3x(x + 1) + 1(x + 1) = 0$

$(x + 1)(3x + 1) = 0$

$x + 1 = 0 \quad , 3x + 1 = 0$

$x = -1 ,\ x = -\frac{1}{3}$

**S.S** $= \left\{-1 , -\frac{1}{3}\right\}$

**Question#2**

$\mathbf{x^2 + 7x + 12 = 0}$

**Solution:**

$x^2 + 4x + 3x + 12 = 0$

$x(x + 4) + 3(x + 4) = 0$

$(x + 3)(x + 4) = 0$

$x + 3 = 0 \quad , x + 4 = 0$

$x = -3 ,\ x = -4$

**S. S** $= \{-3 , -4\}$

**Question#3**

$\mathbf{9x^2 - 12x - 5 = 0}$

**Solution:**

$9x^2 + 3x - 15x - 5 = 0$

$3x(3x + 1) - 5(3x + 1) = 0$

$(3x + 1)(3x - 5) = 0$

$3x + 1 = 0 \quad , 3x - 5 = 0$

$x = -\frac{1}{3} ,\ x = \frac{5}{3}$

**S.S** $= \left\{-\frac{1}{3} , \frac{5}{3}\right\}$

**Question#4**

$\mathbf{x^2 - x = 2}$

**Solution:**

$x^2 - x - 2x - 2 = 0$

$x(x + 1) - 2(x + 1) = 0$

$(x + 1)(x - 2) = 0$

$x + 1 = 0 \quad , x - 2 = 0$

$x = -1 ,\ x = 2$

**S. S** $= \{-1 , 2\}$

**Question#5**

$\mathbf{x(x + 7) = (2x - 1)(x + 4)}$

**Solution:**

$x^2 + 7 = 2x^2 + 8x - x - 4$

$x^2 + 7 = 2x^2 + 7x - 4$

$2x^2 - x^2 + 7x - 7x - 4 = 0$

$x^2 - 4 = 0$

$x^2 - (2)^2 = 0$

$(x + 2)(x - 2) = 0$

$x + 2 = 0 \quad , x - 2 = 0$

$x = -2 ,\ x = 2$

**S. S** $= \{-2 , 2\}$

**Question#6**

$\frac{x}{x+1}+\frac{x+1}{x}=\frac{5}{2}\ ;\ \mathbf{x \neq -1\,,10}$

**Solution:**

**Multiplying by** $2(2x+1)$

$2(2x+1)\,.\frac{x}{x+1}+\ 2(2x+1)\,.\frac{x+1}{x}=\ 2(2x+1)\,.\frac{5}{2}$

$2x^2+2(x+1)(x+1)=5(x+1)$

$2x^2+2(x^2+2x+1)=5x^2+5x$

$4x^2+4x+2=5x^2+5x$

$5x^2-4x^2+5x-4x-2=0$

$x^2+x-2=0$

$x^2-\ x+2x-2=0$

$x(x-1)+2(x-1)=0$

$(x-1)(x+2)=0$

$x-1=0\quad ,x+2=0$

$x=1\,,\ x=-2$

**S. S** $=\{\mathbf{1}\,,-\mathbf{2}\}$

**Question#7**

$\frac{1}{x+1}+\frac{2}{x+2}=\frac{7}{x+5}\ ;\ \mathbf{x\neq -1\,,-2\,,-5}$

**Solution:**

**Multiplying by** $(\mathbf{x+1})(\mathbf{x+2})(\mathbf{x+5})$

$(x+1)(x+2)(x+5).\frac{1}{x+1}+(x+1)(x+2)(x+5)\,.\frac{2}{x+2}$

$=(x+1)(x+2)(x+5)\,.\frac{7}{x+5}$

$(x+2)(x+5)+2(x+1)(x+5)=7(x+1)(x+2)$

$x^2+5x+2x+10+2+2(x^2+5x+x+5)$

$=7(x^2+2x+x+14)$

$x^2+5x+2x+10+2+2x^2+12x+10$

$=7x^2+21x+14$

$3x^2+19x+20=7x^2+21x+14$

$7x^2-3x^2+21x-19x+14-20=0$

$4x^2+2x-6=0$

$2x^2+x-3=0$

$2x^2-2\,x+3x-3=0$

$2x(x-1)+3(x-1)=0$

$(x-1)(2x+3)=0$

$x-1=0\quad ,2x+3=0$

$x=1\,,\ x=-\frac{3}{2}$

**S. S** $=\left\{\mathbf{1}\,,-\frac{\mathbf{3}}{\mathbf{2}}\right\}$

**Question#8**

$\frac{a}{ax-1}+\frac{b}{bx-1}=\mathbf{a+b}\,;\ \ \mathbf{x\neq \frac{1}{a}\ ,\ \frac{1}{b}}$

**Solution:**

$\frac{a}{ax-1}-a+\frac{b}{bx-1}-b=0$

$\frac{a-b(ax-1)}{ax-1}+\frac{b-a(bx-1)}{bx-1}=0$

$\frac{a-abx+b}{ax-1}+\frac{b-abx+a}{bx-1}=0$

$\frac{a+b-abx}{ax-1}+\frac{a+b-abx}{bx-1}=0$

$(a+b-abx)\left\{\frac{1}{ax-1}+\frac{1}{bx-1}\right\}=0$

$(a+b-abx)\left\{\frac{bx-1+ax-1}{(ax-1)(bx-1)}\right\}=0$

$(a+b-abx)(ax+bx-2)=0(ax-1)(bx-1)$

$(a+b-abx)(ax+bx-2)=0$

**Either,**

$abx=a+b\quad ,\ (a+b)x=2$

$\Rightarrow x-\frac{3}{2}=\pm\frac{51}{2}$

$x=\frac{a+b}{ab}\quad ,\ \ x=\frac{2}{(a+b)}$

**S. S** $=\left\{\frac{\mathbf{a+b}}{\mathbf{ab}}\,,\frac{\mathbf{2}}{\mathbf{(a+b)}}\right\}$

**Solve the following equations by completing the square:**

**Question#9**

$\mathbf{x^2-2x-899=0}$

**Solution:**

$x^2-2x=899$

**Adding** $\left(-\frac{2}{2}\right)^2=(-\mathbf{1})^2$ **on both sides**

$x^2-2x+(-1)^2=899+(-1)^2$

$(x-1)^2=899+1$

$(x-1)^2=900$

$\Rightarrow x-1=\pm 30$

$x-1=30\,,\quad x-1=-30$

$x=30+1\,,\ x=-30+1$

$x=31\ ,\ \ x=-29$

**S. S** $=\{31\,,-29\}$

**Question#10**

$\mathbf{x^2+4x-1085=0}$

**Solution:**

$x^2+4x=1085$

**Adding** $\left(\frac{4}{2}\right)^2=(\mathbf{2})^2$ **on both sides**

$x^2+4x+(2)^2=1085+(2)^2$

$(x+2)^2=1085+4$

$(x+2)^2=1089$

$\Rightarrow x+2=\pm 33$

$x+2=33\,,\quad x+2=-33$

$x=33-2\,,\ x=-33-2$

$x=31\ ,\ \ x=-35$

**S. S** $=\{31\,,-35\}$

**Question#11**

$\mathbf{x^2+6x-567=0}$

**Solution:**

$x^2+6x=567$

**Adding** $\left(\frac{6}{2}\right)^2=(\mathbf{3})^2$ **on both sides**

$x^2+6x+(3)^2=567+(3)^2$

$(x+3)^2=567+9$

$(x+3)^2=576$

$\Rightarrow x+3=\pm 24$

$x+3=24\,,\quad x+3=-24$

$x=24-3\,,\ x=-24-3$

$x=21\ ,\ \ x=-27$

**S. S** $=\{21\,,-27\}$

***Question#12***

$\boldsymbol{x^2-3x-648=0}$

***Solution:***

$x^2-3x=648$

***Adding*** $\left(\frac{3}{2}\right)^2$ ***on both sides***

$x^2 - 3x + \left(\frac{3}{2}\right)^2 = 648 + \left(\frac{3}{2}\right)^2$

$\left(x - \frac{3}{2}\right)^2 = 648 + \frac{9}{4}$

$\left(x - \frac{3}{2}\right)^2 = \frac{2592+9}{4}$

$\Rightarrow \left(x - \frac{3}{2}\right)^2 = \pm\frac{2601}{4}$

$x - \frac{3}{2} = \frac{51}{2}$ , $x - \frac{3}{2} = -\frac{51}{2}$

$x = \frac{51}{2} + \frac{3}{2}$ , $x = -\frac{51}{2} + \frac{3}{2}$

$x = \frac{51+3}{2}$ , $x = \frac{-51+3}{2}$

$x = \frac{54}{2}$ , $x = -\frac{48}{2}$

**S. S** $= \left\{\frac{54}{2}, -\frac{48}{2}\right\}$

***Question#13***

$\boldsymbol{x^2 - x - 1806 = 0}$

***Solution:***

$x^2 - x = 1806 = 0$

***Adding*** $\left(\frac{1}{2}\right)^2$ ***on both sides***

$x^2 - x + \left(\frac{1}{2}\right)^2 = 1806 + \left(\frac{1}{2}\right)^2$

$\left(x - \frac{1}{2}\right)^2 = 1806 + \frac{1}{4}$

$\left(x - \frac{1}{2}\right)^2 = \frac{7224+1}{4}$

$\left(x - \frac{1}{2}\right)^2 = \frac{7225}{4}$

$\Rightarrow x - \frac{1}{2} = \pm\frac{85}{2}$

$x - \frac{1}{2} = \frac{85}{2}$ , $x - 1 = -\frac{85}{2}$

$x = \frac{85}{2} + \frac{1}{2}$ , $x = -\frac{85}{2} + \frac{1}{2}$

$x = \frac{85+1}{2}$ , $x = \frac{-85+1}{2}$

$x = \frac{86}{2}$ , $x = -\frac{84}{2}$

$x = 43$ , $x = -42$

**S. S** $= \{43, -42\}$

***Question#14***

$\boldsymbol{2x^2 + 12x - 110 = 0}$

***Solution:***

$2x^2 + 12x - 110 = 0$

***Dividing by 2***

$x^2 + 6x = 55$

***Adding*** $\left(\frac{6}{2}\right)^2 = (3)^2$ ***on both sides***

$x^2 + 6x + (3)^2 = 55 + (3)^2$

$(x+3)^2 = 55 + 9$

$(x+3)^2 = 64$

$\Rightarrow x + 3 = \pm 8$

$x + 3 = 8$ , $x + 3 = -8$

$x = 8 - 3$ , $x = -8 - 3$

$x = 5$ , $x = -11$

$S.\ S = \{5, -11\}$

***Find roots of the following equations by using quadratic formula:***

***Question#15***

$\boldsymbol{5x^2 - 13x + 6 = 0}$

***Solution:***

***Comparing*** $\boldsymbol{ax^2 + bx + c = 0}$

***We have*** $\boldsymbol{a = 5\ \ b = -13, c = 6}$

***Using*** $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-13)\pm\sqrt{(-13)^2-4(5)(6)}}{2(5)}$

$x = \frac{13\pm\sqrt{169-120}}{10}$

$x = \frac{13\pm\sqrt{49}}{10}$

***Question#19***

$x = \frac{13\pm 7}{10}$

$x = \frac{13+7}{10}$ , $x = \frac{13-7}{10}$

$x = \frac{20}{10}$ , $x = \frac{6}{10}$

$x = 2$ , $x = \frac{3}{5}$

**S. S** $= \left\{2, \frac{3}{5}\right\}$

***Question#16***

$\boldsymbol{4x^2 + 7x - 1 = 0}$

***Solution:***

***Comparing*** $\boldsymbol{ax^2 + bx + c = 0}$

***We have*** $\boldsymbol{a = 4\ \ b = 7, c = -1}$

***Using*** $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-7\pm\sqrt{(7)^2-4(7)(-1)}}{2(4)}$

$x = \frac{-7\pm\sqrt{49+16}}{8}$

$x = \frac{-7\pm\sqrt{65}}{8}$

$x = \frac{-7+\sqrt{65}}{8}$ $x = \frac{-7-\sqrt{65}}{8}$

**S. S** $= \left\{\frac{-7+\sqrt{65}}{8}, \frac{-7-\sqrt{65}}{8}\right\}$

***Question#17***

$\boldsymbol{15x^2 + 2ax - a^2 = 0}$

***Solution:***

***Comparing*** $\boldsymbol{ax^2 + bx + c = 0}$

***We have*** $\boldsymbol{a = 15\ \ b = 2a, c = -a^2}$

***Using*** $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-(2a)\pm\sqrt{(2a)^2-4(15)(-a^2)}}{2(5)}$

$x = \frac{-2a\pm\sqrt{4a^2+60a^2}}{30}$

$x = \frac{-2a\pm\sqrt{64a^2}}{30}$

$x = \frac{-2a\pm 8a}{30}$

$x = \frac{-2a+8a}{30}$ , $x = \frac{-2a-8a}{30}$

$x = \frac{6a}{30}$ , $x = \frac{-10a}{30}$

$x = \frac{a}{5}$ , $x = \frac{-a}{3}$

**S. S** $= \left\{\frac{a}{5}, -\frac{a}{5}\right\}$

***Question#18***

$16x^2 + 8x + 1 = 0$

***Solution:***

***Comparing*** $ax^2 + bx + c = 0$

***We have*** $a = 16 \;\; b = 8\,, c = 1$

***Using*** $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-(8)\pm\sqrt{(8)^2-4(16)(1)}}{2(16)}$

$x = \frac{-8\pm\sqrt{64-64}}{32}$

$x = \frac{-8\pm\sqrt{0}}{32}$

$x = \frac{-8}{32}$ ,

$x = \frac{-1}{4}$

**S. S** $= \left\{\frac{-1}{4}\right\}$

$(x-a)(x-b) + (x-b)(x-c) + (x-c)(x-a) = 0$

***Solution:***

$x^2 - bx - ax + ab + x^2 - cx - bx + bc + x^2 - ax - cx + ac = 0$

$3x^2 - 2ax - 2bx - 2cx + ab + bc + ac = 0$

$3x^2 - 2(a+b+c)x + ab + bc + ac = 0$

***Comparing*** $ax^2 + bx + c = 0$

***We have*** $a = 3 \;\; b = -2(a+b+c)\,, c = ab + bc + ac$

***Using*** $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-[-2(a+b+c)]\pm\sqrt{[-2(a+b+c)]^2-4(3)(ab+bc+ac)}}{2(3)}$

$x = \frac{2(a+b+c)\pm2\sqrt{(a+b+c)^2-(3)(ab+bc+ac)}}{6}$

$x = \frac{(a+b+c)\pm\sqrt{a^2+b^2+c^2+2ab+2bc+2ac-3ab-3bc-3ac}}{3}$

$x = \frac{(a+b+c)\pm\sqrt{a^2+b^2+c^2-ab-bc-ac}}{3}$

**S. S** $= \left\{x = \frac{(a+b+c)\pm\sqrt{a^2+b^2+c^2-ab-bc-ac}}{3}\right\}$

***Question#20***

$(a+b)x^2 + (a+2b+c)x + b + c = 0$

***Solution:***

***Comparing*** $ax^2 + bx + c = 0$

***We have*** $a = (a+b)\,, \;\; b = (a+2b+c) \;\;, c = b + c$

***Using*** $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-(a+2b+c)\pm\sqrt{(a+2b+c)^2-4(a+b)(b+c)}}{2(a+b)}$

$x = \frac{-(a+2b+c)\pm\sqrt{(a+2b+c)^2-4(ab+ac+b^2+bc)}}{2(a+b)}$

$x = \frac{-(a+2b+c)\pm\sqrt{a^2+4b^2+c^2+4ab+4bc+2ac-4ab-4ac-4b^2-4bc}}{2(a+b)}$

$x = \frac{-(a+2b+c)\pm\sqrt{a^2+c^2-2ac}}{2(a+b)}$

$x = \frac{-(a+2b+c)\pm\sqrt{(a-c)^2}}{2(a+b)}$

$x = \frac{-(a+2b+c)\pm(a-c)}{2(a+b)}$

$x = \frac{-(a+2b+c)+(a-c)}{2(a+b)}$ , $x = \frac{-(a+2b+c)-(a-c)}{2(a+b)}$

$x = \frac{-(a+2b+c)+a-c}{2(a+b)}$ , $x = \frac{-(a+2b+c)-a+c}{2(a+b)}$

$x = \frac{-a-2b-c+a-c}{2(a+b)}$ , $x = \frac{-a-2b-c-a+c}{2(a+b)}$

$x = \frac{-2b-2c}{2(a+b)}$ , $x = \frac{-2b-2c}{2(a+b)}$

$x = \frac{-2(b+c)}{2(a+b)}$ , $x = \frac{-2(a+b)}{2(a+b)}$

$x = \frac{-(b+c)}{(a+b)}$ , $x = -1$

**S. S** $= \left\{\frac{-(b+c)}{(a+b)} \;, -1\right\}$

**Solution of Equations Reducible to the Quadratic equation:**

**Type 1.**

The equation of the form $ax^{2n} + bx^n + c = 0\,, a \neq 0$

⇨ $a(x^n)^2 + bx^n + c = 0$

Put $x^n = y$ *it becomes as*

$ay^2 + by + c = 0$ *Quadratic eq. in y)*

(Q,1,2,3,4,5)

**Type 2.**

The equation of the form

$$(x+a)(x+b)(x+c)(x+d) = k$$

$$where\ a + b = c + d$$

(Q.6,7,8,9,10,11,12,13)

**Type 3. Exponential Equations:**

Equations in which the variable occurs in exponent, are called exponential Equations.

(Q14,15,16,17)

**Type 4. Reciprocal Equation:**

*An equation which remains unchanged when $x$ is replaced by $\frac{1}{x}$ is called a reciprocal equation.*

# Exercise 4.2

***Solve the following equations:***

***Question#1***

$x^4 - 6x^2 + 8 = 0$

***Solution:***

$(x^2)^2 - 6x^2 + 8 = 0$

***Put*** $x^2 = y$ ***then,***

$y^2 - 6y + 8 = 0$

$y^2 - 2y - 4y + 8 = 0$

$y(y-2) - 4(y-2) = 0$

$(y-2)(y-4) = 0$

$y - 2 = 0 \quad , y - 4 = 0$

$y = 2\,, y = 4$

***If*** $y = 2$ ***then,*** ***if*** $y = 4$ ***then,***

$x^2 = 2 \qquad x^2 = 4$

$x = \pm\sqrt{2} \qquad x = \pm 2$

**S. S** $= \{\pm\sqrt{2}\,, \pm 2\}$

***Question#2***

$x^{-2} - 10 = 3x^{-1}$

$(x^{-1})^2 - 3x^{-1} - 10 = 0$

***Put*** $x^{-1} = y$ ***then,***

$y^2 - 3y - 10 = 0$

$y^2 + 2y - 5y - 10 = 0$

$y(y+2)-5(y+2)=0$
$(y+2)(y-5)=0$
$y+2=0 \quad ,y-5=0$
$y=-2\,,\ \ y=5$

***If $y=-2$ then,*** ***if $y=5$ then,***

$x^{-1}=-2 \qquad x^{-1}=5$
$\frac{1}{x}=-2 \qquad \frac{1}{x}=5$
$x=-\frac{1}{2} \qquad x=\frac{1}{5}$

**S. S** $=\left\{-\frac{1}{2},\frac{1}{5}\right\}$

***Question#3***

$\boldsymbol{x^6-9x^3+8=0}$

***Solution:***

$(x^3)^2-9x^3+8=0$

***Put $x^3=y$ then,***

$y^2-9y+8=0$
$y^2-y-8y+8=0$
$y(y-1)-8(y-1)=0$
$(y-1)(y-8)=0$
$y-1=0 \quad ,y-8=0$
$y=1\,,\ \ y=8$

***If $y=1$ then***

$x^3=1$
$x^3-1=0$
$(x)^3-(1)^3=0$
$\because\ a^3-b^3=(a-b)(a^2+ab+b^2)$
$(x-1)(x^2+x+1)=0$
$x-1=0$

***Using*** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(1)\pm\sqrt{(1)^2-4(1)(1)}}{2(1)}$
$x=\frac{-1\pm\sqrt{1-4}}{2}$
$x=\frac{-1\pm\sqrt{-3}}{2}$

***if $y=8$ then,***

$x^3=8$
$x^3-8=0$
$(x)^3-(2)^3=0$
$(x-2)(x^2+2x+4)=0$
$x-2=0\ ,\ x^2+2x+4=0$
$x=2\,,\ \ x^2+2x+4=0$

***Using*** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-(2)\pm\sqrt{(2)^2-4(1)(4)}}{2(1)}$$
$$x=\frac{-2\pm\sqrt{4-16}}{2}$$
$$x=\frac{-2\pm\sqrt{-12}}{2}$$
$$x=\frac{-2\pm\sqrt{4}\sqrt{-3}}{2}$$
$$x=\frac{-2\pm2\sqrt{-3}}{2}$$
$$x=\frac{2(-1\pm\sqrt{-3})}{2}$$
$$x=-1\pm\sqrt{-3}$$

**S. S** $=\left\{1\,,2\,,\frac{-1\pm\sqrt{-3}}{2},-1\pm\sqrt{-3}\right\}$

***Question#4***

$\boldsymbol{8x^6-19x^3-27=0}$

***Solution:***

$\boldsymbol{8x^6-19x^3-27=0}$
$8(x^3)^2-19x^3-27=0$

***Put $x^3=y$ then,***

$8y^2-19y-27=0$
$8y^2+8y-27y-27=0$
$8y(y+1)-27(y+1)=0$
$(y+1)(8y-27)=0$
$y+1=0 \quad ,8y-27=0$
$y=-1\,,\ \ y=\frac{27}{8}$

***If $y=-1$ then,***

$x^3=-1$
$x^3+1=0$
$(x)^3+(1)^3=0$
$\because\ a^3-b^3=(a-b)(a^2+ab+b^2)$
$(x+1)(x^2-x+1)=0$
$x+1=0$
$x=-1\,,\ \ x^2-x+1=0$

***Using*** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(1)}}{2(1)}$
$x=\frac{1\pm\sqrt{1-4}}{2}$
$x=\frac{1\pm\sqrt{-3}}{2}$

***if $y=\frac{27}{8}$ then,***

$x^3=\frac{27}{8}$
$8x^3=27$
$(2x)^3-(3)^3=0$
$(2x-3)(4x^2+6x+9)=0$
$2x-3=0\ ,\ 4x^2+6x+9=0$
$x=\frac{3}{2},\ \ 4x^2+6x+9=0$

***Using*** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-(6)\pm\sqrt{(6)^2-4(4)(9)}}{2(4)}$$
$$x=\frac{-6\pm\sqrt{36-144}}{8}$$
$$x=\frac{-6\pm\sqrt{-108}}{8}$$
$$x=\frac{-6\pm\sqrt{36}\sqrt{-3}}{8}$$
$$x=\frac{-6\pm6\sqrt{-3}}{8}$$
$$x=\frac{6(-1\pm\sqrt{-3})}{8}$$
$$x=\frac{3(-1\pm\sqrt{-3})}{4}$$

**S. S** $=\left\{-1\,,\frac{3}{2}\,,\frac{1\pm\sqrt{-3}}{2}\,,\frac{3(-1\pm\sqrt{-3})}{4}\right\}$

***Question#5***

$\boldsymbol{x^{\frac{2}{5}} + 8 = 6x^{\frac{1}{5}}}$

**Solution:**

$\boldsymbol{x^{\frac{2}{5}} - 6x^{\frac{1}{5}} + 8 = 0}$

$\left(x^{\frac{1}{5}}\right)^2 - \left(x^{\frac{1}{5}}\right)^2 + 8 = 0$

***Put*** $x^{\frac{1}{5}} = y$ ***then,***

$y^2 - y + 8 = 0$

$y^2 - 2y - 4y + 8 = 0$

$y(y-2) - 4(y-2) = 0$

$(y-2)(y-4) = 0$

$y - 2 = 0 \quad , \ y - 4 = 0$

$y = 2 \, , \ \ y = 4$

***If*** $y = 2$ ***then,*** ***if*** $y = 4$ ***then,***

$x^{\frac{1}{5}} = 2$ $\qquad x^{\frac{1}{5}} = 4$

$\left(x^{\frac{1}{5}}\right)^5 = (2)^5$ $\qquad \left(x^{\frac{1}{5}}\right)^5 = (4)^5$

$x = 2^5$ $\qquad x = 4^5$

$x = 32$ $\qquad x = 1024$

**S. S** $= \{32, 1024\}$

***Question#6***

$\boldsymbol{(x+1)(x+2)(x+3)(x+4) = 24}$

**Solution:**

***Rearranging it***

$(x+1)(x+4)(x+2)(x+3) = 24$

$(x^2 + 5x + 4)(x^2 + 5x + 6) = 24$

$x^2 + 5x + 4 = 24 \qquad x^2 + 5x + 6 = 24$

***Put*** $\boldsymbol{x^2 + 5x = y}$ ***then,***

$(y+4)(y+6) = 24$

$y^2 + 6y + 4y + 24 = 24$

$y^2 + 10y + 24 - 24 = 0$

$y(y+10) = 0$

$y = 0 \quad , \ y + 10 = 0$

$y = 0 \, , \ \ y = -10$

***If*** $y = 0$ ***then,*** ***if*** $y = 4$ ***then,***

$x^2 + 5x = 0$ $\qquad x^2 + 5x = -10$

$x(x+5) = 0$ $\qquad x^2 + 5x + 10 = 0$

$x = 0 \, , x = -5$ $\qquad$ ***Using*** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(10)}}{2(1)}$

$x = \frac{-5 \pm \sqrt{25 - 40}}{2}$

$x = \frac{-5 \pm \sqrt{-15}}{2}$

**S. S** $= \left\{0 \, , -5 \, , \frac{-5 \pm \sqrt{-15}}{2}\right\}$

***Question#11***

$\boldsymbol{(2x-7)(x^2-9)(2x+5) - 91 = 0}$

**Solution:**

$(2x-7)(x-3)(x+3)(2x+5) - 91 = 0$

***Rearranging it***

$(2x-7)(x+3)(x-3)(2x+5) = 91$

$(2x^2 + 6x - 7x - 21)(2x^2 + 5x - 6x - 15) = 91$

$(2x^2 - x - 21)(2x^2 - x - 15) = 91$

***Put*** $\boldsymbol{2x^2 - x = y}$ ***then,***

$(y-21)(y-15) = 91$

$y^2 - 15y - 21y + 315 - 91 = 0$

$y^2 - 36y + 224 = 0$

$y^2 - 8y - 28y + 224 = 0$

$y(y-8) - 28(y-8) = 0$

$(y-8)(y-28) = 0$

$y - 8 = 0 \quad , \ y - 28 = 0$

$y = 8 \, , \quad y = 28$

***If*** $y = 8$ ***then,*** ***if*** $y = 28$ ***then,***

$2x^2 - x = 8$ $\qquad 2x^2 - x = 28$

$2x^2 - x - 8 = 0$ $\qquad 2x^2 - x - 28 = 0$

***Using*** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad 2x^2 - 8x + 7x - 28 = 0$

$x = \frac{-1 \pm \sqrt{(-1)^2 - 4(2)(-8)}}{2(2)}$ $\qquad 2x(x-4) + 7(x-4) = 0$

$x = \frac{-(-1) \pm \sqrt{1+64}}{4}$ $\qquad (x-4)(2x+7) = 0$

$x = \frac{1 \pm \sqrt{65}}{4}$ $\qquad x - 4 = 0 \, , 2x + 7 = 0$

$x = 4 \, , x = -7/2$

**S. S** $= \left\{4 \, , -\frac{7}{2}, \frac{1 \pm \sqrt{65}}{4}\right\}$

***Question#12***

$\boldsymbol{(x^2 + 6x + 8)(x^2 + 14x + 48) = 105}$

**Solution:**

***Factorizing it***

$(x^2 + 2x + 4x + 8)(x^2 + 6x + 8x + 48) = 105$

$[x(x+2) + 4(x+2)][x(x+6) + 8(x+6)] = 105$

$(x+2)(x+4)(x+6)(x+8) = 105$

***Rearranging it***

$(x+2)(x+8)(x+4)(x+6) = 105$

$(x^2 + 8x + 2x + 16)(x^2 + 6x + 4x + 24) = 105$

$(x^2 + 10x + 16)(x^2 + 10x + 24) = 105$

***Put*** $\boldsymbol{x^2 + 10x = y}$ ***then,***

$(y+16)(y+24) = 105$

$y^2 + 24y + 16y + 384 - 105 = 0$

$y^2 + 40y + 279 = 0$

$y^2 + 9y + 31y + 279 = 0$

$y(y+9) + 31(y+9) = 0$

$y + 9 = 0 \quad , \ y + 31 = 0$

$y = -9 \, , \ \ y = -31$

***If*** $y = -9$ ***then,*** ***if*** $y = -31$ ***then,***

$x^2 + 10x = -9$ $\qquad x^2 + 10x = -31$

$x^2 + x + 9x + 9 = 0$ $\qquad x^2 + 10x + 31 = 0$

$x(x+1) + 9(x+1) = 0$ $\qquad$ ***Using*** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$(x+1)(x+9) = 0$ $\qquad x = \frac{-10 \pm \sqrt{(10)^2 - 4(1)(31)}}{2(1)}$

$x + 1 = 0 \qquad x + 9 = 0$ $\qquad x = \frac{-10 \pm \sqrt{100 - 124}}{2(1)}$

$x = -1 \qquad x = -9$

$(y+2)(y-40) = -360$
$y^2 - 40y + 2y - 80 + 360 = 0$
$y^2 - 38y + 280 = 0$
$y^2 - 10y - 28y + 280 = 0$
$y(y-10) - 28(y-10) = 0$
$(y-10)(y-28) = 0$
$y - 10 = 0 \quad , \; y - 28 = 0$
$y = 10 \;, \; y = 28$

***If $y = 10$ then,***
$x^2 - 3x = 10$
$x^2 - 3x - 10 = 0$
$x^2 + 2x - 5x - 10 = 0$
$x(x+2) - 5(x+2) = 0$
$(x+2)(x-5) = 0$
$x + 2 = 0 \quad , \; x - 5 = 0$
$x = -2 \quad , \; x = 5$

***if $y = 28$ then,***
$x^2 - 3x = 28$
$x^2 - 3x - 28 = 0$
$x^2 + 4x - 7x - 28 = 0$
$x(x-7) + 4(x-7) = 0$
$(x-7)(x+4) = 0$
$x - 7 = 0 \;, \; x + 4 = 0$
$x = 7 \quad , \; x = -4$

**S. S** $= \{5, -2, 7, 4\}$

***Question#10***
$$(x+1)(2x+3)(2x+5)(x+3) = 945$$
***Hint:*** $(x+1)(2x+5)(2x+3)(x+3) = 945$
***Solution:***
***Rearranging it***
$(x+1)(x+3)(2x+3)(2x+5) = 945$
$(x^2 + 3x + x + 3)(4x^2 + 10x + 6x + 15) = 945$
$(x^2 + 4x + 3)(4x^2 + 16x + 15) = 945$
$(x^2 + 4x + 3)(4(x^2 + 4x) + 15) = 945$
***Put $x^2 + 4x = y$ then,***
$(y+3)(4y+15) = 945$
$4y^2 + 15y + 12y + 45 - 945 = 0$
$4y^2 - 48y + 75y - 900 = 0$
$4y(y-12) + 75(y-12) = 0$
$(y-12)(4y+75) = 0$
$y - 12 = 0 \quad , \; 4y + 75 = 0$
$y = 12 \quad , \; y = -\frac{75}{4}$

***If $y = 12$ then,***
$x^2 + 4x = 12$
$x^2 + 4x - 12 = 0$
$x^2 - 2x + 6x - 12 = 0$
$x(x-2) + 6(x-12) = 0$
$(x-2)(x+6) = 0$
$x - 2 = 0 \quad , \; x + 6 = 0$
$x = 2 \quad , \; x = -6$

***if $y = -\frac{75}{4}$ then,***
$x^2 + 4x = -\frac{75}{4}$
$4x^2 + 16x = -75$
$4x^2 + 16x + 75 = 0$
***Using*** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-16 \pm \sqrt{(16)^2 - 4(4)(75)}}{2(4)}$
$x = \frac{-16 \pm \sqrt{256 - 1200}}{8}$
$x = \frac{-16 \pm \sqrt{16 \times (-59)}}{8}$
$x = \frac{4\left(-4 \pm \sqrt{(-59)}\right)}{8}$
$x = \frac{-4 \pm \sqrt{(-59)}}{2}$
$x = \frac{-4 \pm \sqrt{59}\sqrt{-1}}{2}$
$x = \frac{-4 \pm \sqrt{59}i}{2}$

**S. S** $= \left\{-6, 2, \frac{-4 \pm \sqrt{59}i}{2}\right\}$

***Question#11***
$$(2x-7)(x^2-9)(2x+5) - 91 = 0$$
***Solution:***
$(2x-7)(x-3)(x+3)(2x+5) - 91 = 0$
***Rearranging it***
$(2x-7)(x+3)(x-3)(2x+5) = 91$
$(2x^2 + 6x - 7x - 21)(2x^2 + 5x - 6x - 15) = 91$
$(2x^2 - x - 21)(2x^2 - x - 15) = 91$
***Put $2x^2 - x = y$ then,***
$(y-21)(y-15) = 91$
$y^2 - 15y - 21y + 315 - 91 = 0$
$y^2 - 36y + 224 = 0$
$y^2 - 8y - 28y + 224 = 0$
$y(y-8) - 28(y-8) = 0$
$(y-8)(y-28) = 0$
$y - 8 = 0 \quad , \; y - 28 = 0$
$y = 8 , \quad y = 28$

***If $y = 8$ then,***
$2x^2 - x = 8$
$2x^2 - x - 8 = 0$
***Using*** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-1 \pm \sqrt{(-1)^2 - 4(2)(-8)}}{2(2)}$
$x = \frac{-(-1) \pm \sqrt{1+64}}{4}$
$x = \frac{1 \pm \sqrt{65}}{4}$

***if $y = 28$ then,***
$2x^2 - x = 28$
$2x^2 - x - 28 = 0$
$2x^2 - 8x + 7x - 28 = 0$
$2x(x-4) + 7(x-4) = 0$
$(x-4)(2x+7) = 0$
$x - 4 = 0 \;, \; 2x + 7 = 0$
$x = 4 \;, x = -7/2$

**S. S** $= \left\{4, -\frac{7}{2}, \frac{1 \pm \sqrt{65}}{4}\right\}$

***Question#12***
$$\left(x^2 + 6x + 8\right)\left(x^2 + 14x + 48\right) = 105$$
***Solution:***
***Factorizing it***
$(x^2 + 2x + 4x + 8)(x^2 + 6x + 8x + 48) = 105$
$[x(x+2) + 4(x+2)][x(x+6) + 8(x+6)] = 105$
$(x+2)(x+4)(x+6)(x+8) = 105$
***Rearranging it***
$(x+2)(x+8)(x+4)(x+6) = 105$
$(x^2 + 8x + 2x + 16)(x^2 + 6x + 4x + 24) = 105$
$(x^2 + 10x + 16)(x^2 + 10x + 24) = 105$
***Put $x^2 + 10x = y$ then,***
$(y+16)(y+24) = 105$
$y^2 + 24y + 16y + 384 - 105 = 0$
$y^2 + 40y + 279 = 0$
$y^2 + 9y + 31y + 279 = 0$
$y(y+9) + 31(y+9) = 0$
$y + 9 = 0 \quad , \; y + 31 = 0$
$y = -9 \;, \; y = -31$

***If $y = -9$ then,*** ***if $y = -31$ then,***

$x^2+10x=-9$

$x^2+x+9x+9=0$

$x(x+1)+9(x+1)=0$

$(x+1)(x+9)=0$

$x+1=0 \quad x+9=0$

$x=-1 \quad x=-9$

$x^2+10x=-31$

$x^2+10x+31=0$

***Using*** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-10\pm\sqrt{(10)^2-4(1)(31)}}{2(1)}$

$x=\frac{-10\pm\sqrt{100-124}}{2(1)}$

$x=\frac{-10\pm\sqrt{4(-6)}}{2}$

$x=\frac{2\left(-5\pm\sqrt{(-6)}\right)}{2}$

$x=-5\pm\sqrt{-6}$

$x=-5\pm\sqrt{6}\sqrt{-1}$

$x=-5\pm\sqrt{6}i$

***S. S*** $=\{-1,-9,-5\pm\sqrt{6}i\}$

***Question#13***

$(\boldsymbol{x^2+6x-27})(\boldsymbol{x^2-2x-35})=\mathbf{385}$

***Solution:***

***Factorizing it***

$(x^2-2x+9x-27)(x^2+5x-7x-35)=385$

$[x(x-3)+9(x-3)][x(x+5)-7(x+5)]=385$

$(x-3)(x+9)(x-7)(x+5)=385$

***Rearranging it***

$(x-3)(x+5)(x-7)(x+9)=385$

$(x^2+5x-3x-15)(x^2+9x-7x-63)=385$

$(x^2+2x-15)(x^2+2x-63)=385$

***Put*** $x^2+2x=y$ ***then,***

$(y-15)(y-63)=385$

$y^2-63y-15y+945-385=0$

$y^2-78y+560=0$

$y^2-8y-70y+560=0$

$y(y-8)-70(y-8)=0$

$y-8=0 \quad , y-70=0$

$y=8, \; y=70$

***If*** $y=8$ ***then,***

$x^2+2x=8$

$x^2-2x+4x-8=0$

$x(x-2)+4(x-2)=0$

$(x-2)(x+4)=0$

$x-2=0 \quad x+4=0$

$x=2 \quad x=-4$

***if*** $y=70$ ***then,***

$x^2+2x=70$

$x^2+2x-70=0$

***Using*** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-2\pm\sqrt{(2)^2-4(1)(-70)}}{2(1)}$

$x=\frac{-2\pm\sqrt{4+280}}{2}$

$x=\frac{-2\pm\sqrt{4(71)}}{2}$

$x=\frac{2(-1\pm\sqrt{71})}{2}$

$x=-1\pm\sqrt{71}$

***S. S*** $=\{2,-4,-1\pm\sqrt{71}\}$

***Question#14***

$\mathbf{4.2^{2x+1}-9.2^{x}+1=0}$

***Solution:***

$4.2^{2x}.2^1-9.2^{2x}+1=0$

$4.(2^x)^2.2-9.2^{2x}+1=0$

$8.(2^x)^2.-9.2^{2x}+1=0$

***Put*** $2^x=y$ ***then,***

$8y^2-9y+1=0$

$8y^2-8y-y+1=0$

$8y(y-1)-1(y-1)=0$

$(y-1)(8y-1)=0$

$y-1=0 \quad ,8y-1=0$

$y=1, \; y=\frac{1}{8}$

***If*** $y=1$ ***then,***

$2^x=1$

$2^x=2^0$

$x=0$

***if*** $y=\frac{1}{8}$ ***then,***

$2^x=\frac{1}{8}$

$2^x=\frac{1}{2^3}$

$2^x=2^{-3}$

$x=-3$

***S. S*** $=\{0,-3\}$

***Question#15***

$\mathbf{2^x-2^{-x+6}-20=0}$

***Solution:***

$2^x+2^{-x}.2^6-20=0$

$2^x+2^{-x}.64-20=0$

$2^x+\frac{64}{2^x}-20=0$

***Put*** $2^x=y$ ***then,***

$y^2+\frac{64}{y}-20=0$

$y^2+64-20y=0$

$y^2-20y+64=0$

$y^2-4y-16y+64=0$

$y(y-4)-16(y-4)=0$

$(y-4)(y-16)=0$

$y-4=0 \quad ,y-16=0$

$y=4, \; y=16$

***If*** $y=4$ ***then,***

$2^x=4$

$2^x=2^2$

$x=2$

***if*** $y=16$ ***then,***

$2^x=16$

$2^x=2^4$

$x=4$

***S. S*** $=\{2,4\}$

***Question#16***

$\mathbf{4^x-3.2^{x+3}+128=0}$

***Solution:***

$(2^2)^x-3.2^x.2^3+128=0$

$(2^x)^2-3.2^x.8+128=0$

$(2^x)^2.-24.2^x+128=0$

***Put*** $2^x=y$ ***then,***

$y^2-24y+128=0$

$y^2-\mathbf{8}y-16y+128=0$

$y(y-8)-16(y-8)=0$

$(y-8)(y-16)=0$

$y-8=0 \quad ,y-16=0$

$y=8, \; y=16$

***If*** $y=8$ ***then,***

$2^x=8$

$2^x=2^3$

$x=3$

***if*** $y=16$ ***then,***

$2^x=16$

$2^x=2^4$

$x=4$

***S. S*** $=\{3,4\}$

***Question#17***

$\mathbf{3^{2x-1}-12.3^x+81=0}$

***Solution:***

$(3^x)^2.3^{-1}-12.3^x+81=0$

$(3^x)^2.\frac{1}{3}-36.3^x+243=0$

***Put*** $3^x=y$ ***then,***

$y^2-36y+243=0$

$y^2-9y-27y+243=0$
$y(y-9)-27(y-9)=0$
$(y-9)(y-27)=0$
$y-9=0 \quad ,y-27=0$
$y=9,\ y=27$

***If*** $y=9$ ***then,*** ***if*** $y=27$ ***then,***

$3^x=9$ $\quad 3^x=27$
$3^x=3^2$ $\quad 3^x=3^3$
$x=2$ $\quad x=3$

**S. S** $=\{2,3\}$

***Question#18***

$$\left(x+\frac{1}{x}\right)^2-3\left(x+\frac{1}{x}\right)-4=0$$

***Solution:***

***Put*** $x+\frac{1}{x}=y$ ***then,***

$y^2-3y-4=0$
$y^2+y-4y-4=0$
$y(y+1)-4(y+1)=0$
$(y+1)(y-4)=0$
$y+1=0 \quad , y-4=0$
$y=-1,\ y=4$

***If*** $y=-1$ ***then,***

$x+\frac{1}{x}=-1$
$x^2+1=-x$
$x^2+x+1=0$
***Using*** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$x=\frac{-1\pm\sqrt{(1)^2-4(1)(1)}}{2(1)}$
$x=\frac{-1\pm\sqrt{1-4}}{2}$
$x=\frac{-1\pm\sqrt{-3}}{2}$
$x=\frac{-1\pm\sqrt{3}\sqrt{-1}}{2}$
$x=\frac{-1\pm\sqrt{3}i}{2}$

***if*** $y=4$ ***then,***

$x+\frac{1}{x}=4$
$x^2+1=4x$
$x^2-4x+1=0$
***Using*** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(1)}}{2(1)}$
$x=\frac{4\pm\sqrt{16-4}}{2}$
$x=\frac{4\pm\sqrt{12}}{2}$
$x=\frac{4\pm\sqrt{4\times3}}{2}$
$x=\frac{4\pm2\sqrt{3}}{2}$
$x=2\pm\sqrt{3}$

**S. S** $=\left\{\frac{-1\pm\sqrt{3}i}{2},2\pm\sqrt{3}\right\}$

***Question#19***

$$x^2+x-4+\frac{1}{x}+\frac{1}{x^2}=0$$

***Solution:***

$x^2+\frac{1}{x^2}+x+\frac{1}{x}-4=0$

***Put*** $x+\frac{1}{x}=y$ ***then,***

$\left(x+\frac{1}{x}\right)^2=y^2$
$x^2+\frac{1}{x^2}=y^2-2$

***Given equation takes form***

$y^2-2+y-4=0$
$y^2+y-6=0$
$y^2-2y+3y-6=0$
$y(y-2)+3(y-2)=0$
$(y-2)(y+3)=0$
$y-2=0 \quad , y+3=0$
$y=2,\ y=-3$

***If*** $y=2$ ***then,***

$x+\frac{1}{x}=2$
$x^2+1=2x$
$x^2-2x+1=0$
$(x-1)^2=0$
$x-1=0$
$x=1$

***if*** $y=-3$ ***then,***

$x+\frac{1}{x}=-3$
$x^2+1=-3x$
$x^2+3x+1=0$
***Using*** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$x=\frac{-(3)\pm\sqrt{(3)^2-4(1)(1)}}{2(1)}$
$x=\frac{-3\pm\sqrt{9-4}}{2}$
$x=\frac{-3\pm\sqrt{5}}{2}$
$x=\frac{-3\pm\sqrt{5}}{2}$

**S** ***Question#20***

$$\left(x-\frac{1}{x}\right)^2+3\left(x+\frac{1}{x}\right)=0$$

***Solution:***

$x^2+\frac{1}{x^2}-2+3\left(x+\frac{1}{x}\right)=0$

***Put*** $x+\frac{1}{x}=y$ ***then,***

$\left(x+\frac{1}{x}\right)^2=y^2$
$x^2+\frac{1}{x^2}=y^2-2$

***Given equation takes form***

$y^2-2-2+3y=0$
$y^2+3y-4=0$
$y^2-y+4y-4=0$
$y(y-1)+4(y-1)=0$
$(y-1)(y+4)=0$
$y-1=0 \quad , y+4=0$
$y=1,\ y=-4$

***If*** $y=1$ ***then,***

$x+\frac{1}{x}=1$
$x^2+1=x$
$x^2-x+1=0$
***Using*** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(1)}}{2(1)}$
$x=\frac{1\pm\sqrt{1-4}}{2}$
$x=\frac{1\pm\sqrt{3}\sqrt{-1}}{2}$
$x=\frac{1\pm\sqrt{3}i}{2}$

***if*** $y=-4$ ***then,***

$x+\frac{1}{x}=-4$
$x^2+1=-4x$
$x^2+4x+1=0$
***Using*** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$x=\frac{-(4)\pm\sqrt{(4)^2-4(1)(1)}}{2(1)}$
$x=\frac{-4\pm\sqrt{16-4}}{2}$
$x=\frac{-4\pm\sqrt{12}}{2}$
$x=\frac{-4\pm2\sqrt{3}}{2}$
$x=\frac{2(-2\pm\sqrt{3})}{2}$
$x=-2\pm\sqrt{3}$

**S. S** $=\left\{\frac{1\pm\sqrt{3}i}{2},-2\pm\sqrt{3}\right\}$

***Question#21***

$$2x^4-3x^3-x^2-3x+2=0$$

***Solution:***

***Dividing by*** $x^2$

$\frac{2x^4}{x^2}-\frac{3x^3}{x^2}-\frac{x^2}{x^2}-\frac{3x}{x^2}+\frac{2}{x^2}=0$
$2x^2-3x-1-\frac{3}{x}+\frac{2}{x^2}=0$
$2x^2+\frac{2}{x^2}-3x-\frac{3}{x}-1=0$
$2\left(x^2+\frac{1}{x^2}\right)-3\left(x+\frac{1}{x}\right)-1=0$

***Put*** $x+\frac{1}{x}=y$ ***then,***

$\left(x+\frac{1}{x}\right)^2 = y^2$

$x^2+\frac{1}{x^2} = y^2-2$

***Given equation takes form***

$2(y^2-2)-3y-1=0$

$2y^2-4-3y-1=0$

$2y^2-3y-5=0$

$2y^2+2y-5y-5=0$

$2y(y+1)-5(y+1)=0$

$(y+1)(2y-5)=0$

$y+1=0 \;,\; 2y-5=0$

$y=-1\,,\;\; y=\frac{5}{2}$

***If $y=-1$ then,***

$x+\frac{1}{x}=-1$

$x^2+1=-x$

$x^2+x+1=0$

**S. S** $=\left\{1\,,\frac{-3\pm\sqrt{5}}{2}\right\}$

***Using*** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-1\pm\sqrt{(1)^2-4(1)(1)}}{2(1)}$

$x=\frac{-1\pm\sqrt{1-4}}{2}$

$x=\frac{-1\pm\sqrt{3}\sqrt{-1}}{2}$

$x=\frac{-1\pm\sqrt{3}i}{2}$

***if $y=\frac{5}{2}$ then,***

$x+\frac{1}{x}=\frac{5}{2}$

$2x^2+2=5x$

$2x^2-5x+2=0$

$2x^2-x-4x+2=0$

$x(2x-1)-2(2x-1)=0$

$(2x-1)(x-2)=0$

$2x-1=0 \;,\; x-2=0$

$x=\frac{1}{2} \qquad x=2$

**S. S** $=\left\{2,\frac{1}{2},\frac{1\pm\sqrt{3}i}{2}\right\}$

***Question#22***

$\boldsymbol{2x^4+3x^3-4x^2-3x+2=0}$

***Solution:***

$\frac{2x^4}{x^2}+\frac{3x^3}{x^2}-\frac{4x^2}{x^2}-\frac{3x}{x^2}+\frac{2}{x^2}=0$

$2x^2+3x-4-\frac{3}{x}+\frac{2}{x^2}=0$

$2x^2+\frac{2}{x^2}+3x-\frac{3}{x}-4=0$

$2\left(x^2+\frac{1}{x^2}\right)+3\left(x-\frac{1}{x}\right)-4=0$

***Put*** $x-\frac{1}{x}=y$ ***then,***

$\left(x-\frac{1}{x}\right)^2=y^2$

$x^2+\frac{1}{x^2}-2=y^2$

***Given equation takes form***

$2(y^2+2)+3y-4=0$

$2y^2+4+3y-4=0$

$2y^2+3y=0$

$y(2y+3)=0$

$y=0\,,\;\; y=\frac{-3}{2}$

***If $y=0$ then,***

$x-\frac{1}{x}=0$

$x^2-1=0$

$x^2+x+1=0$

$(x-1)(x+1)=0$

$x-1=0 \;,\; x+1=0$

$x=1\,, \qquad x=-1$

***if $y=\frac{-3}{2}$ then,***

$x-\frac{1}{x}=-\frac{3}{2}$

$2x^2-2=-3x$

$2x^2+3x-2=0$

$2x^2-x+4x-2=0$

$x(2x-1)+2(2x-1)=0$

$(2x-1)(x+2)=0$

$2x-1=0 \;,\; x+2=0$

$x=\frac{1}{2} \;,\; x=-2$

**S. S** $=\{\mathbf{1,-1,2,-2}\}$

***Question#23***

$\boldsymbol{6x^4-35x^3+62x^2-35x+6=0}$

***Solution:***

$\frac{6x^4}{x^2}-\frac{35x^3}{x^2}+\frac{62x^2}{x^2}-\frac{35x}{x^2}+\frac{6}{x^2}=0$

$6x^2-35x+62-\frac{35}{x}+\frac{6}{x^2}=0$

$6x^2+\frac{6}{x^2}-35x-\frac{35}{x}+62=0$

$6\left(x^2+\frac{1}{x^2}\right)-3\left(x+\frac{1}{x}\right)+62=0$

***Put*** $x+\frac{1}{x}=y$ ***then,***

$\left(x+\frac{1}{x}\right)^2=y^2$

$x^2+\frac{1}{x^2}-2=y^2$

***Given equation takes form***

$6(y^2-2)-35y+62=0$

$6y^2-12-35y+62=0$

$6y^2-15y-20y+50=0$

$3y(2y-5)-10(2y-5)=0$

$(2y-5)(3y-10)=0$

$2y-5=0\,,\;\; 3y-10=0$

$y=\frac{5}{2}\,,\;\; y=\frac{10}{3}$

***If $y=\frac{5}{2}$ then,***

$x+\frac{1}{x}=\frac{5}{2}$

$2x^2+2=5x$

$2x^2-5x+2=0$

$2x^2-x-4x+2=0$

$x(2x-1)-2(2x-1)=0$

$(2x-1)(x-2)=0$

$2x-1=0 \;,\; x-2=0$

$x=\frac{1}{2}\,, \qquad x=2$

***if $y=\frac{10}{3}$ then,***

$x+\frac{1}{x}=\frac{10}{3}$

$3x^2+3=10x$

$3x^2-10x+3=0$

$3x^2-x-9x+3=0$

$x(3x-1)-3(3x-1)=0$

$(3x-1)(x-3)=0$

$3x-1=0\,, x-3=0$

$3x-1=0 \,, x-3=0$

$3x-1=0 \;,\; x-3=0$

$x=\frac{1}{3} \;,\; x=3$

**S. S** $=\left\{\frac{1}{2},2,\frac{1}{3},3\right\}$

***Question#24***

$\boldsymbol{x^4-6x^2+10-\frac{6}{x^2}+\frac{1}{x^4}=0}$

***Solution:***

$x^4+\frac{1}{x^4}-6x^2-\frac{6}{x^2}+10=0$

$x^4+\frac{1}{x^4}-6\left(x^2+\frac{1}{x^2}\right)+10=0$

***Put*** $x^2+\frac{1}{x^2}=y$ ***then,***

$\left(x^2+\frac{1}{x^2}\right)^2=y^2$

$x^4+\frac{1}{x^4}+2=y^2$

$x^4+\frac{1}{x^4}=y^2-2$

***Given equation takes form***

$(y^2-2)-6y+10=0$

$y^2-2-6y+10=0$

$y^2-6y+8=0$

$y^2-2y-4y+8=0$

$y(y-2)-4(y-2)=0$

$(y-2)(y-4)=0$

$y-2=0\ ,\ \ y-4=0$

$y=2\ ,\ \ y=4$

***If*** $y=2$ ***then,***

$x^2+\frac{1}{x^2}=y$

$x^2+\frac{1}{x^2}=2$

$x^4+1=2x^2$

$x^4-2x^2+1=0$

$(x^2-1)^2=0$

$x^2-1=0$

$(x-1)(x+1)=$

$x-1=0\quad ,x+1=0$

$x=1\ ,\qquad x=-1$

$As,\ \ ,x^2=z$

***if*** $y=4$ ***then,***

$x^2+\frac{1}{x^2}=y$

$x^2+\frac{1}{x^2}=4$

$x^4+1=x^2$

$x^4-4x^2+1=0$

$(x^2)^2-4x^2+1=0$

$Let\ x^2=z\ ,\ then$

$z^2-4z+1=0$

$z=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$z=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(1)}}{2(1)}$

$z=\frac{4\pm\sqrt{16-4}}{2}$

$z=\frac{4\pm2\sqrt{3}}{2}$

$z=\frac{2(2\pm\sqrt{3})}{2}$

$z=2\pm\sqrt{3}$

$So,\ x^2=2\pm\sqrt{3}$

$x=\sqrt{2\pm\sqrt{3}}$

**S. S** $=\left\{1\ ,-1\ ,\sqrt{2\pm\sqrt{3}}\right\}$

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**Type v. Radical Equation:**

**Equation** involving radical expression of the variable are called radical equations. e.g.;

$$\sqrt{2x+8}+\sqrt{x+5}-7=0$$

**Extraneous roots:**

A root that does not satisfied given equation is called an extraneous root.

**i)** The equation of the form

$l(ax^2+bx)+m\sqrt{ax^2+bx+c}=0$

**Question rerated of the form**

**Q1,2,10**

**ii)** The equations of the form

$\sqrt{x+a}+\sqrt{x+b}=\sqrt{x+c}$

**Question related to this type Q3,4,5**

**iii)** The equation of the form

$\sqrt{ax^2+bx+c}+\sqrt{px^2+qx+r}=\sqrt{lx^2+mx+n}$

Where $ax^2+bx+c, px^2+qx+r$ and $x^2+mx+n$ Have common factor.

**Questions related to this type Q7,8,9**

**iv)** The equation of the form

$\sqrt{ax^2+bx+c}+\sqrt{px^2+qx+r}=mx+n$

Where $mx+n\ is\ a\ factor\ of$

$$ax^2+bx+c-(px^2+qx+r)$$



# Exercise 4.3

***Solve the following equations:***

***Question#1***

$\mathbf{3x^2+2x-\sqrt{3x^2+2x-1}=3}$

***Solution:***

***Put*** $\sqrt{3x^2+2x-1}=y$

$3x^2+2x-1=y^2$

$3x^2+2x=y^2+1$

***Then given equation takes form***

$y^2+1-y=7$

$y^2-y+1-7=0$

$y^2-y\ -6=0$

$y^2+2y-3y\ -6=0$

$y(y+2)-3(y+2)=0$

$(y+2)(y-3)=0$

$y+2=0\qquad ,y-3=0$

$y=-2\ ,\quad y=3$

***If*** $y=-2$ ***then,***

$\sqrt{3x^2+2x-1}=-2$

$3x^2+2x-1=4$

$3x^2+2x-1-4=0$

$3x^2-3x+5x-5=0$

$3x(x-1)+5(x-1)=0$

$(x-1)(3x+5)=0$

$x-1=0\quad ,3x+5=0$

$x=-1\ ,\ x=-\frac{5}{3}$

***If*** $y=3$ ***then,***

$\sqrt{3x^2+2x-1}=3$

$3x^2+2x-1=9$

$3x^2+2x-1-9=0$

$3x^2+2x-10=0$

***Using*** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(2)\pm\sqrt{(2)^2-4(3)(-10)}}{2(3)}$

$x=\frac{-2\pm\sqrt{4+120}}{6}$

$x=\frac{-2\pm\sqrt{124}}{6}$

$x=\frac{-2\pm\sqrt{4\times13}}{6}$

$x=\frac{-2\pm2\sqrt{13}}{6}$

$x=\frac{-2(-1\pm\sqrt{13})}{6}$

$x=\frac{-1\pm\sqrt{13}}{3}$

*On checking we found that* 1 *and* $-\frac{5}{3}$ *are Extraneous roots. Hence,*

**S. S** $=\left\{\frac{-1\pm\sqrt{13}}{3}\right\}$

***Question#2***

$\mathbf{x^2-\frac{x}{2}-7=x-3\sqrt{2x^2-3x+2}}$

***Solution:***

*Multiplying by 2*

$2x^2-2x-14=2x-6\sqrt{2x^2-3x+2}$

$2x^2-2x-14-2x+6\sqrt{2x^2-3x+2}=0$

**Put** $\sqrt{2x^2-3x+2}=y$

$2x^2-3x+2=y^2$

$2x^2-3x=y^2-2$

***Then given equation takes form***

$y^2-2-14+6y=0$

$y^2+6y-16=0$

$y^2-2y+8y-16=0$

$y(y-2)+8(y-2)=0$

$(y-2)(y+8)=0$

$y-2=0 \quad ,y+8=0$

$y=2,\quad y=-8$

***If*** $y=2$ ***then,***

$\sqrt{2x^2-3x+2}=2$

$2x^2-3x+2=4$

$2x^2-3x+2-4=0$

$2x^2-3x-2=0$

$2x^2+x-3x-2=0$

$x(2x+1)-2(2x+1)=0$

$(2x+1)(x-2)=0$

$2x+1=0 \quad ,x-2=0$

$x=\frac{-1}{2},\ x=2$

***If*** $y=-8$ ***then,***

$\sqrt{2x^2-3x+2}=-8$

$2x^2-3x+2=64$

$2x^2-3x+2-64=0$

$2x^2-3x-62=0$

***Using*** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(2)(-62)}}{2(2)}$

$x=\frac{3\pm\sqrt{9+496}}{4}$

$x=\frac{3\pm\sqrt{505}}{4}$

*On checking we found that* $\frac{3\pm\sqrt{505}}{4}$ *are extraneous roots. Hence,*

**S. S** $=\left\{2,\frac{-1}{2}\right\}$

***Question#3***

$\mathbf{\sqrt{2x+8}+\sqrt{x+5}=7}$

***Solution:***

***Squaring on both sides***

$\left(\sqrt{2x+8}+\sqrt{x+5}\right)^2=(7)^2$

$\left(\sqrt{2x+8}\right)^2+\left(\sqrt{x+5}\right)^2+2\sqrt{2x+8}.\sqrt{x+5}=49$

$2x+8+x+5+2\sqrt{(2x+8)(x+5)}=49$

$3x+13+2\sqrt{2x^2+10x+8x+40}=49$

$2\sqrt{2x^2+18x+40}=49-3x-13$

$2\sqrt{2x^2+18x+40}=36-3x$

***Squaring on both sides***

$4(2x^2+18x+40)=(36-3x)^2$

$8x^2+72x+160=1296+9x^2-216x$

$9x^2-8x^2-216x-72x+1296-160=0$

$x^2-288x+1136=0$

$x^2-4x-284x+1136=0$

$x(x-4)-284(x-4)=0$

$(x-4)(x-284)=0$

$x-4=0 \quad ,x-284=0$

$x=4,\ x=284$

*On checking we found that* 284 *is an extraneous root. Hence,*

*S. S* $=\{4\}$

***Question#4***

$\mathbf{\sqrt{3x+4}=2+\sqrt{2x-4}}$

***Solution:***

$\sqrt{3x+4}-\sqrt{2x-4}=2$

***Squaring on both sides***

$\left(\sqrt{3x+4}-\sqrt{2x-4}\right)^2=(2)^2$

$\left(\sqrt{3x+4}\right)^2+\left(\sqrt{2x-4}\right)^2+2\sqrt{3x+4}.\sqrt{2x-4}=4$

$3x+4+2x-4+2\sqrt{(3x+4)(2x-4)}=4$

$5x-2\sqrt{6x^2-12x+8x-16}=4$

$-2\sqrt{6x^2-12x+8x-16}=4-5x$

$-2\sqrt{6x^2-4x-16}=4-5x$

***Squaring on both sides***

$4(6x^2-4x-16)=(4-5x)^2$

$24x^2-16x-64=16+25x^2-40x$

$25x^2-24x^2-40x+16x+16+64=0$

$x^2-24x+80=0$

$x^2-4x-20x+80=0$

$x(x-4)-20(x-4)=0$

$(x-4)(x-20)=0$

$x-4=0 \quad , x-20=0$
$x=4\,,\ x=20$
*On checking we found that no root is an extraneous root. Hence,*
**S. S** $=\{4,20\}$

**Question#5**

$\sqrt{x+7}+\sqrt{x+2}=\sqrt{6x+13}$

**Solution:**

**Squaring on both sides**

$\left(\sqrt{x+7}+\sqrt{x+2}\right)^2=\left(\sqrt{6x+13}\right)^2$
$\left(\sqrt{x+7}\right)^2+\left(\sqrt{x+2}\right)^2+2\sqrt{x+7}.\sqrt{x+2}=6x+13$
$x+7+x+2+2\sqrt{(x+7)(x+2)}=6x+13$
$2x+9+2\sqrt{x^2+2x+7x+14}=6x+13$
$2\sqrt{x^2+9x+14}=6x-2x+13-9$
$2\sqrt{x^2+9x+14}=4x+4$
$\sqrt{x^2+9x+14}=2x+2$

**Squaring on both sides**

$x^2+9x+14=(2x+2)^2$
$x^2+9x+14=4x^2+8x+4$
$4x^2-x^2+8x-9x+4-14=0$
$3x^2-x-10=0$
$3x^2-6x+5x-10=0$
$3x(x-2)+5(x-2)=0$
$(x-2)(3x+5)=0$
$x-2=0 \quad , 3x+5=0$
$x=2\,,\ x=-\frac{5}{3}$
*On checking we found that* $-\frac{5}{3}$ *is an extraneous root. Hence,*
**S. S** $=\{2\}$

**Question#6**

$\sqrt{x^2+x+1}+\sqrt{x^2+x-1}=1$

**Solution:**

**Put** $\sqrt{x^2+x+1}=a\,,\ \sqrt{x^2+x-1}=b$
**Then given equation takes form**
$a-b=1 \quad \ldots(1)$
**To find a and b we find that**
$a^2-b^2=\left(\sqrt{x^2+x+1}\right)^2-\left(\sqrt{x^2+x-1}\right)^2$
$a^2-b^2=x^2+x+1-(x^2+x-1)$
$a^2-b^2=x^2+x+1-x^2-x+1$
$a^2-b^2=2 \qquad \because a^2-b^2=(a+b)(a-b)$
$(a+b)(a-b)=2 \ \ldots(2)$
**Putting values of** $a-b$ **from** $(1)$ **in** $(2)$
$1(a+b)=2$
$a+b=2\ldots(3)$
*Adding* $(1)$ *and* $(3)$
$a-b=1$
$\underline{a+b=2}$
$2a=3$
$\Rightarrow a=\frac{3}{2}$
**Putting values of** $a$ **in** $(1)$
$\frac{3}{2}-1=b \Rightarrow b=\frac{3-2}{2}=\frac{1}{2}$

**If** $a=\frac{3}{2}$ **then ,**
$\sqrt{x^2+x+1}=\frac{3}{2}$
$2\sqrt{x^2+x+1}=3$
**Squaring on both sides**
$4(x^2+x+1)=9$
$4x^2+4x+4-9=0$
$4x^2+4x-5=0$

**If** $b=\frac{1}{2}$ **then ,**
$\sqrt{x^2+x-1}=\frac{1}{2}$
$2\sqrt{x^2+x-1}=3$
**Squaring on both sides**
$4(x^2+x-1)=9$
$4x^2+4x+4-9=0$
$4x^2+4x-5=0$

**Solving any one**
$4x^2+4x-5=0$
**Using** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$x=\frac{-(4)\pm\sqrt{(4)^2-4(4)(-5)}}{2(4)}$
$x=\frac{-4\pm\sqrt{16+80}}{8}$
$x=\frac{-4\pm\sqrt{96}}{8}$
$x=\frac{-4\pm\sqrt{16\times 6}}{8}$
$x=\frac{-4\pm 4\sqrt{6}}{8}$
$x=\frac{4(-1\pm\sqrt{13})}{8}$
$x=\frac{-1\pm\sqrt{6}}{2}$
*On checking we found that no root is an extraneous root. Hence,*
**S. S** $=\left\{\frac{-1\pm\sqrt{6}}{2}\right\}$

**Question#7**

$\sqrt{x^2+2x-3}+\sqrt{x^2+7x-8}=\sqrt{5(x^2+3x-4)}$

**Solution:**

**Factorizing the factor under the radical sign**
$\sqrt{x^2-x+3x-3}+\sqrt{x^2-x+8x-8}=\sqrt{5(x^2-x+4x-4)}=0$
$\sqrt{x(x-1)+3(x-1)}+\sqrt{x(x-1)+8(x-1)}=\sqrt{5[x(x-1)+3(x-1)]}$
$\sqrt{(x-1)(x+3)}+\sqrt{(x-1)(x+8)}-\sqrt{5[(x-1)(x+3)]}=0$
$\sqrt{(x-1)}\left\{\sqrt{(x+3)}+\sqrt{(x+8)}-\sqrt{5(x+4)}\right\}=0$
$\sqrt{(x-1)}=0\,,\quad \sqrt{(x+3)}+\sqrt{(x+8)}-\sqrt{5(x+4)}=0$
$x-1=0 \quad , \quad \sqrt{(x+3)}+\sqrt{(x+8)}=\sqrt{5(x+4)}$
$x=1 \quad$ **,** **Squaring on both sides**
$\left(\sqrt{(x+3)}+\sqrt{(x+8)}\right)^2=\left(\sqrt{5(x+4)}\right)^2$
$x+3+x+8+2\sqrt{(x+3)}+\sqrt{(x+8)}=5(x+4)$
$2x+11+2\sqrt{x^2+8x+3x+24}=5x+20$
$2\sqrt{x^2+11x+24}=5x-2x+20-11$
$2\sqrt{x^2+11x+24}=3x+9$
**Squaring again**
$4(x^2+11x+24)=(3x+9)^2$
$4x^2+44x+96=9x^2+54x+81$
$9x^2-4x^2+54x-44x+88-96=0$

$5x^2+10x-15=0$

$x^2+2x-3=0$

$x^2-x+3x-3=0$

$x(x-1)+3(x-1)=0$

$(x-1)(x+3)=0$

$x-1=0 \quad ,x+3=0$

$x=1\,,\; x=-3$

*On checking we found that no root is an extraneous root. Hence,*

**S. S** $=\{1\,,-3\,\}$

**Question#8**

$\boldsymbol{\sqrt{2x^2-5x-3}+3\sqrt{2x+1}=\sqrt{2x^2+25x+12}}$

**Solution:**

***Factorizing the factor under the radical sign***

$\sqrt{2x^2+x-6x-3}+3\sqrt{2x+1}=$

$\sqrt{2x^2+x+24x+12}=0$

$\sqrt{x(2x+1)-3(2x+1)}+3\sqrt{2x+1}=$

$\sqrt{x(2x+1)+12(2x+1)}$

$\sqrt{(2x+1)(x-3)}+3\sqrt{2x+1}-\sqrt{(2x+1)(x+12)}=0$

$\sqrt{(2x+1)}\left\{\sqrt{(x-3)}+3-\sqrt{(x+12)}\right\}=0$

$\sqrt{2x+1}=0\,, \quad \sqrt{(x-3)}+3-\sqrt{(x+12)}=0$

$2x+1=0 \quad , \quad \sqrt{(x-3)}+3=\sqrt{5(x+12)}$

$\boldsymbol{x=\frac{-1}{2}}$ **,** ***Squaring on both sides***

$\left(\sqrt{(x-3)}+3\right)^2=\left(\sqrt{(x+12)}\right)^2$

$x-3+9+2(3)\sqrt{x-3}=x+12$

$x+6+6\sqrt{x-3}=x+12$

$6\sqrt{x-3}=x-x+12-6$

$6\sqrt{x-3}=6$

$\sqrt{x-3}=1$

$x-3=1$

$x=4$

*On checking we found that no root is an extraneous root. Hence,*

**S. S** $=\{4\,\}$

**Question#9**

$\boldsymbol{\sqrt{3x^2-5x+2}+\sqrt{6x^2-11x+5}=\sqrt{5x^2-9x+4}}$

**Solution:**

***Factorizing the factor under the radical sign***

$\sqrt{x^2-3x-2x+2}+\sqrt{6x^2-6x-5x+5}=\sqrt{5x^2-5x-4x+4}=0$

$\sqrt{3x(x-1)-2(x-1)}+\sqrt{6x(x-1)-5(x-1)}=$

$\sqrt{5x(x-1)-4(x-1)}$

$\sqrt{(x-1)(3x-2)}+\sqrt{(x-1)(6x-5)}-\sqrt{(x-1)(5x-4)}=0$

$\sqrt{(x-1)}\left\{\sqrt{(3x-2)}+\sqrt{(6x-5)}-\sqrt{5x-4}\right\}=0$

$\sqrt{(x-1)}=0 \;\;, \;\sqrt{(3x-2)}+\sqrt{(6x-5)}-\sqrt{5x-4}$

$x-1=0 \quad , \quad \sqrt{(3x-2)}+\sqrt{(6x-5)}=\sqrt{5x-4}$

$\boldsymbol{x=1}$ **,** ***Squaring on both sides***

$\left(\sqrt{(3x-2)}+\sqrt{(6x-5)}\right)^2=\left(\sqrt{5x-4}\right)^2$

$\left(\sqrt{(3x-2)}\right)^2+\left(\sqrt{(6x-5)}\right)^2+2\sqrt{(3x-2)(6x-5)}=5x-4$

$3x-2+6x-5+2\sqrt{(x+3)(x+8)}=5x-4$

$9x-7+2\sqrt{18x^2-15x-12x+10}=5x-4$

$2\sqrt{18x^2-27x+10}=5x-9x-4+7$

$2\sqrt{18x^2-27x+10}=3-4x$

***Squaring again***

$4(18x^2-27x+10)=(3-4x\,)^2$

$72x^2-16x^2-108x+24x+40-9=0$

$56x^2-84x+31=0$

***Using*** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-84)\pm\sqrt{(-84)^2-4(56)(31)}}{2(3)}$

$x=\frac{84\pm\sqrt{7056-6994}}{112}$

$x=\frac{84\pm\sqrt{112}}{112}$

$x=\frac{84\pm\sqrt{16\times 7}}{112}$

$x=\frac{84\pm 4\sqrt{7}}{112}$

$x=\frac{4(21\pm 4\sqrt{7})}{112}$

$x=\frac{21\pm\sqrt{7}}{28}$

*On checking we found that no root is an extraneous root. Hence,*

**S. S** $=\left\{\frac{21\pm\sqrt{7}}{28}\right\}$

**Question#10**

$\boldsymbol{(x+4)(x+1)=\sqrt{x^2+2x-15}+3x+31}$

**Solution:**

$x^2+x+4x+4=\sqrt{x^2+2x-15}+3x+31$

$x^2+5x+4=\sqrt{x^2+2x-15}+3x+31$

$x^2+5x-3x+4-31-\sqrt{x^2+2x-15}=0$

$x^2+2x-27-\sqrt{x^2+2x-15}=0$

***Put*** $\boldsymbol{\sqrt{x^2+2x-15}=y}$

$x^2+2x=y^2+15$

***Then given equation takes form***

$y^2+15-27-y=0$

$y^2-y-12=0$

$y^2+3y-4y-12=0$

$y(y+3)-4(y+3)=0$

$(y+3)(y-4)=0$

$y+3=0 \quad ,y-4=0$

$y=-3\,, \;\; y=4$

***If*** $\boldsymbol{y=4}$ ***then,***

$\sqrt{x^2+2x-15}=-3$

$x^2+2x-15+9=0$

$x^2+2x-24=0$

***Using*** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(2)\pm\sqrt{(2)^2-4(2)(-31)}}{2(2)}$

$x=\frac{-2\pm\sqrt{4+124}}{4}$

$x=\frac{-2\pm\sqrt{128}}{4}$

$x=\frac{-2\pm 8\sqrt{2}}{2}$

$x=\frac{-2\pm 8\sqrt{64\times 2}}{2}$

$x=\frac{2(-1\pm 4\sqrt{2})}{2}$

$x=-1\pm 4\sqrt{2}$

***If*** $\boldsymbol{y=-3}$ ***then,***

$x^2+2x-15=9$

$x^2+2x-15-9=0$

$x^2+2x-124=0$

$x^2-4x+6x-124=0$

$x(x-4)+6(x-4)=0$

$(x-4)(x+6)=0$

$x-4=0 \quad ,x+6=0$

$x=4\,, \;\; x=-6$

*On checking we found that* $4, -6,$ *are extraneous roots. Hence,*

$\textbf{S. S} = \{-1 \pm 4\sqrt{2}\}$

**Question#11**

$\sqrt{3x^2 - 2x + 9} + \sqrt{3x^2 - 2x - 4} = 13$

**Solution:**

**Put** $\sqrt{3x^2 - 2x + 9} = a$ , $\sqrt{3x^2 - 2x - 4} = b$

**Then given equation takes form**

$a + b = 13 \quad \ldots(1)$

**To find a and b we find that**

$a^2 - b^2 = \left(\sqrt{3x^2 - 2x + 9}\right)^2 - \left(\sqrt{3x^2 - 2x - 4}\right)^2$

$a^2 - b^2 = 3x^2 - 2x + 9 - 3x^2 + 2x + 4$

$a^2 - b^2 = 13$

$\because a^2 - b^2 = (a + b)(a - b)$

$(a + b)(a - b) = 13 \ldots(2)$

**Putting values of** $a - b$ **from** $(1)$ **in** $(2)$

$13(a - b) = 13$

$a - b = 1 \ldots(3)$

**Adding** $(1)$ **and** $(3)$

$a + b = 13$

$\underline{a - b = 1}$

$2a = 14$

$\Rightarrow a = 7$

**Putting values of** $a$ **in** $(1)$

$7 + b = 13 \Rightarrow b = 6$

*If* $a = 7$ *then* ,

$3x^2 - 2x + 9 = 7$

$3x^2 - 2x + 9 = 49$

$3x^2 - 2x - 40 = 0$

*If* $b = 6$ *then* ,

$\sqrt{3x^2 - 2x - 4} = 6$

$3x^2 - 2x - 4 = 36$

$3x^2 - 2x - 40 = 0$

**Solving any one**

$3x^2 - 2x - 40 = 0$

$3x^2 - 12x + 10x - 40 = 0$

$3x(x - 4) + 10(x - 4) = 0$

$(x - 4)(3x + 10) = 0$

$x - 4 = 0 \quad ,3x + 10 = 0$

$x = 4$ , $x = \frac{-10}{3}$

*On checking we found that no root is an extraneous root. Hence,*

$\textbf{S. S} = \left\{4, \frac{-10}{3}\right\}$

*Question 12.*

$\sqrt{5x^2 + 7x + 2} - \sqrt{4x^2 + 7x + 18} = x - 4$

**Solution:**

**Put** $\sqrt{5x^2 + 7x + 2} = a$ , $\sqrt{4x^2 + 7x + 18} = b$

**Then given equation takes form**

$a - b = x - 4 \quad \ldots(1)$

**To find a and b we find that**

$a^2 - b^2 = \left(\sqrt{5x^2 + 7x + 2}\right)^2 - \left(\sqrt{4x^2 + 7x + 18}\right)^2$

$a^2 - b^2 = 5x^2 + 7x + 2 - (4x^2 + 7x + 18)$

$a^2 - b^2 = 5x^2 + 7x + 2 - 4x^2 - 7x - 18$

$a^2 - b^2 = x^2 - 16 \qquad \because a^2 - b^2 = (a + b)(a - b)$

$a^2 - b^2 = (x)^2 - (4)^2$

$(a + b)(a - b) = (x - 4)(x + 4) \ldots(2)$

**Putting values of** $a - b$ **from** $(1)$ **in** $(2)$

$(x - 4)(a + b) = (x - 4)(x + 4)$

$a + b = x + 4 \ldots(3)$

**Adding** $(1)$ **and** $(3)$

$a - b = x - 4$

$\underline{a + b = x + 4}$

$2a = 2x$

$\Rightarrow a = x$

**Putting values of** $a$ **in** $(1)$

$x - b = x - 4 \Rightarrow b = 4$

**If** $a = x$ **then,**

$\sqrt{5x^2 + 7x + 2} = x^2$

$5x^2 + 7x + 2 = x^2$

$5x^2 - 4x^2 + 7x + 2 = x^2$

$4x^2 + 7x + 2 = 0$

**If** $b = 4$ **then,**

$\sqrt{4x^2 + 7x + 18} = 4$

$4x^2 + 7x + 18 = 16$

$4x^2 + 7x + 18 - 16 = 0$

$4x^2 + 7x + 2 = 0$

**Solving any one**

$4x^2 + 7x + 2 = 0$

**Using** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(4)(2)}}{2(4)}$

$x = \frac{-7 \pm \sqrt{49 - 32}}{8}$

$x = \frac{-4 \pm \sqrt{17}}{8}$

*On checking we found that no root is an extraneous root. Hence,*

$\textbf{S. S} = \left\{\frac{-4 \pm \sqrt{17}}{8}\right\}$



**Important Note:**

We know that the numbers containing $i$ are called complex nos. so $\frac{-1+\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}$ are complex or imaginary roots of cube roots of unity."

- Every non-zero real number has one real and two complex cube roots.
- Let $w = \frac{-1+\sqrt{3}i}{2}, w^2 = \frac{-1-\sqrt{3}i}{2}$

So

$1, w, w^2$ *are also called cube roots of unity.*

**Properties of Cube Roots of Unity:**

**i) Each complex cube of unity is square of the other.**

**Proof:**

**We know** that complex cube roots of unity are

$$\frac{-1+\sqrt{3}i}{2} \text{ and } \frac{-1-\sqrt{3}i}{2}$$

Let $w = \frac{-1+\sqrt{3}i}{2}$

⇨ $w^2 = \left(\frac{-1+\sqrt{3}i}{2}\right)^2$ *squaring*

$$w^2 = \frac{(-1)^2 + (\sqrt{2}i)^2 + 2(-1)(\sqrt{3}\,i)}{4}$$

$$w^2 = \frac{1+3i^2-2\sqrt{3}i}{4} = \frac{1+3(-1)-\sqrt{3}i}{4}$$

$$w^2 = \frac{1-3-2\sqrt{3}i}{4} = \frac{-2-2\sqrt{3}i}{2}$$

$$w^2 = 2\left(\frac{-1-\sqrt{3}i}{4}\right) = \frac{-1-\sqrt{3}i}{2}$$

Which is other root

Hence proved.

ii) ***The sum of all the three roots of unity is zero. i.e***

$1 + w + w^2 = 0$

Proof :

We know that three cube roots of unity are 1.

$w = \frac{-1+\sqrt{3}i}{2}, w^2 = \frac{-1-\sqrt{3}i}{2}$

Now

$$1 + w + w^2 = 1 + \frac{-1+\sqrt{3}i}{2} + \frac{-1-\sqrt{3}i}{2}$$

$$= \frac{2-1+\sqrt{3}i-1-\sqrt{3}i}{2} = \frac{2-2}{2} = 0$$

Hence proved.

⇨ $1 + w + w^2 = 0$

iii) **Product of all the three cube roots of unity is unity.** $1.w.w^2 = 1$

Proof:

We know that three cube roots of unity are

$1, w = \frac{-1+\sqrt{3}i}{2}, w^2 = \frac{-1-\sqrt{3}i}{2}$

So,

$$1.w.w^2 = 1 \times \frac{-1+\sqrt{3}i}{2} \times \frac{-1-\sqrt{3}i}{2}$$

$$w^3 = \frac{(-1+\sqrt{3}i)(-1-\sqrt{3}i)}{2} = \frac{(-1)^2 - (\sqrt{3}i)^2}{4}$$

$$w^3 = \frac{1-3i^2}{4} = \frac{1+3}{4} = \frac{4}{4} = 1$$

$w^3 = 1$ *hence proved*

iv) **For any $n \in Z$ , $w^n$ is equivalent to one of the cube roots of unity.**

Proof:

$$w^4 = w63\,.w = (1)w = w$$
$$w^5 = w^3.w^2 = (1).w^2 = w^2$$
$$w^6 = w^3.w^3 = (1)(1) = 1$$
$$w^{15} = (w^3)^5 = (1)^5 = 1$$
$$w^{17} = (w^3)^5.w^2 = (1)^5.w^2 = w^2$$
$$w^{10} = w^9.w = (w^3)^3.w = (1)^3.w = w$$
$$w^{-1} = w^{-3}(w^2)(w^3)^{-1}(w^2) = (1)^{-1}.w^2 = w$$
$$w^{-5} = w^{-6}.w = (w^3)^{-2} = (1)^{-2}.w = w$$
$$w^{-12} = (w^3)^{-4} = (1)^{-4} = 1 \text{ hence proved}\mathbf{c}$$

**Four fourth roots of Unity:**

***let x be the fourth root of unity.***

$$\because x = \sqrt[4]{1} = (1)^{\frac{1}{4}}$$

⇨ $x^4 = \left[(1)^{\frac{1}{4}}\right]^4 \Rightarrow x^4 = 1$

⇨ $x^4 - 1 = 0 \Rightarrow (x^2)^2 - (1)^2 = 0$

⇨ $(x^2 - 1)(x^2 + 1) = 0$

⇨ $x^2 - 1 = 0, \quad x^2 + 1 = 0$

⇨ $x^2 = 1, \quad x^2 = -1$

⇨ $x = \pm 1, \quad x = \pm\sqrt{-1} = \pm i$

⇨ So four roots of unity are $1, -1, i, -i$

**Properties of four fourth Roots of unity.**

**i)** Sum of four forth roots of unity is zero.

$\boldsymbol{i.e\ 1 + (-1) + i(-i) = 0}$

ii) *The real fourth roots of unity are additive inverse of each other. i.e*

$\mathbf{1 + (-1) = (-1) + 1 = 0}$

**iii)** Both the complex fourth roots of unity are conjugate of each other.

*i.e conjugate of* $i = -i$

*conjuagte of* $-i = i$

**iv)** Product of all the fourth roots of unity is -1

$\boldsymbol{i.e\ 1 \times (-1) \times i \times (-i)}$

$\boldsymbol{-1 \times (-i^2) = i^2 = -1}$

# Exercise 4.4

$$(x-3)(x^2+3x+9)=0$$
$$(x-3)=0 \quad , \quad x^2+3x+9=0$$
$$x=3 \quad using\ x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(3)\pm\sqrt{(3)^2-4(1)(9)}}{2(1)}$$
$$x=\frac{(-3)\pm\sqrt{9-36}}{2}\Rightarrow x=\frac{-(-2)\pm\sqrt{-27}}{2}$$
$$x=\frac{-(-2)\pm\sqrt{9\times(-3)}}{2}$$
$$x=\frac{(-3)\pm 3\sqrt{\times(-3)}}{2}$$
$$x=3\left(\frac{-1\pm\sqrt{3i}}{2}\right)$$
$$x=3\left(\frac{-1+\sqrt{3i}}{2}\right) \quad ,x=3\left(\frac{-1-\sqrt{3i}}{2}\right)$$
$$x=3w, \quad x= \quad 3w^2$$

Hence cube roots of 27 $are 3, 3w, 3w^2$

$(iv)$. . ***Let x be a cube root of 8 then,***

$x=(-27)^{\frac{1}{3}}\Rightarrow x^3=-27$

$\Rightarrow x^3+27=0 \Rightarrow (x)^3+(3)^3$

$\Rightarrow (x+3)(x^2-3x+9)=0$

$\Rightarrow x+3=0 \quad , \quad x^2-3x+9=0$

$\Rightarrow x=-3 \ , \quad x^2-3x+9=0$

$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(9)}}{2(1)}$$
$$=\frac{3\pm\sqrt{9-36}}{2}$$
$$=\frac{3\pm\sqrt{-27}}{2}$$
$$=\frac{3\pm 3\sqrt{-3}}{2}$$
$$x=-3\left[\frac{-1-\sqrt{-3}}{2}\right] \quad or\ x=-3\left[\frac{-1+\sqrt{-3}}{2}\right]$$
$$\Rightarrow x=-3\omega \quad or \quad x=-3\omega^2$$

*Hence the cube roots of* $-27$ *are*

$-3, -3\omega, -3\omega^2$

$(v)$.

$x=(64)^{\frac{1}{3}}\Rightarrow x^3=64$

$\Rightarrow x^3-64=0 \Rightarrow (x)^3-(4)^3$

$\Rightarrow (x-4)(x^2+4x+16)=0$

$\Rightarrow x-4=0 \ , x^2+2x+4=0$

$\Rightarrow x=4 \ , \quad x^2+4x+16=0$

$$x=\frac{-(4)\pm\sqrt{(4)^2-4(1)(16)}}{2(1)}$$
$$=\frac{-4\pm\sqrt{16-64}}{2}$$
$$=\frac{-4\pm\sqrt{-48}}{2}$$
$$=\frac{-4\pm 4\sqrt{-3}}{2}$$
$$x=4\left[\frac{-1+\sqrt{-3}}{2}\right] \quad or\ x=4\left[\frac{-1-\sqrt{-3}}{2}\right]$$
$$\Rightarrow x=4\omega \quad or \quad x=4\omega^2$$

*Hence the cube roots of* $64$ *are* $4, 4\omega, 4\omega^2$

***Question#2***

***Evaluate:***

$(i)$. $\boldsymbol{(1+\omega-\omega^2)^8}$

***Solution:***

$(1+\omega-\omega^2)^8 = (1+\omega+\omega^2-2\omega^2)^8$

$= (0-2\omega^2)^8 \quad \because 1+\omega+\omega^2=0$

$= (2)^8.(\omega^2)^8$

$=256\omega^{16}$

$=256\omega^{15}.\omega$

$=256(\omega^3)^5.\omega$

$\because \omega^3=1$

$=256(1)^5.\omega$

$=256\omega$

$(ii)$. $\boldsymbol{\omega^{28}+\omega^{29}+1}$

***Solution:***

$\omega^{28}+\omega^{29}+1=\omega^{27}.\omega+\omega^{27}.\omega^2+1$

$=(\omega^3)^9.\omega+(\omega^3)^9.\omega^2 \ +1$

$\because \omega^3=1$

$=(1)^9.\omega+(1)^9.\omega^2 \ +1$

$=\omega+\ \omega^2 \ +1$

$\because 1+\omega+\omega^2=0$

$=0$

$(iii)$. $\boldsymbol{(1+\omega-\omega^2)(1-\omega+\omega^2)}$

***Solution:***

$=(1+\omega+\omega^2-2\omega^2)(1+\omega+\omega^2-2\omega)$

$\because 1+\omega+\omega^2=0$

$=(0-2\omega^2)(0-2\omega)$

$=(-2\omega^2)(-2\omega)$

$=4\omega^3$

$\because \omega^3=1$

$=4(1)=4$

$(iv)$. $\boldsymbol{\left(\frac{-1+\sqrt{-3}}{2}\right)^7+\left(\frac{-1-\sqrt{-3}}{2}\right)^7}$

***Solution:***

$=\omega^7+(\omega^2)^7 \quad \because \omega=\frac{-1+\sqrt{-3}}{2} \ ,\omega^2=\frac{-1-\sqrt{-3}}{2}$

$=\omega^7+\omega^{14}$

$=\omega^6.\omega+\omega^{12}.\omega^2$

$=(\omega^3)^9.\omega+(\omega^3)^9.\omega^2 \ +1$

$\because \omega^3=1$

$=(1)^9.\omega+(1)^9.\omega^2 \ +1$

$=\omega+\ \omega^2 \ +1$

$\because 1+\omega+\omega^2=0$

$=0$

$(v)$. $\boldsymbol{(-1+\sqrt{-3})^5+(-1-\sqrt{-3})^5}$

***Solution:***

$\left(2.\frac{-1+\sqrt{-3}}{2}\right)^5+\left(2.\frac{-1-\sqrt{-3}}{2}\right)^5$

$=(2\omega)^5+(2\omega^2)^5 \quad \because \omega=\frac{-1+\sqrt{-3}}{2} \ ,\omega^2=\frac{-1-\sqrt{-3}}{2}$

$=32\omega^5+32\omega^{10}$

$=32\omega^3.\omega^2+32\omega^9.\omega^1$

$=32(1).\omega^2+32(1).\omega$

$=32(\omega+\ \omega^2)$

$\because 1+\omega+\omega^2=0$

$=32(-1)$

$= -32$

**Question#3**

**Show that:**

$(i).\ \boldsymbol{x^3 - y^3 = (x-y)(x-\omega y)(x-\omega^2 y)}$

**Solution:**

$R.H.S = (x-y)(x-\omega y)(x-\omega^2 y)$

$= (x-y)[x(x-\omega^2 y) - \omega y(x-\omega^2 y)]$

$= (x-y)[x^2 - \omega^2 xy - \omega xy + \omega^3 y^2]$

$\because \omega^3 = 1$

$\because 1+\omega+\omega^2 = 0$

$= (x-y)[x^2 - (\omega^2+\omega)xy + (1)y^2]$

$\because \omega+\omega^2 = -1$

$= (x-y)[x^2 - (-1)xy + y^2]$

$= (x-y)[x^2 + xy + y^2] = L.H.S$

$(ii).\ \boldsymbol{x^3+y^3+z^3-3xyz = (x+y+z)(x+\omega y+\omega^2 z)(x+\omega^2 y + \omega z)}$

**Solution:**

$\boldsymbol{R.H.S} = (x+y+z)(x+\omega y+\omega^2 z)(x+\omega^2 y+\omega z)$

$= (x+y+z)(x^2+\omega^2 xz+\omega xz+\omega xy+\omega^3 y^2+\omega^2 yz+\omega^2 xz+\omega^4 yz+\omega^3 z^2)$

$= (x+y+z)(x^2+(\omega^2+\omega)xy+(\omega^2+\omega)xz+(\omega^2+\omega^4)yz+\omega^3 y^2+\omega^3 z^2)$

$= (x+y+z)(x^2+(-1)xy+(-1)xz+(\omega^2+\omega)yz+(1)y^2+(1)z^2)$

$\because \omega^4 = \omega \quad , \omega+\omega^2 = -1$

$= (x+y+z)(x^2+y^2+z^2-xy+-xz+(-1)yz)$

$= (x+y+z)(x^2+y^2+z^2-xy-xz-yz)$

$= x^3+y^3+z^3-3xyz = L.H.S$

$(iii).\ \boldsymbol{(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)\ldots 2n\ factors = 1}$

**Hint:** $\boldsymbol{1+\omega^4 = 1+\omega^3.\omega = +\omega^2\ ,1 =}$

**Solution:**

$\boldsymbol{L.H.S} = (1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)\ldots 2n\ factors$

$= [(1+\omega)(1+\omega^2)][(1+\omega^4)(1+\omega^8)]\ldots n\ factors$

$= [(1+\omega)(1+\omega^2)][(1+\omega^3.\omega)(1+\omega^6.\omega^2)]\ldots n\ factors$

$= [(1+\omega)(1+\omega^2)][(1+\omega^3.\omega)(1+(\omega^3)^2.\omega^2)]\ldots n\ factors$

$= [(1+\omega)(1+\omega^2)][(1+1.\omega)(1+(1)^2.\omega^2)]\ldots n\ factors$

$= [(1+\omega)(1+\omega^2)][(1+\omega)(1+\omega^2)]\ldots n\ factors$

$= [(1+\omega)(1+\omega^2)]^n$

$\because 1+\omega+\omega^2 = 0, \quad \omega^3 = 1$

$= [0+1]^n$

$= [1]^n = 1 = \boldsymbol{R.H.S}$

**Question#4**

***If $\omega$ is a root of $x^2 + x + 1 = 0$, show that its other root is $\omega^2$ and prove that $\omega^3 = 1$.***

**Solution:**

***Let*** $x^2 + x + 1 = 0 \quad \ldots(i)$

***Since $\omega$ is the root of $(i)$, therefore***

$\omega^2 + \omega + 1 = 0 \quad \ldots.(ii)$

***To prove $\omega^2$ is the root of $(i)$,***

*Consider*

$(\omega^2)^2 + \omega^2 + 1 = \omega^4 + 2\omega^2 + 1 - \omega^2$

$= (\omega^2+1)^2 - \omega^2$

$= (\omega^2+1+\omega)(\omega^2+1-\omega)$

$= (0)(\omega^2+1-\omega)$ *from* $(i)$

$\Rightarrow (\omega^2)^2 + \omega^2 + 1 = 0 \ \ldots(iii)$

***$\omega^2$ is the root of $(i)$***

***Now subtracting $(ii)$ from $(iii)$***

$(\omega^2)^2 + \omega^2 + 1 = 0$

$\quad\ \ \omega^2 + \omega + 1 = 0$

$\quad\ \ - \quad - \quad\ -$

$\omega^4 \quad\ - \omega \quad = 0$

$\Rightarrow \omega(\omega^3 - 1) = 0$

$\Rightarrow \omega^3 - 1 = 0 \ , \Rightarrow \omega \neq 0$

$\Rightarrow \omega^3 = 1$

Hence proved.

**Question#5**

***Prove that complex cube roots of $-1$ are $\frac{1+\sqrt{3}i}{2}$ and $\frac{1-\sqrt{3}i}{2}$ hence prove that***

$\boldsymbol{\left(\frac{1+\sqrt{-3}}{2}\right)^9 + \left(\frac{1-\sqrt{-3}}{2}\right)^9 = 2}$

**Solution:**

*. Let $x$ be a cube root of 8 then,*

$x = (-1)^{\frac{1}{3}} \Rightarrow x^3 = -1$

$\Rightarrow x^3 + 1 = 0 \Rightarrow (x)^3 + (1)^3$

$\Rightarrow (x+1)(x^2 - x + 1) = 0$

$\Rightarrow x + 1 = 0 \ , \quad x^2 - 3x + 9 = 0$

$\Rightarrow x = -1 \ , \quad x^2 - x + 1 = 0$

$x = \frac{-(-1)\pm\sqrt{(-1)^2-4(1)(1)}}{2(1)}$

$= \frac{1\pm\sqrt{1-4}}{2}$

$= \frac{1\pm\sqrt{-3}}{2}$

$= \frac{1\pm\sqrt{-3}}{2}$

$\Rightarrow x = \frac{1+\sqrt{-3}}{2} \quad or \quad x = \frac{1-\sqrt{-3}}{2}$

$\Rightarrow x = \frac{1+\sqrt{3}i}{2} \quad or \quad x = \frac{1-\sqrt{3}i}{2}$

*Hence the complex cube roots of $-1$ are* $\frac{1+\sqrt{3}i}{2}\ , \ \frac{1-\sqrt{3}i}{2}$

Now

$$L.H.S = \left(\frac{1+\sqrt{3}i}{2}\right)^9 + \left(\frac{1-\sqrt{3}i}{2}\right)^9$$

$$\left[-\left(\frac{1+\sqrt{3}i}{2}\right)\right]^9 + \left[\frac{-1-\sqrt{3}i}{2}\right]^9$$

$$= (-w)^9 + (-w^2)^9 = -w^9 - w^{18}$$

$$= -(w^9 + w^{18}) = -(w^3)^3 + (w^3)^6$$

$$= -(1)^3 + (1)^6 = -(1+1) = -2 = R.H.S$$

Hence proved.

***Question#6***

***If $\omega$ is a cube root of unity, form an equation whose roots are $2\omega$ and $2\omega^2$.***

***Solution:***

***Since $2\omega$ and $2\omega^2$ are the roots of 16 then,***

$(x-2\omega)(x-2\omega^2)$

$\Rightarrow x^2-2\omega x-2\omega^2 x+4\omega^3=0$

$\Rightarrow x^2-2x(\omega+\omega^2)x+4\omega^3=0$

$\Rightarrow x^2-2x(-1)+4(1)=0$

$\Rightarrow x^2+2x+4(1)=0$

$\Rightarrow x^2+2x+4=0$

***Is the required equation***

***Question#7***

***Find four fourth roots of 16, 81, 625.***

***Solution:***

$(i)$

*Let $x$ be a cube root of 8 then,*

$x=(16)^{\frac{1}{4}} \Rightarrow x^4=16$

$\Rightarrow x^4-16=0 \Rightarrow (x^2)^2-(4)^2$

$\Rightarrow (x^2-4)(x^2+4)=0$

$\Rightarrow x^2-4=0 \quad or \quad x^2+4=0$

$\Rightarrow x^2=4 \quad or \quad x^2=-4$

$\Rightarrow x=\pm\sqrt{4} \quad or \quad x=\pm\sqrt{-4}$

$\Rightarrow x=\pm 2 \quad or \quad x=\pm 2i$

*Hence the four fourth roots of 16 are*

$2, -2, -2i, 2i$

$(ii)$

*Let $x$ be a fourth root of 81 then,*

$x=(81)^{\frac{1}{4}} \Rightarrow x^4=81$

$\Rightarrow x^4-81=0 \Rightarrow (x^2)^2-(9)^2$

$\Rightarrow (x^2-9)(x^2+9)=0$

$\Rightarrow x^2-9=0 \quad or \quad x^2+9=0$

$\Rightarrow x^2=9 \quad or \quad x^2=-9$

$\Rightarrow x=\pm 3 \quad or \quad x=\pm\sqrt{-9}, \pm\sqrt{9}i$

$\Rightarrow x=\pm 23 \quad or \quad x=\pm 3i$

*Hence the four fourth roots of 81 are*

$3, -3, -3i, 3i$

$(iii)$.

***Let $x$ be a cube root of 8 then,***

$x=(625)^{\frac{1}{4}} \Rightarrow x^4=625$

$\Rightarrow x^4-625=0 \Rightarrow (x^2)^2-(25)^2$

$\Rightarrow (x^2-25)(x^2+25)=0$

$\Rightarrow x^2-25=0 \quad or \quad x^2+25=0$

$\Rightarrow x^2=25 \quad or \quad x^2=-25$

$\Rightarrow x=\pm\sqrt{25} \quad or \quad x=\pm\sqrt{-25}$

$\Rightarrow x=\pm 5 \quad or \quad x=\pm 5i$

*Hence the four fourth roots of 16 are*

$5, -5, -5i, 5i$

***Question#8***

***Solve the following equations:***

$(i)$. $\mathbf{2x^4-32=0}$

***Solution:***

$2(x^4-16)=0$

$x^4-16=0$

$(x^2)^2-(4)^2=0$

$(x^2-4)(x^2+4)=0$

$x^2=4, x^2=-4$

$x=\pm 2, x=\pm 2i$

$S.S\{\pm 2, \pm 2\}$

$(ii)$. $\mathbf{3y^5-243y=0}$

***Solution:***

$3y(y^4-81)=0$

$\Rightarrow 3y=0 \quad y^4-81=0$

$\Rightarrow y=0$

$\Rightarrow y^2-81=0 \Rightarrow (y^2)^2-(9)^2$

$\Rightarrow (y^2-9)(y^2+9)=0$

$\Rightarrow y^2-9=0 \quad or \quad y^2+9=0$

$\Rightarrow y=\pm\sqrt{9} \quad or \quad y=\pm 9$

$\Rightarrow y=\pm 3 \quad or \quad xy=\pm 3i$

*Hence* $S.S=\{0, \pm 3, \pm 3i\}$

$(iii)$. $\mathbf{x^3+x^2+x+1=0}$

***Solution:***

$x^2(x+1)+1(x+1)=0$

$(x+1)(x^2+1)=0$

$\Rightarrow x+1=0 \quad , \quad x^2+1=0$

$\Rightarrow x=-1 \quad , \quad x^2=-1$

$x=\pm\sqrt{-1}$

$x=\pm 1i$

*Hence* $\mathbf{S.S}=\{-\mathbf{1}, \pm\mathbf{1}i\}$

$\Rightarrow y^2=9 \quad or \quad y^2=-9$

$(iv)$. $\mathbf{5x^5-5x=0}$

***Solution:***

$5(x^4-1)=0$

$\Rightarrow 5x=0 \quad , x^4-1=0$

$\Rightarrow x=0$

$\Rightarrow x^4-1=0 \Rightarrow (x^2)^2-(1)^2$

$\Rightarrow (x^2-1)(x^2+1)=0$

$\Rightarrow x^2-1=0 \quad or \quad x^2+1=0$

$\Rightarrow x^2=1 \quad or \quad x^2=-1$

$\Rightarrow x=\pm\sqrt{1} \quad or \quad x=\pm\sqrt{-1}$

$\Rightarrow x=\pm 1 \quad or \quad x=\pm 1i$

*Hence* $S.S=\{0, \pm 1, \pm 1i\}$

**Polynomial function:**

A polynomial in x is an expression of the form

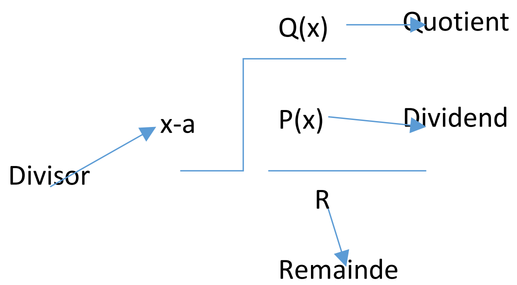
$\boldsymbol{a_n x^n + a_{n-1} x^{+} \ldots + a_1 x + a_0, a_n \neq 0 \rightarrow (i)}$

Where n is non- negative integer and $a_n, a_{n-1} + \cdots + a_1$ $are\ real\ no.s$

*the highest power of polynomial is $x$ are called degree of polynomial so expression (i) is polynomial of degree n.

**Note:**

Consider a polynomial p(x) is divided by $x - a\ and\ Q(x)$ be quotient while R be remainder then




**Theorem:**

**Remainder theorem:**

"if a polynomial f(x) of degree $n \geq 1$ nis non negative integer is dividend by $x - a$ till no $x - a\ term\ exist\ in\ the\ remainder\ then\ f(a) is\ the\ remainder"$

**Proof:**

Let us divide a polynomial $f(x) by\ x - a\ then\ we\ get$ $a\ unique\ quotient\ q(x) and\ a\ unique\ remaindor$ $R\ such\ that\ f(x) = (x - a)q(x) + R$

$f(a) = R\ hence\ f(a) = R\ remainder.$

**Note:**

**Remainder obtained when**

$\boldsymbol{f(x) is\ dividend\ by\ (x - a) is\ same\ as\ the\ valve\ of\ the\ polynomial\ f(x) at x = a}$

**Factor theorem:**

the polynomial $x - a$ is factor of the polynomial $f(x)$

$if\ and\ only\ if\ f(a) = 0 \quad OR$

$x - a\ is\ a\ factor\ of\ f(x) if\ and\ omly\ if$

$x = a\ is\ root\ is\ root\ of\ polynoimal\ equation$

$f(x) = 0$

**Proof:**

Suppose g(x) is quotient and R is the remainder when a polynomial $f(x)$ is divided $by\ x - a$ then by Remainder theorem. .

$$f(x) = (x - a)g(x) + R \rightarrow (1)$$

$\because x - a\ is\ faster\ of\ f(x)$

⇨ Remainder $= R = o$

⇨ $f(x) = x - ag(x) + 0$

⇨ $f(x) = x - ag(x)$

Take $x = a$

⇨ $f(a) = (a - a)g(x) + 0$

⇨ $f(a) = 0\ hence\ proved.$

$Remainder = f(a)$

Conversely, suppose $f(a) = 0$

We prove

$x - a\ is\ factor\ of\ f(x) by\ reaminder\ theorm\ f(x) = (x - a)g(x) + R \rightarrow (ii)$

$\because f(a) = 0\ so$

$f(a) = (a - 0)g(a) + R \rightarrow R = 0$

$ii\ f(x) = (x - a)g(x) this\ sjow\ taht\ x - a\ is$ Factor of $f(x). hence\ proved.$

Note:

**To determine if a given linear polynomial $(\boldsymbol{x - a})$**

**Is a factor of**

$\boldsymbol{f(x) all\ we\ need\ to\ check\ whehter}$

$\boldsymbol{f(a) = 0}$

**Synthetic Division:**

**This is shortest method for long division of a polynomial $\boldsymbol{f(x) by\ polynomial\ of\ the\ form}$ $\boldsymbol{x - a}$**

**Outline of the method:**

**i) write down the coefficient of the dividend $\boldsymbol{f(x) from\ left\ to\ right\ in\ decreasingorder\ of\ powers\ of\ x. insert\ 0\ for\ any\ missing\ term}$**

**ii) to the left of the first line, write a of the divisor $(\boldsymbol{x - a})$.**

**iii) Use the following patterns to write the second and third lines . Vertical pattern (↓)**

**And diagonal pattern (↗) multiply by a.**

# Exercise 4.5

***Use the remainder theorem to find the remainder when the first polynomial is divided by the second polynomial:***

***Question#1***

$x^2 + 3x + 7$ , $x + 1$

***Solution:***

*Let* $f(x) = x^2 + 3x + 7$

$x + a = x + 1 \Rightarrow a = -1$

*Remainder* = $f(a)$

***By Remainder theorem***

$R = f(-1) = (-1)^2 + 3(-1) + 7$

$= 1 - 3 + 7 = 5$

***Question#2***

$x^3 - x^2 + 5x + 4$ , $x - 2$

***Solution:***

*Let* $f(x) = x^3 - x^2 + 5x + 4$

$x - a = x - 2 \Rightarrow a = 2$

*Remainder* = $f(a)$

***By Remainder theorem***

$R = f(2) = (2)^3 - (2)^2 + 5(2) + 4$

$= 8 - 4 + 10 + 4 = 18$

***Question#3***

$3x^4 + 4x^3 + x - 5$ , $x + 1$

***Solution:***

*Let* $f(x) = 3x^4 + 4x^3 + x - 5$

***By Remainder theorem***

$R = f(-1) = 3(-1)^4 + 4(-1)^3 + (-1) - 5$

$= 3 - 4 - 1 - 5$

$= -7$

***Question#4***

$x^3 - 2x^2 + 3x + 3$ , $x - 3$

***Solution:***

*Let* $f(x) = x^3 - 2x^2 + 3x + 3$

$x - a = x - 3 \Rightarrow a = 3$

*Remainder* = $f(a)$

***By Remainder theorem***

$R = f(3) = (3)^3 - 2(3)^2 + 3(3) + 3$

$= 27 - 18 + 9 + 3 = 21$

***Use the factor theorem to determine if the first polynomial is a factor of the second polynomial.***

***Question#5***

$x - 1$ , $x^2 + 4x - 5$

***Solution:***

*Let* $f(x) = x^2 + 4x - 5$

$x - a = x - 1 \Rightarrow a = 1$

*Remainder* = $f(a)$

***By Remainder theorem***

$R = f(1) = (1)^2 + 4(1) - 5$

$= 5 - 5 = 0$

***Hence*** $(x - 1)$ ***is a factor of*** $f(x)$ ***by factor theorem***

***Question#6***

$x - 2$ , $x^3 + x^2 - 7x + 1 = 0$

***Solution:***

*Let* $f(x) = x^3 + x^2 - 7x + 1 = 0$

$x - a = x - 2 \Rightarrow a = 2$

*Remainder* = $f(a)$

***By Remainder theorem***

$R = f(2) = (2)^3 + (2)^2 - 7(2) + 1$

$= 8 + 4 - 14 + 1 \neq 0$

***Hence*** $(x - 2)$ ***is not a factor of*** $f(x)$ ***by factor theorem***

***Question#7***

$\omega + 2$ , $2\omega^3 + \omega^2 - 4\omega + 7$

***Solution:***

*Let* $f(\omega) = 2\omega^3 + \omega^2 - 4\omega + 7$

$\omega - a = \omega - 2 \Rightarrow a = 2$

*Remainder* = $f(a)$

***By Remainder theorem***

$R = f(2) = 2(2)^3 + 2(2)^2 - 4(2) + 7$

$= -16 + 4 + 8 + 7 = 3 \neq 0$

***Hence*** $(\omega + 2)$ ***is not a factor of*** $f(\omega)$ ***by factor theorem***

***Question#8***

$x - a$ , $x^n - a^n$ ***,where n is a positive integer.***

***Solution:***

*Let* $f(x) = x^n - a^n$

*where n is a positive integer.*

$x - a = x - a \Rightarrow a = a$

***By Remainder theorem***

$R = f(a) = (a)^n - (a)^n = a^n - a^n = 0$

***Hence*** $(x + a)$ ***is a factor of*** $f(x)$ ***by factor theorem***

$x + a = x + 1 \Rightarrow a = -1$

***Question#9***

$x + a$ , $x^n + a^n$ ***where n is an odd integer.***

***Solution:***

*Let* $f(x) = x^n + a^n$

*where n is an odd integer.*

$x - a = x + a \Rightarrow a = -a$

***By Remainder theorem***

$R = f(-a) = (-a)^n + (a)^n = -a^n + a^n = 0$

***Hence*** $(x + a)$ ***is not a factor of*** $f(x)$ ***by factor theorem***

***Question#10***

***When*** $x^4 + 2x^3 + kx^2 + 3$ ***is divided by*** $x - 2$ ***the remainder is 1. Find the value of k.***

***Solution:***

*Let* $f(x) = x^4 + 2x^3 + kx^2 + 3$

$k = ?$ , $R = 1$

$x - a = x - 2 \Rightarrow a = 2$

*Remainder* = $f(a)$

***By Remainder theorem***

$R = f(2)$

$1 = (2)^4 + 2(2)^3 + k(2)^2 + 3$

$1 = 16 + 16 + 4k + 3$

$4k = -34$

$\Rightarrow k = -\frac{17}{2}$

***Question#11***

***When $x^4 + 2x^3 + kx + 4$ the polynomial is divided by $x - 2$ the remainderis 14. Find the value of k.***

***Solution:***

*Let* $f(x) = x^4 + 2x^3 + kx + 4$

$k = ?\ ,\ R = 14$

$x - a = x - 2 \Rightarrow a = 2$

*Remainder* $= f(a)$

***By Remainder theorem***

$R = f(2)$

$14 = (2)^3 + 2(2)^2 + 2k + 4$

$2k = -6$

$\Rightarrow k = -3$

***Use Synthetic division to show that x is the solution of the polynomial and use the result to factorize the polynomial completely.***

***Question#12***

$\boldsymbol{x^3 - 7x + 6 = 0\ ,\ x = 2}$

***Solution:***

$f(x) = x^3 + ox^2 + 6$

$x = 2$

| | | | | |
|---|---|---|---|---|
| | 1 | 0 | −7 | 6 |
| 2 | 0 | 2 | 4 | −6 |
| | 1 | 2 | −3 | 0 |

*Quotient* $= x^2 + 2x - 3$

$= x^2 + 3x - x - 3$

$x(x + 3) - 1(x + 3) = 0$

$(x + 3)(x - 1) = 0$

*Hence* $x^3 - 7x + 6 = (x - 2)(x + 3)(x - 1)$

***Question#13***

$\boldsymbol{x^3 - 28x - 48 = 0\ ,\ x = 4}$

***Solution:***

$f(x) = x^3 + ox^2 - 28x - 48$

$x = -4$

| | | | | |
|---|---|---|---|---|
| | 1 | 0 | −28 | −48 |
| −4 | 0 | −4 | 16 | 48 |
| | 1 | 2 | −12 | 0 |

*Quotient* $= x^2 - 4x - 12$

$= x^2 - 6x + 2x - 12$

$x(x - 6) + 2(x - 6) = 0$

$(x - 6)(x + 2) = 0$

*Hence* $x^3 - 28x - 48 = (x + 4)(x - 6)(x + 2)$

***Question#14***

$\boldsymbol{2x^4 + 7x^3 - 4x^2 - 27x - 18 = 0\ \ ,x = 2\ ,\ x = 3}$

***Solution:***

$f(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18$

$x = -3$

| | | | | | |
|---|---|---|---|---|---|
| | 1 | 7 | −4 | −27 | −18 |
| 2 | 0 | 4 | 22 | 36 | 18 |
| | 2 | 11 | 18 | 9 | 0 |
| −3 | 0 | −6 | −15 | −9 | |
| | 2 | 5 | 3 | 0 | |

*Quotient* $= 2x^2 + 5x + 3$

$= 2x^2 + 2x + 3x + 3$

$2x(x + 1) + 3(x + 1) = 0$

$(x + 1)(2x + 3) = 0$

***Hence*** $\boldsymbol{2x^4 + 7x^3 - 4x^2 - 27x - 18 = (x - 2)(x + 3)(x + 1)(2x + 3)}$

***Question#15***

***Use synthetic division to find the values of p and q if $x + 1$ and $x - 2$ are the factors o the polynomial $x^3 + px^2 + qx + 6$ .***

***Solution:***

$f(x) = x^3 + px^2 + qx + 6$

$x - a = x + 1 \Rightarrow a = -1$

$x - a = x - 2 \Rightarrow a = 2$

| | | | | |
|---|---|---|---|---|
| | 1 | $p$ | $q$ | 6 |
| −1 | 0 | − | $-p + 1$ | $-q + p - 1$ |
| | 1 | $p - 1$ | $q - p + 1$ | $-q + p + 5$ |
| 2 | 0 | −6 | $2p + 2$ | |
| | 2 | $p + 1$ | $q + p + 3$ | |

***Since $(x + 1)$ and $(x - 2)$ are the factors of $f(x)$***

***Then remainder* $= 0$**

$-q + p + 5 = 0 \quad ....(1)$

$q + p + 3 = 0 \quad ....(2)$

$2p + 8 = 0$

$\Rightarrow p = -4$

*Put* $p = -4$ *in* $(2)$

$\Rightarrow q - 4 + 3 = 0$

$\Rightarrow q = 1$

$\boldsymbol{\Rightarrow p = -4\ ,\Rightarrow q = 1}$

***Question#16***

***Find the values of a and b if $-2$ and $2$ are the roots of the polynomial***

$\boldsymbol{x^3 - 4x^2 + ax + b}$ .

***Solution:***

$f(x) = x^3 - 4x^2 + ax + b$

$\Rightarrow a = -2$

$\Rightarrow a = 2$

| | | | | |
|---|---|---|---|---|
| | 1 | $-4$ | $a$ | $b$ |
| $-1$ | 0 | $-2$ | 12 | $-2a-24$ |
| | 1 | $-6$ | $a+12$ | $b-2a-24$ |
| 2 | 0 | 2 | $-8$ | |
| | 1 | $-4$ | $a+12-8$ | |

***Since*** $-2$ ***and*** $2$ ***are the roots of*** $f(x)$

*Then* $remainder = 0$

$\Rightarrow a + 4 = 0 \Rightarrow a = -4$

$b - 2a - 24 = 0$

$\Rightarrow b - 2(-4) - 24 = 0$

$\Rightarrow b + 8 - 24 = 0$

$\Rightarrow b - 16 = 0$

$\Rightarrow b = 16$

$\Rightarrow \boldsymbol{a = -4}$ **,** $\Rightarrow \boldsymbol{b = 16}$

**Relation between the Roots and the coefficients of a quadratic Equation:**

*let \alpa and* $\beta$ *be the roots of* $ax^2 + bx + c = 0$

$a \neq 0$

*such that*

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

Sum of the Roots $= \boldsymbol{\alpha + \beta}$

$$\boldsymbol{S} = \frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$\boldsymbol{S} = \frac{-b+\sqrt{b^2-4ac} \; -b-\sqrt{b^2-4ac}}{2a}$$

$$S = -\frac{2b}{2a} = -\frac{b}{a}$$

Product of Roots$= \boldsymbol{\alpha\beta}$

$$\boldsymbol{P} = \left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$$

$$\boldsymbol{P} = \frac{(-\boldsymbol{b})^2 - \left(\sqrt{b^2-4ac}\right)^2}{(\boldsymbol{2a})^2}$$

$$= \frac{\boldsymbol{b^2 - (b^2 - 4ac)}}{\boldsymbol{4a^2}} = \frac{\boldsymbol{b^2 = b^2 + 4ac}}{\boldsymbol{4a^2}}$$

$$\boldsymbol{P = \frac{c}{a}}$$

**Thus**

**Sum of roots** $\boldsymbol{S == -\frac{b}{a} = \frac{coefficeints\ of\ x}{coefficients\ of\ x^2}}$

**Product of roots** $\boldsymbol{P == \frac{c}{a} = \frac{constant\ term}{coefficients\ of\ x^2}}$

**Formation of an Equation whose roots are Given:**

$\because$ ***the quadratic equation in standard form is***

$$\boldsymbol{ax^2 + bx + c = 0 \; a \neq 0 \rightarrow (1)}$$

**Then** $\boldsymbol{S = \alpha + \beta = -\frac{b}{a}}$

$$\boldsymbol{P = \alpha\beta = \frac{c}{a}}$$

**Dividing eq(1) by a**

$$\boldsymbol{\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = \frac{0}{a}}$$

$\Rightarrow \boldsymbol{x^2 + \frac{b}{a}x + \frac{c}{a} = 0}$

$\Rightarrow \boldsymbol{x^2 - \left(\frac{b}{a}\right)x + \frac{c}{a} = 0}$

$\Rightarrow \boldsymbol{x^2 - \left(-\frac{b}{a}\right)x + \frac{c}{a} = 0}$

$\Rightarrow \boldsymbol{\because -\frac{b}{a} = S, P = \frac{c}{a}}$

$\Rightarrow$ ***which is requied equation.***

# Exercise 4.6

***<u>Question#1</u>***

***If*** $\boldsymbol{\alpha, \beta}$ ***are the root of*** $\boldsymbol{3x^2 - 2x + 4 = 0,}$ ***find the values of***

$(\boldsymbol{i}).\ \boldsymbol{\frac{1}{\alpha^2} + \frac{1}{\beta^2}}$

***<u>Solution:</u>***

$3x^2 - 2x + 4 = 0$

$a = 3 \quad , b = -2 \quad , c = 4$

$\alpha + \beta = -\frac{b}{a} = \frac{-(-2)}{3} = \frac{2}{3}$

$\alpha\ \beta = \frac{c}{a} = \frac{4}{3} = \frac{4}{3}$

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\beta^2+\alpha^2+2\alpha\beta-2\alpha\beta}{(\alpha\beta)^2}$

$= \frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$

$= \frac{\left(\frac{2}{3}\right)^2-2\left(\frac{4}{3}\right)}{\left(\frac{4}{3}\right)^2}$

$= \left(\frac{4}{9} - \frac{8}{3}\right) \times \frac{9}{16}$

$= \frac{4-24}{16} \times \frac{9}{16}$

$= \frac{-20}{16}$

$= -\frac{5}{4}$

$(\boldsymbol{ii}).\boldsymbol{\frac{\alpha}{\beta} + \frac{\beta}{\alpha}}$

***<u>Solution:</u>***

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$\frac{\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta}{\alpha\beta}$

$= \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

$= \frac{\left(\frac{2}{3}\right)^2-2\left(\frac{4}{3}\right)}{\frac{4}{3}}$

$= \left(\frac{4}{9} - \frac{8}{3}\right) \times \frac{3}{4}$

$= \frac{4-24}{16} \times \frac{3}{4}$

$= \frac{-20}{9} \times \frac{3}{4}$

$= -\frac{5}{3}$

$(\boldsymbol{iii}).\ \boldsymbol{\alpha^4 + \beta^4}$

***<u>Solution:</u>***

$\alpha^4 + \beta^4 = \{(\alpha^2)^2 + (\beta^2)^2 + 2\alpha^2\beta^2 - 2\alpha^2\beta^2\}$

$= (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$

$= [(\alpha^2 + \beta^2 + 2\alpha\beta) - 2\alpha\beta]^2 - 2(\alpha\beta)^2$

$= [(\alpha^2+\beta^2)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2$

$= \left[\left(\frac{2}{3}\right)^2 - 2\left(\frac{4}{3}\right)\right]^2 - 2\left(\frac{4}{3}\right)^2$

$= \left[\frac{4}{9} - \frac{8}{3}\right]^2 - \frac{32}{9}$

$= \left[\frac{4-24}{9}\right]^2 - \frac{32}{9}$

$= \frac{400}{81} - \frac{32}{9}$

$= \frac{400-288}{81}$

$= \frac{112}{81}$

$(iv)$. $\boldsymbol{\alpha^3 + \beta^3}$

**Solution:**

$\alpha^3 + \beta^3 = [\alpha^3 + \beta^3 + 3\alpha\beta(\alpha+\beta) - 3\alpha\beta(\alpha+\beta)]$

$= (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$= \left(\frac{2}{3}\right)^3 - 3\left(\frac{4}{3}\right)\left(\frac{2}{3}\right)$

$= \frac{8}{27} - \frac{8}{3}$

$= \frac{8-72}{27}$

$= -\frac{64}{27}$

$(v)$. $\boldsymbol{\frac{1}{\alpha^3} + \frac{1}{\beta^3}}$

**Solution:**

$\frac{1}{\alpha^3} + \frac{1}{\beta^3} = \frac{\beta^3+\alpha^3}{\alpha^3\beta^3}$

$= \frac{[(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)]}{\alpha^3\beta^3}$

$= \frac{\left[\left(\frac{2}{3}\right)^3 - 3\left(\frac{4}{3}\right)\left(\frac{2}{3}\right)\right]}{\left(\frac{4}{3}\right)^3}$

$= \left[\frac{8}{27} - \frac{8}{3}\right] \times \frac{27}{64}$

$= \left[-\frac{64}{27}\right] \times \frac{27}{64}$

$= -1$

$(vi)$. $\boldsymbol{\alpha^2 - \beta^2}$

**Solution:**

$\alpha^2 - \beta^2 = (\alpha+\beta)(\alpha-\beta)$

$= (\alpha+\beta)\sqrt{(\alpha-\beta)^2}$

$= (\alpha+\beta)\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$= \left(\frac{2}{3}\right)\sqrt{\left(\frac{2}{3}\right)^2 - 4\left(\frac{4}{3}\right)}$

$= \left(\frac{2}{3}\right)\sqrt{\frac{4}{9} - \frac{16}{3}}$

$= \frac{2}{3}\sqrt{\frac{4-48}{9}}$

$= \frac{2}{3}\sqrt{\frac{-44}{9}}$

$= \frac{2\sqrt{11}i}{3}$

***Question#2***

***If $\alpha$, $\beta$ are the root of $x^2 - px - p - c = 0$, prove that***

$\boldsymbol{(1+\alpha)(1+\beta) = 1 - c}$

**Solution:**

$x^2 - px - p - c = 0$

$A = 1$ , $B = -p$ , $C = -p - c$

$\alpha + \beta = \frac{-B}{A} = \frac{-(-p)}{1} = p$

$\alpha\beta = \frac{C}{A} = \frac{-p-c}{1} = -p - c$

$L.H.S = (1+\alpha)(1+\beta)$

$= 1 + \beta + \alpha + \alpha\beta$

$= 1 + p - p - c$

$= 1 - c = R.H.S$

***Question#3***

***Find the condition that one root of $x^2 + px + q = 0$ is***

$(i)$. ***double the other***

**Solution:**

$x^2 + px + q = 0$

$a = 1$ , $b = p$ , $c = q$

***Let $\alpha$ and $2\alpha$ be the roots of the equation***

$S = \alpha + 2\alpha = \frac{-b}{a} = -\frac{p}{1} = -p$

$3\alpha = -p \Rightarrow \alpha = \frac{-p}{3}$

$P = \alpha(2\alpha) = \frac{c}{a} = \frac{q}{1} = q$

$2\alpha^2 = q \Rightarrow 2\left(\frac{-p}{3}\right)^2 = q$

$\Rightarrow 2\frac{p^2}{q} = q \Rightarrow 2p^2 = 9q$

***Which is required condition***

$(ii)$. ***square of the other***

**Solution:**

*Let $\alpha$ and $\alpha^2$ be the roots of the equation*

$S = \alpha + \alpha^2 = \frac{-b}{a} = -\frac{p}{1} = -p$

$P = \alpha(\alpha^2) = \frac{c}{a} = \frac{q}{1} = q$

$(\alpha+\alpha^2)^3 = (-p)^3$

$\alpha^3 + (\alpha^2)^3 + 3\alpha\alpha^2(\alpha+\alpha^2) = -p^3$

$\alpha^3 + (\alpha^2)^3 + 3\alpha^3(\alpha+\alpha^2) = -p^3$

$q + q^2 + 3q(-p) + p^3 = 0$

$q + q^2 - 3pq + p^3 = 0$

***Which is required condition***

$(iii)$. ***additive inverse of the other***

**Solution:**

***Let $\alpha$ and $-\alpha$ be the roots of the equation***

$S = \alpha + (-\alpha) = \frac{-b}{a} = -\frac{p}{1} = -p \Rightarrow p = 0$

$P = \alpha(-\alpha) = \frac{c}{a} = \frac{q}{1} = q \Rightarrow -\alpha^2 = q$

*So,*

$p = 0$

***Which is required condition***

$(iv)$. ***multiplicative inverse of the other.***

**Solution:**

***Let $\alpha$ and $\frac{1}{\alpha}$ be the roots of the equation***

$S = \alpha + \left(\frac{1}{\alpha}\right) = \frac{-b}{a} = -\frac{p}{1} = -p$

$P = \alpha\left(\frac{1}{\alpha}\right) = \frac{c}{a} = \frac{q}{1} = q$

*So,*

$\Rightarrow q = 1$ *Which is required condition*

***Question#4***

***If the roots of the equation $x^2 - px + q = 0$ differ by unity, prove that***

$\boldsymbol{p^2 = 4q + 1}$.

**Solution:**

$x^2 - px + q = 0$

$A = 1\ , B = -p\ \ , C = q$

$\alpha + \beta = \frac{-B}{A} = \frac{-(-p)}{1} = p$

$\alpha\beta = \frac{C}{A} = \frac{q}{1} = q$

***By given condition***

$\alpha - \beta = 1 \Rightarrow (\alpha - \beta)^2 = 1$

$(\alpha + \beta)^2 - 4\alpha\beta = 1$

$p^2 - 4q = 1$

$\Rightarrow p^2 = 4q + 1$

**Question#5**

***Find the condition that $\frac{a}{x-a} + \frac{b}{x-b}$ 5 may have roots equal in magnitude but opposite in signs.***

**Solution:**

***Multiplying both sides by*** $(x-a)(x-b)$

$\boldsymbol{a(x-b) + b(x-a) = 5(x-a)}$

$\Rightarrow \boldsymbol{ax - ab + bx - ab = 5x^2 - 5bx - 5ax + 5bx}$

$\Rightarrow \boldsymbol{-5x^2 + 6ax + 6bx - 7ab = 0}$

$\Rightarrow \boldsymbol{5x^2 - 6(a+b)x + 7ab = 0}$

$\boldsymbol{A = 5\ , B = -6(a+b)\ \ , C = 7ab}$

***Let $\alpha$ and $-\alpha$ be the roots of the equation***

$S = \alpha + (-\alpha) = -\left[\frac{-6(a+b)}{5}\right]$

$0 = \frac{6(a+b)}{5} \Rightarrow a + b = 0$

$P = \alpha(-\alpha) = \frac{7ab}{5} \Rightarrow \alpha^2 = -\frac{7ab}{5}$

*So , $a + b = 0$ is the required equation*

**Question#6**

***If the roots of $px^2 + qx + q = 0$ are $\alpha$ and $\beta$ then prove that*** $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}} = 0$

**Solution:**

$a = p\ , b = q\ \ , c = q$

***Let $\alpha$ and $\beta$ be the roots of the equation***

$\alpha + \beta = \frac{-q}{p}$

$\alpha\beta = \frac{q}{p}$

L.H.S $= \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}}$

$= \frac{\alpha+\beta}{\sqrt{\alpha\beta}} + \sqrt{\frac{q}{p}}$

$= \frac{\frac{-q}{p}}{\sqrt{\frac{-q}{p}}} + \sqrt{\frac{q}{p}}$

$-\sqrt{\frac{q}{p}} + \sqrt{\frac{q}{p}} = 0 = R.H.S$

**Question#7**

***If $\alpha$ , $\beta$ are the roots of the equation $ax^2 + bx + c = 0$, form the equations whose roots are***

$(i)$. $\boldsymbol{\alpha^2, \beta^2}$

**Solution:**

$\alpha + \beta = -\frac{b}{a}$

$\alpha\beta = \frac{c}{a}$

$S = \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= \left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right)$

$= \frac{b^2}{a^2} - \frac{2c}{a} = \frac{b^2 - 2ac}{a^2}$

$P = (\alpha\beta)^2$

$= \left(\frac{c}{a}\right)^2$

$= \frac{c^2}{a^2}$

***Required quadratic equation***

$\boldsymbol{y^2 - Sy + p = 0}$

$\Rightarrow \boldsymbol{y^2 - \left(\frac{b^2 - 2ac}{a^2}\right)y + \frac{c^2}{a^2} = 0}$

$\Rightarrow \boldsymbol{a^2y^2 - (b^2 - 2ac)y + c^2 = 0}$

$(ii)$. $\boldsymbol{\frac{1}{\alpha}, \frac{1}{\beta}}$

**Solution:**

$S = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta} = \frac{-\frac{b}{a}}{\frac{c}{a}} = -\frac{b}{c}$

$P = \left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right) = \frac{1}{\alpha\beta} = \frac{1}{\frac{c}{a}} = \frac{a}{c}$

***Required quadratic equation***

$\boldsymbol{y^2 - Sy + p = 0}$

$\Rightarrow \boldsymbol{y^2 - \left(-\frac{b}{c}\right)y + \frac{a}{c} = 0}$

$\Rightarrow \boldsymbol{cy^2 + by + a = 0}$

$(iii)$. $\boldsymbol{\frac{1}{\alpha^2}, \frac{1}{\beta^2}}$

**Solution:**

$\boldsymbol{S = \frac{1}{\alpha^2} + \frac{1}{\beta^2}}$

$= \boldsymbol{\frac{\beta^2 + \alpha^2}{\alpha^2\beta^2}}$

$= \boldsymbol{\frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}}$

$= \boldsymbol{\frac{\left[\left(\frac{-b}{a}\right)^2 - \frac{2c}{a}\right]}{\left(\frac{c}{a}\right)^2}}$

$= \boldsymbol{\left[\left(\frac{-b}{a}\right)^2 - \frac{2c}{a}\right] \cdot \frac{a^2}{c^2}}$

$= \boldsymbol{\left[\frac{b^2 - 2ac}{a^2}\right] \cdot \frac{a^2}{c^2}}$

$\boldsymbol{P = \left(\frac{1}{\alpha^2}\right)\left(\frac{1}{\beta^2}\right) = \frac{1}{(\alpha\beta)^2} = \frac{1}{\left(\frac{c}{a}\right)^2} = \frac{a^2}{c^2}}$

***Required quadratic equation***

$\boldsymbol{y^2 - Sy + p = 0}$

$\Rightarrow \boldsymbol{y^2 - \left[\frac{b^2 - 2ac}{c^2}\right]y + \frac{a^2}{c^2} = 0}$

$\Rightarrow \boldsymbol{c^2y^2 - (b^2 - 2ac)y + a^2 = 0}$

$(iv)$. $\boldsymbol{\alpha^3, \beta^3}$

**Solution:**

$S = \alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) =$

$\left(\frac{-b}{a}\right)^3 - 3\left(\frac{c}{a}\right)\left(\frac{-b}{a}\right) = \frac{-b^3}{a^3} + \frac{3bc}{a^2} = \frac{-b^3 + 3bc}{a^3}$

$P = \alpha^3\beta^3 = (\alpha\beta)^3 = \left(\frac{c}{a}\right)^3 = \frac{c^3}{a^3}$

*Required quadratic equation*

$\boldsymbol{y^2 - Sy + p = 0}$

$\Rightarrow y^2-\left[\frac{-b^3+3bc}{a^3}\right]y+\frac{c^3}{a^3}=0$

$\Rightarrow a^3y^2-(3bc-b^3)y+c^3=0$

(v). $\frac{1}{\alpha^3},\frac{1}{\beta^3}$

**Solution:**

$S=\frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{\beta^3+\alpha^3}{\alpha^3\beta^3}$

$=\frac{[(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)]}{\alpha^3\beta^3}$

$=\frac{[(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)]}{(\alpha\beta)^3}$

$=\frac{\left[\left(\frac{-b}{a}\right)^3-3\left(\frac{c}{a}\right)\left(\frac{-b}{a}\right)\right]}{\left(\frac{c}{a}\right)^3}$

$=\frac{\left[\frac{-b^3}{a^3}+\frac{3bc}{a^2}\right]}{\left(\frac{c^3}{a^3}\right)^3}$

**Question#5**

***Find the condition that*** $\frac{a}{x-a}+\frac{b}{x-b}$ ***5 may have roots equal in magnitude but opposite in signs.***

**Solution:**

***Multiplying both sides by*** $(x-a)(x-b)$

$a(x-b)+b(x-a)=5(x-a)$

$\Rightarrow ax-ab+bx-ab=5x^2-5bx-5ax+5bx$

$\Rightarrow -5x^2+6ax+6bx-7ab=0$

$\Rightarrow 5x^2-6(a+b)x+7ab=0$

$A=5\ ,B=-6(a+b)\ ,C=7ab$

***Let $\alpha$ and $-\alpha$ be the roots of the equation***

$S=\alpha+(-\alpha)=-\left[\frac{-6(a+b)}{5}\right]$

$0=\frac{6(a+b)}{5}\Rightarrow a+b=0$

$P=\alpha(-\alpha)=\frac{7ab}{5}\Rightarrow \alpha^2=-\frac{7ab}{5}$

*So , $a+b=0$ is the required equation*

**Question#6**

***If the roots of*** $px^2+qx+q=0$ ***are*** $\alpha$ ***and*** $\beta$ ***then prove that*** $\sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}+\sqrt{\frac{q}{p}}=0$

**Solution:**

$a=p\ ,b=q\ ,c=q$

***Let $\alpha$ and $\beta$ be the roots of the equation***

$\alpha+\beta=\frac{-q}{p}$

$\alpha\beta=\frac{q}{p}$

$L.H.S=\sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}+\sqrt{\frac{q}{p}}$

$=\frac{\alpha+\beta}{\sqrt{\alpha\beta}}+\sqrt{\frac{q}{p}}$

$=\frac{\frac{-q}{p}}{\sqrt{\frac{-q}{p}}}+\sqrt{\frac{q}{p}}$

$-\sqrt{\frac{q}{p}}+\sqrt{\frac{q}{p}}=0=R.H.S$

**Question#7**

***If $\alpha$ , $\beta$ are the roots of the equation $ax+bx+c=0$, form the equations whose roots are***

(i). $\alpha^2,\beta^2$

**Solution:**

$\alpha+\beta=-\frac{b}{a}$

$\alpha\beta=\frac{c}{a}$

$S=\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=\left(-\frac{b}{a}\right)^2-2\left(\frac{c}{a}\right)$

$=\frac{b^2}{a^2}-\frac{2c}{a}=\frac{b^2-2ac}{a^2}$

$P=(\alpha\beta)^2$

$=\left(\frac{c}{a}\right)^2$

$=\frac{c^2}{a^2}$

***Required quadratic equation***

$y^2-Sy+p=0$

$\Rightarrow y^2-\left(\frac{b^2-2ac}{a^2}\right)y+\frac{c^2}{a^2}=0$

$\Rightarrow a^2y^2-(b^2-2ac)y+c^2=0$

(ii). $\frac{1}{\alpha},\frac{1}{\beta}$

**Solution:**

$S=\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}=\frac{-\frac{b}{a}}{\frac{c}{a}}=-\frac{b}{c}$

$P=\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right)=\frac{1}{\alpha\beta}=\frac{1}{\frac{c}{a}}=\frac{a}{c}$

***Required quadratic equation***

$y^2-Sy+p=0$

$\Rightarrow y^2-\left(-\frac{b}{c}\right)y+\frac{a}{c}=0$

$\Rightarrow cy^2+by+a=0$

(iii). $\frac{1}{\alpha^2},\frac{1}{\beta^2}$

**Solution:**

$S=\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

$=\frac{\beta^2+\alpha^2}{\alpha^2\beta^2}$

$=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$

$=\frac{\left[\left(\frac{-b}{a}\right)^2-\frac{2c}{a}\right]}{\left(\frac{c}{a}\right)^2}$

$=\left[\left(\frac{-b}{a}\right)^2-\frac{2c}{a}\right].\frac{a^2}{c^2}$

$=\left[\frac{b^2-2ac}{a^2}\right].\frac{a^2}{c^2}$

$P=\left(\frac{1}{\alpha^2}\right)\left(\frac{1}{\beta^2}\right)=\frac{1}{(\alpha\beta)^2}=\frac{1}{\left(\frac{c}{a}\right)^2}=\frac{a^2}{c^2}$

***Required quadratic equation***

$y^2-Sy+p=0$

$\Rightarrow y^2-\left[\frac{b^2-2ac}{c^2}\right]y+\frac{a^2}{c^2}=0$

$\Rightarrow c^2y^2-(b^2-2ac)y+a^2=0$

(iv). $\alpha^3,\beta^3$

**Solution:**

$S = \alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = \left(\frac{-b}{a}\right)^3 - 3\left(\frac{c}{a}\right)\left(\frac{-b}{a}\right) = \frac{-b^3}{a^3} + \frac{3bc}{a^2} = \frac{-b^3+3bc}{a^3}$

$P = \alpha^3\beta^3 = (\alpha\beta)^3 = \left(\frac{c}{a}\right)^3 = \frac{c^3}{a^3}$

*Required quadratic equation*

$y^2 - Sy + p = 0$

$\Rightarrow y^2 - \left[\frac{-b^3+3bc}{a^3}\right]y + \frac{c^3}{a^3} = 0$

$\Rightarrow a^3y^2 - (3bc - b^3)y + c^3 = 0$

(v). $\frac{1}{\alpha^3}, \frac{1}{\beta^3}$

**Solution:**

$S = \frac{1}{\alpha^3} + \frac{1}{\beta^3} = \frac{\beta^3+\alpha^3}{\alpha^3\beta^3}$

$= \frac{[(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)]}{\alpha^3\beta^3}$

$= \frac{[(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)]}{(\alpha\beta)^3}$

$= \frac{\left[\left(\frac{-b}{a}\right)^3 - 3\left(\frac{c}{a}\right)\left(\frac{-b}{a}\right)\right]}{\left(\frac{c}{a}\right)^3}$

$= \frac{\left[\frac{-b^3}{a^3} + \frac{3bc}{a^2}\right]}{\left(\frac{c^3}{a^3}\right)^3}$

$= \left[\frac{-b^3+3bc}{a^3}\right] \cdot \frac{a^3}{c^3}$

$P = \frac{1}{\alpha^3} \cdot \frac{1}{\beta^3} = \frac{1}{(\alpha\beta)^3} = \frac{1}{\left(\frac{c}{a}\right)^3} = \frac{a^3}{c^3}$

**Required quadratic equation**

$y^2 - Sy + p = 0$

$\Rightarrow y^2 - \left[\frac{-b^3+3bc}{c^3}\right]y + \frac{a^3}{c^3} = 0$

$\Rightarrow c^3y^2 - (3bc - b^3)y + a^3 = 0$

(vi). $\alpha + \frac{1}{\alpha}$ , $\beta + \frac{1}{\beta}$

**Solution:**

$S = \alpha + \frac{1}{\alpha} + \beta + \frac{1}{\beta} = \alpha + \beta + \frac{\beta+\alpha}{\alpha\beta} = \left(-\frac{b}{a}\right) + \frac{-\frac{b}{a}}{\frac{c}{a}} = \frac{-bc-ab}{ac}$

$P = \left(\alpha + \frac{1}{\alpha}\right)\left(\beta + \frac{1}{\beta}\right)$

$= \alpha\beta + \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + \frac{1}{\alpha\beta}$

$= \frac{(\alpha\beta)^2 + (\alpha+\beta)^2 - 2\alpha\beta + 1}{\alpha\beta}$

$= \frac{\left[\left(\frac{c}{a}\right)^2 + \left(\frac{-b}{a}\right)^2 - \frac{2c}{a} + 1\right]}{\frac{c}{a}}$

$= \left[\frac{a^2+b^2-2ac+c^2}{a^2}\right] \cdot \frac{a}{c}$

**Required quadratic equation**

$y^2 - Sy + p = 0$

$\Rightarrow y^2 - \left[\frac{-bc-ab}{ac}\right]y + \left[\frac{a^2+b^2-2ac+c^2}{ac}\right] = 0$

$\Rightarrow acy^2 - b(a+c)y + b^2 + (a-c)^2 = 0$

(vii). $(\alpha-\beta)^2, (\alpha+\beta)^2$

**Solution:**

$S = (\alpha-\beta)^2 \ (\alpha+\beta)^2$

$= \alpha^2 + \beta^2 - 2\alpha\beta + (\alpha+\beta)^2$

$= \alpha^2 + \beta^2 + 2\alpha\beta - 4\alpha\beta + (\alpha+\beta)^2$

$= (\alpha+\beta)^2 - 4\alpha\beta + (\alpha+\beta)^2$

$= 2(\alpha+\beta)^2 - 4\alpha\beta$

$= 2\left(-\frac{b}{a}\right)^2 - 4\left(\frac{c}{a}\right) = 2\frac{b^2}{a^2} - 4\left(\frac{c}{a}\right) = \frac{2b^2-4ac}{a^2}$

$P = (\alpha-\beta)^2 \ (\alpha+\beta)^2$

$= [(\alpha+\beta)^2 - 4\alpha\beta](\alpha+\beta)^2$

$= \left[\left(-\frac{b}{a}\right)^2 - 4\left(\frac{c}{a}\right)\right]\left(-\frac{b}{a}\right)^2 = \left(\frac{b^2-4ac}{a^2}\right)\frac{b^2}{a^2}$

**Required quadratic equation**

$y^2 - Sy + p = 0$

$\Rightarrow y^2 - \left[\frac{2b^2-4ac}{a^2}\right]y + \left(\frac{b^2-4ac}{a^2}\right)\frac{b^2}{a^2} = 0$

$\Rightarrow a^4y^2 - 2a^2(b^2-2ac)y + b^2(b^2-4ac) = 0$

(viii). $-\frac{1}{\alpha^3}$ , $-\frac{1}{\beta^3}$

**Solution:**

$S = \frac{1}{-\alpha^3} + \left(-\frac{1}{\beta^3}\right) = -\left[\frac{\beta^3+\alpha^3}{\alpha^3\beta^3}\right]$

$= -\frac{[(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)]}{\alpha^3\beta^3}$

$= -\frac{[(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)]}{(\alpha\beta)^3}$

$= -\frac{\left[\left(\frac{-b}{a}\right)^3 - 3\left(\frac{c}{a}\right)\left(\frac{-b}{a}\right)\right]}{\left(\frac{c}{a}\right)^3}$

$= -\frac{\left[\frac{-b^3}{a^3} + \frac{3bc}{a^2}\right]}{\left(\frac{c^3}{a^3}\right)^3}$

$= \left[\frac{b^3-3bc}{a^3}\right] \cdot \frac{a^3}{c^3}$

$P = \left(-\frac{1}{\alpha^3}\right) \cdot \left(-\frac{1}{\beta^3}\right)$

$= \frac{1}{(\alpha\beta)^3} = \frac{1}{\left(\frac{c}{a}\right)^3}$

$= \frac{a^3}{c^3}$

**Required quadratic equation**

$y^2 - Sy + p = 0$

$\Rightarrow y^2 - \left[\frac{b^3-3bc}{c^3}\right]y + \frac{a^3}{c^3} = 0$

$\Rightarrow c^3y^2 - (b^3 - 3bc)y + a^3 = 0$

**Question#8**

***If $\alpha$ , $\beta$ are the roots of the $5x^2 - x - 2 = 0$, form the equation whose roots are $\frac{3}{\alpha}$ , $\frac{3}{\beta}$***

**Solution:**

$5x^2 - x - 2 = 0$

$a = 5$ , $b = -1$ , $c = -2$

***Let $\alpha$ and $\beta$ be the roots of the equation***

$\alpha + \beta = \frac{-b}{a} = \frac{-(-1)}{5} = \frac{1}{5}$

$\alpha\beta = \frac{c}{a} = \frac{-2}{5}$

$S = \frac{3}{\alpha} + \frac{3}{\beta} = \frac{3(\beta+\alpha)}{\alpha\beta} = \frac{3\left(\frac{1}{5}\right)}{\left(\frac{-2}{5}\right)} = \frac{-3}{2}$

$P = \left(\frac{3}{\alpha}\right)\left(\frac{3}{\beta}\right) = \frac{9}{\alpha\beta} = \frac{9}{\frac{-2}{5}} = \frac{-45}{2}$

***Required equation***

$y^2 - Sy + p = 0$

$\Rightarrow y^2 - \left(\frac{-3}{2}\right)y + \frac{-45}{2} = 0$

$\Rightarrow y^2 + 3y - 45 = 0$

**Question#9**

***If $\alpha$ , $\beta$ are the roots of the $x^2 - 3x + 5 = 0$, form the equation whose roots are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$***

**Solution:**

$x^2 - 3x + 5 = 0$
$a = 1\ , b = -3\ \ , c = 5$
***Let $\alpha$ and $\beta$ be the roots of the equation***
$\alpha + \beta = \frac{-b}{a} = \frac{-(-3)}{5} = \frac{3}{5}$
$\alpha\beta = \frac{c}{a} = \frac{5}{1} = 5$
$S = \frac{1-\alpha}{1+\alpha} + \frac{1-\beta}{1+\beta}$
$= \frac{(1-\alpha)(1+\beta)+(1+\alpha)(1-\beta)}{(1+\alpha)(1+\beta)}$
$= \frac{1+\beta-\alpha-\alpha\beta+1-\beta+\alpha-\alpha\beta}{1+\alpha+\beta+\alpha\beta}$
$= \frac{2-2\alpha\beta}{1+(\alpha+\beta)+\alpha\beta}$
$= \frac{2(1-\alpha\beta)}{1+(\alpha+\beta)+\alpha\beta} = \frac{2(1-5)}{1+3+5} = \frac{-8}{9}$
$P = \frac{(1-\alpha)(1-\beta)}{(1+\alpha)(1+\beta)}$
$= \frac{1-\beta-\alpha+\alpha\beta}{1+\alpha+\beta+\alpha\beta}$
$= \frac{1-(\alpha+\beta)+\alpha\beta}{1+(\alpha+\beta)+\alpha\beta}$
$= \frac{1-3-5}{1+3+5} = \frac{3}{9} = \frac{1}{3}$
***Required equation***
$\boldsymbol{y^2 - Sy + p = 0}$
$\Rightarrow \boldsymbol{y^2 - \left(\frac{-8}{9}\right)y + \frac{1}{3} = 0}$
$\Rightarrow \boldsymbol{9y^2 + 8y + 3 = 0}$

**Nature of the roots of a Quadratic equation:**
**We know that the roots of the quadratic equation**
$ax^2 + bx + c = 0$
Are given by quadratic formula.
$as\ x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$
*the expression* $b^2 - 4ac$ *is called Discriminent (Disc.)*
*nature of roots depends on the value of expression
$b^2 - 4ac$

Case 1
*if* $b^2 - 4ac > 0$
Then roots are real and unequal.
Case11.
*if* $b^2 - 4ac > 0$ *but perfect square then roots*
Are real rational and unequal.
Case 111:
*if*
$b^2 - 4ac > 0$ *but not perfect square then roots*. Are real , irrational and unequal.
Case iv.
*if* $b^2 - 4ac < 0$ *then roots are complex and* Unequal.
Case V.
*if* $b^2 - 4ac = 0$ *then roots are real and equal.*

# Exercise 4.7

***<u>Discuss the nature of the roots of the following equations:</u>***
***<u>Question#1</u>***
$(i).\ \boldsymbol{4x^2 + 6x + 1 = 0}$
***<u>Solution:</u>***
$4x^2 + 6x + 1 = 0$
*Here*
$a = 4\ ,\quad b = 6\ ,\ \ c = 1$
$Disc.= b^2 - 4ac$
$= (6)^2 - 4(4)(1)$
$= 36 - 16 = 20 > 0$
***Discriminant is not perfect square therefore the roots are irrational (real) and unequal.***
$(ii).\ \ \boldsymbol{x^2 - 5x + 6 = 0}$
***<u>Solution:</u>***
$x^2 - 5x + 6 = 0$
*Here*
$a = 1\ ,\quad b = -5\ ,\ \ c = 6$
$Disc.= b^2 - 4ac$
$= (-5)^2 - 4(4)(6)$
$= 25 - 24 = 4 > 0$
***Discriminant is not perfect square therefore the roots are irrational (real) and unequal.***
$(iii).\ \boldsymbol{2x^2 - 5x + 1 = 0}$
***<u>Solution:</u>***
$\boldsymbol{2x^2 - 5x + 1 = 0}$
***here*** $\boldsymbol{a = 2, b = -5, c = 1}$
$Disc. b^2 - 4ac = (-5)^2 - 4(2)(1)$
Disc.$= 25 - 18 = 17 >$
$0$ *but not perfect square.*
Thus roots are real, irrational and unequal.
$(iv).\ \ \boldsymbol{25x^2 - 30x + 9 = 0}$
***<u>Solution:</u>***
$25x^2 - 30x + 9 = 0$
*Here*
$a = 25\ ,\quad b = -30\ ,\ \ c = 9$
$Disc.= b^2 - 4ac$
$= (-30)^2 - 4(25)(9)$
$= 900 - 900 = 0$
$\therefore$ ***roots are rational (real) and equal.***
***<u>Question#2</u>***
***Show that the roots of the following equations will be real:***
$(i).\ \boldsymbol{x^2 - 2\left(m + \frac{1}{m}\right)x + 3 = 0\ ;\ m = 0}$
***<u>Solution:</u>***
$x^2 - 2\left(m + \frac{1}{m}\right)x + 3 = 0$
*Here*
$a = 1\ ,\quad b = -2\left(m + \frac{1}{m}\right)\ \ ,\ \ c = 3$
$Disc.= b^2 - 4ac$

$= \left(-2\left(m+\frac{1}{m}\right)\right)^2 - 4(1)(3)$
$= 4\left(m^2+\frac{1}{m^2}\right) - 12$
$= 4\left(m^2+\frac{1}{m^2} - 12\right)$
$= 4\left(m^2+\frac{1}{m^2} + 2 - 3\right)$
$= 4\left(m^2+\frac{1}{m^2} - 1\right)$
$= 4\left(m^2+\frac{1}{m^2} - 2 + 1\right)$
$= 4\left(\left(m-\frac{1}{m}\right)^2 + 1\right) > 0$

***Hence roots are real.***

**(ii).** $(b-c)x^2 - (c-a)x + (a-b) = 0\,;\ a, b, c \in Q$

***Solution:***

*Here,*
$A = b - c\,,\quad B = c - a\,,\quad C = a - b$
$Disc. = B^2 - 4AC$
$= (c-a)^2 - 4(b-c)(a-b)$
$= c^2 + a^2 - 2ca - 4(ab - b^2 - ac + bc)$
$= c^2 + a^2 - 2ca - 4ab + 4b^2 + 4ac - 4bc$
$= a^2 + c^2 + 2ca - 4ab + 4b^2 + 4ac - 4bc$
$= (a^2+c^2)^2 - 4b(a+c) + (2b)^2$
$= (a+c+2b)^2 > 0$

***Hence roots are real.***

***Question#3***

***Show that the roots of the following equations will be rational:***

**(i).** $(p+q)x^2 - px - q = 0$

***Solution:***

$(p+q)x^2 - px - q = 0$
*Here*
$a = p + q\,,\quad b = -p\,,\quad c = -q$
$Disc. = b^2 - 4ac$
$= (p+q)^2 - 4(p+q)(-q)$
$p^2 + 4pq + 4q^2 = 0$
$= (p+2q)^2$

***Hence roots are rational.***

**(ii).** $px^2 - (p-q)x - q = 0$

***Solution:***

$$px^2 - (p-q)x - q = 0$$
$$here\ a = p, b = -(p-q), c = -q$$
$$Disc = b^2 - 4ac$$
$$[-(p-q)]^2 - 4(p)(-q)$$
$$= (p-q)^2 + 4pq$$
$$p^2 + q^2 - 2pq + 4pq$$
$$p^2 + q^2 + 2pq$$
$$Disc = (p+q)^2 > 0\ and\ perfect\ square$$

***Hence roots are rational.***

***Question#4***

***For what values of m will the roots of the following equations be equal?***

**(i).** $(m+1)x^2 + 2(m+3)x + m + 8 = 0$

***Solution:***

$(m+1)x^2 + 2(m+3)x + m + 8 = 0m$
*Here*
$a = m + 3\,,\quad b = 2(m+3)\,,\quad c = m + 8$
$Disc. = b^2 - 4ac$
$(2(m+3)^2) - 4(m+1)(m+8)$
$= 4(m^2 + 6m + 9) - 4(m^2 + 8m + m + 8)$
$= 4(m^2 + 6m + 9 - m^2 - 8m - m - 8)$
$= 4(-3m + 1)$

***For equal roots, we have***
***Disc.* = 0**
$\Rightarrow 4(-3m+1) = 0$
$\Rightarrow -3m + 1 = 0$
$\Rightarrow 3m = -1$
$\Rightarrow m = \frac{-1}{3}$

**(ii).** $x^2 - 2(1+3m)x + 7(3+2m) = 0$

***Solution:***

$$x^2 - 2(1+3m)x + 7(3+2m) = 0$$
*Here a=1* $b = -2(1+3m), c = 7(3+2m)$
*Disc.*$=b^2 - 4ac$
$$= [-2(1+3m]^2 - 4(1)[7(3+2m)]$$
$$= 4(1+3m)^2 - 4(21+14m)$$
$$= 4(1+9m^2+6m) - 4(21+14m)$$
$$4 + 36m^2 + 24m - 84 - 56m$$
$$Dis = 36m^2 - 32m - 80$$



Given roots are equal
$$i.e\ disc = 0$$
⇨ $36m^2 - 32m - 80 = 0$
⇨ $9m^2 - 8m - 20 = 0(\div by\ 4)$
⇨ $9m^2 - 18m + 10m - 20 = 0$
⇨ $9m(m-2) + 10(m-2) = 0$
⇨ $m - 2 = 0, 9m + 10 = 0$
$$m = 2\ 9m = -10$$
$$m = 2, m = -\frac{10}{9}$$

**(iii).** $(1+m)x^2 - 2(1+3m)x + (1+8m) = 0$

***Solution:***

$\boldsymbol{here\ a = 1 + m, b = -2(1+3m), c = 1 + 8m}$
$$\boldsymbol{Disc = b^2 - 4ac}$$
$$= [-2(1+3m)]^2 - 4(1+m)(1+8m)$$
$$= 4(1+3m)^2 - 4(1+3m)^2$$
$$- 4(1 + m + 8m + 8m^2)$$
$$= 4(1+9m^2+6m) - 4(1+9m+8m^2)$$
$$4 + 36m^2 + 24m - 4 - 36m - 32m^2$$
$$Disc.\, 4m^2 - 12m$$
*Given roots are equal.*
$$i.e\ disc. = 0$$
⇨ $4m^2 - 12m = 0$
⇨ $4m(m-3) = 0$
⇨ $4m = 0, m - 3 = 0$
⇨ $m = 0, m = 3$

***Question#5***

***Show that the roots of*** $x^2 + (mx + c)^2 = a^2$ ***will be equal, if*** $c^2 = a^2(1 + m^2)$

***Solution:***

$x^2 + (mx + c)^2 = a^2$
$\Rightarrow x^2 + m^2 + 2mcx + c^2 = a^2$
$\Rightarrow x^2(1 + m^2) + 2mcx + c^2 - a^2 = 0$
*Here*
$A = 1 + m^2, \quad B = 2mc, \quad C = c^2 - a^2$
$Disc. = B^2 - 4AC$
$= (2mc)^2 - 4(1 + m^2)(c^2 - a^2)$
$= 4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2)$
$= 4(m^2c^2 - c^2 + a^2 - m^2c^2 + m^2a^2)$
***For equal roots, we have***
$\mathbf{Disc. = 0}$
$\mathbf{-c^2 + a^2 + m^2a^2 = 0}$
$\mathbf{\Rightarrow c^2 = a^2 + m^2a^2}$
$\mathbf{\Rightarrow c^2 = a^2(1 + m^2)}$
***As required.***

***Question#6***

***Show that the roots of*** $\mathbf{(mx + c)^2 = 4ax}$ ***will be equal, if*** $\mathbf{c = \frac{a}{m}\ ;\ m = 0}$

***Solution:***
$(mx + c)^2 = 4ax$
$\Rightarrow m^2x^2 + 2mcx + c^2 - 4ax = 0$
$\Rightarrow m^2x^2 + 2(mc - 2a)x + c^2 = 0$
*Here*
$A = m^2, \quad B = 2(mc - 2a), \quad C = c^2$
$Disc. = B^2 - 4AC$
$= [2(mc - 2a)]^2 - 4(m^2)(c^2)$
$= 4(m^2c^2 + 4a^2 - 4amc - m^2c^2)$
$= 4(4a^2 - 4amc)$
***For equal roots, we have***
$\mathbf{Disc. = 0}$
$4(4a^2 - 4amc) = 0$
$\Rightarrow 16a(4a - mc)$
$\Rightarrow a - mc = 0$
$\Rightarrow a = mc$
$\Rightarrow \frac{a}{m} = c$
$\Rightarrow c = \frac{a}{m}$
*As required.*

***Question#7***

***Prove that*** $\mathbf{\frac{x^2}{a^2} + \frac{(mx+c)^2}{b^2} = 1}$ ***will have equal roots, if*** $\mathbf{c^2 = a^2m^2 + b^2\ ;\ a \neq 0, b \neq 0}$

***Solution:***
$b^2x^2 + a^2(mx + c)^2 = a^2b^2$
$\Rightarrow b^2x^2 + a^2(m^2x^2 + c^2 + 2mcx) - a^2b^2 = 0$
$\Rightarrow b^2x^2 + a^2m^2x^2 + a^2c^2 + 2a^2mcx - a^2b^2 = 0$
$\Rightarrow (b^2 + a^2m^2)x^2 + 2a^2mcx - a^2(c^2 - b^2) = 0$
***Here***
$A = b^2 + a^2m^2, \quad B = 2a^2mc, \quad C = a^2(c^2 - b^2)$
$Disc. = B^2 - 4AC$
$= (2a^2mc)^2 - 4(b^2 + a^2m^2)(a^2(c^2 - b^2))$
$= 4a^4m^2c^2 - 4a^2(c^2b^2 + b^4 - a^2m^2c^2 + a^2b^2m^2)$
$= 4a^2b^2(-c^2 + b^2 + a^2m^2)$
*For equal roots, we have*
$Disc. = 0$
$4a^2b^2(-c^2 + b^2 + a^2m^2) = 0$
$\Rightarrow c^2 = a^2 + m^2a^2$
$\Rightarrow c^2 = a^2m^2 + b^2 \quad ; a \neq 0, b$

***Question#8***

***Show that the roots of the equation*** $\mathbf{(a^2 - bc)x^2 + 2(b^2 - ca)x + c^2 - ab = 0}$ ***will be equal, if either*** $\mathbf{a^3 + b^3 + c^3 = 3abc}$ ***or*** $\mathbf{b = 0}$.

***Solution:***
$(a^2 - bc)x^2 + 2(b^2 - ca)x + c^2 - ab = 0$
*Here*
$A = a^2 - bc, \quad B = 2(b^2 - ca), \quad C = (c^2 - b^2)$
$Disc. = B^2 - 4AC$
$= [2(b^2 - ca)]^2 - 4(a^2 - bc)(c^2 - ab)$
$= 4(b^4 + a^2c^2 - 2ab^2c) - 4(a^2c^2 - a^3b + bc^3 - ab^2c)$
$= 4(b^4 + a^2c^2 - 2ab^2c - a^2c^2 + a^3b + bc^3 - ab^2c)$
$= 4(a^3b + b^4 + bc^3 - 3ab^2c)$
$= 4b(a^3 + b^3 + c^3 - 3abc)$
***For equal roots, we have***
$\mathbf{Disc. = 0}$
$4b(a^3 + b^3 + c^3 - 3abc) = 0$
$\Rightarrow 4b = 0 \quad or \quad a^3 + b^3 + c^3 - 3abc = 0$
$\Rightarrow b = 0 \quad or \quad a^3 + b^3 + c^3 = 3abc$

**System of two Equations involving two variables:**

**Simultaneous Equation:**

A set of two or more than two equations in which the values of variables satisfied all the equation are called simultaneous equations or a system of equation.

Case 1.**when one equation is linear and other is quadratic:**

If one of the equation is linear. We can find value of one variable in term of the other variable from linear equation.

Put this value of one variable to the quadratic equations. **We solve it see example 1.Q1,2,3,4,5,6,7**

**Note :**

Two quadratic equations in which xy term is missing and the coefficients of $x^2$ and$y^2$ *are equal, given a linear equations by*

Subtracting *see example* 2. *Q*8,9,10

# Exercise 4.8

***Solve the following systems of equations:***

***Question#1***

$\mathbf{2x - y = 4 \;;\; 2x^2 - 4xy - y^2 = 6}$

***Solution:***

$2x - y = 4 \quad \ldots(1)$

$2x^2 - 4xy - y^2 = 6 \quad \ldots(2)$

***From*** $(1) \quad \mathbf{y = 2x - 4} \quad \ldots(3)$

***Putting value of y in eq.*** $(2)$

$2x^2 - 4x(2x - 4) - (2x - 4)^2 = 6$

$2x^2 - 8x^2 + 16x - (4x^2 - 16x + 16) - 6 = 0$

$-6x^2 + 16x - 4x^2 + 16x - 16 - 6 = 0$

$-10x^2 + 32x - 22 = 0$

***Dividing by*** $-2$ ***we get***

$5x^2 - 16x + 11 = 0$

$5x^2 - 5x - 11x + 11 = 0$

$5x(x - 1) - 11(x - 1) = 0$

$(x - 1)(5x - 11) = 0$

$x - 1 = 0 \quad , 5x - 11 = 0$

$x = 1 , \quad x = \frac{11}{5}$

***If*** $x = 1$ ***then, from*** $(3)$

$y = 2(1) - 4$

$y = 2 - 4$

$\Rightarrow y = -2$

***If*** $x = \frac{11}{5}$ ***then,***

$y = 2\left(\frac{11}{5}\right) - 4$

$y = \frac{22}{5} - 4 = \frac{22-20}{5}$

$\Rightarrow y = \frac{2}{5}$

**S. S** $= \left\{ (1, -2), \left(\frac{11}{5}, \frac{2}{5}\right) \right\}$

***Question#2***

$\mathbf{x + y = 5 \;;\; x^2 + 2y^2 = 17}$

***Solution:***

$x + y = 5 \ldots(1)$

$x^2 + 2y^2 = 17 \quad \ldots(2)$

***From*** $(1) \quad \mathbf{y = 5 - x} \quad \ldots(3)$

***Putting value of y in eq.*** $(2)$

$x^2 + 2(5 - x)^2 = 17$

$x^2 + 2(x^2 - 10x + 25) - 17 = 0$

$x^2 + 2x^2 - 20x + 50 = 0$

$3x^2 - 20x + 33 = 0$

$3x^2 - 9x - 11x + 33 = 0$

$3x(x - 3) - 11(x - 3) = 0$

$(x - 3)(3x - 11) = 0$

$x - 3 = 0 \quad , 3x - 11 = 0$

$x = 3 , \quad x = \frac{11}{3}$

***If*** $x = 3$ ***then, from*** $(3)$

$y = 5 - 3$

$\Rightarrow y = 2$

***If*** $x = \frac{11}{3}$ ***then,***

$y = 5 - \frac{11}{3} = \frac{15-11}{3}$

$\Rightarrow y = \frac{4}{3}$

**S. S** $= \left\{ (3, 2), \left(\frac{11}{3}, \frac{4}{3}\right) \right\}$

***Question#3***

$\mathbf{3x + 2y = 7 \;;\; 3x^2 = 25 + 2y^2}$

***Solution:***

$3x + 2y = 7 \ldots(1)$

$3x^2 - 2y^2 = 25 \ldots(2)$

***From*** $(1) \quad \mathbf{y = \frac{7-3x}{2}} \quad \ldots(3)$

***Putting value of y in eq.*** $(2)$

$3x^2 - 2\left(\frac{7-3x}{2}\right)^2 - 25 = 0$

$3x^2 - 2\left(\frac{49-42x+9x^2}{4}\right) - 25 = 0$

$3x^2 - \left(\frac{49-42x+9x^2}{2}\right) - 25 = 0$

***Dividing by*** $2$ ***we get***

$6x^2 - (49 - 42x + 9x^2) - 50 = 0$

$6x^2 - 49 + 42x - 9x^2 - 50 = 0$

$-3x^2 + 42x - 90 = 0$

***Dividing by*** $-3$ ***we get***

$x^2 - 14x + 33 = 0$

$x^2 - 3x - 11x + 33 = 0$

$x(x - 3) - 11(x - 3) = 0$

$(x - 3)(x - 11) = 0$

$x - 3 = 0 \quad , x - 11 = 0$

$x = 3 , \quad x = 11$

***If*** $x = 3$ ***then, from*** $(3)$

$y = \frac{7-3(3)}{2}$

$y = \frac{7-9}{2} = \frac{-2}{2}$

$\Rightarrow y = -1$

***If*** $x = \frac{11}{5}$ ***then,***

$y = \frac{7-3(11)}{2}$

$y = \frac{7-33}{2} = \frac{-26}{2}$

$\Rightarrow y = -13$

$S.\ S = \{ (3, -1), (11, -13) \}$

***Question#4***

$\mathbf{x + y = 5 \;;\; \frac{2}{x} + \frac{3}{y} = 2 \;, x \neq 0 , y \neq 0}$

***Solution:***

$x + y = 5 \quad \ldots(1)$

$\frac{2}{x} + \frac{3}{y} = 2$

***Multiplying by xy we get***

$2y + 3x = 2xy \quad \ldots(2)$

*From* $(1) \quad y = 5 - x \quad \ldots(3)$

***Putting value of y in eq.*** $(2)$

$2(5 - x) + 3x = 2x(5 - x)$

$10 - 2x + 3x = 10x - 2x^2$

$10 + x = 10x - 2x^2$

$2x^2 - 10x + x + 10 = 0$

$2x^2 - 9x + 10 = 0$

$2x^2 - 4x - 5x + 10 = 0$

$2x(x - 2) - 5(x - 2) = 0$

$(x - 2)(2x - 5) = 0$

$x - 2 = 0 \quad , 2x - 5 = 0$

$x = 1\,, \quad x = \frac{5}{2}$

***If*** $x = 2$ ***then, from*** $(3)$

$y = 5 - 2$

$\Rightarrow y = 3$

***If*** $x = \frac{5}{2}$***then,***

$y = 5 - \frac{5}{2} = \frac{10-5}{2}$

$\Rightarrow y = \frac{5}{2}$

**S. S** $= \left\{ (2\,,3)\,, \left(\frac{5}{2}\,, \frac{5}{2}\right) \right\}$

***Question#5***

$x + y = a + b \quad ; \quad \frac{a}{x} + \frac{b}{y} = 2$

***Solution:***

$x + y = a + b \;....(i)$

$\frac{a}{x} + \frac{b}{y} = 2 \quad ....(ii)$

*From* $(1) \quad y = a + b - x \quad ...(3)$

***Putting value of y in eq.*** $(2)$

$a(a + b - x) + bx = 2x(a + b - x)$

$a^2 + ab - ax + bx = 2ax + 2bx - 2x^3$

$2x^3 - 2ax - 2bx - ax + bx + a^2 + ab = 0$

$2x^3 - 3ax - bx + a^2 + ab = 0$

***Using*** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-3a+b) \pm \sqrt{(-3a+b)^2 - 4(2)(a^2+b^2)}}{2(2)}$

$x = \frac{(3a+b) \pm \sqrt{(3a+b)^2 - 8(a^2+b^2)}}{4}$

$x = \frac{(3a+b) \pm \sqrt{9a^2+b^2+6ab-8a^2-8ab}}{4}$

$x = \frac{(3a+b) \pm \sqrt{a^2+b^2-2ab}}{4}$

$x = \frac{(3a+b) \pm \sqrt{(a-b)^2}}{4}$

$x = \frac{(3a+b) \pm (a-b)}{4}$

$x = \frac{3a+b+a-b}{4} \qquad x = \frac{3a+b-a+b}{4}$

$x = \frac{4a}{4} \;, x = \frac{3a+b-a+b}{4}$

$x = a, \; x = \frac{2a+2b}{4} = \frac{a+b}{2}$

***If*** $x = a$ ***then, from*** $(3)$

$y = a + b - a$

$\Rightarrow y = b$

***If*** $x = \frac{a+b}{2}$ ***then,***

$y = a + b - \frac{a+b}{2} = \frac{a+b}{2}$

$\Rightarrow y = \frac{5}{2}$

**S. S** $= \left\{ (a\,, b)\,, \left(\frac{a+b}{2}\,, \frac{a+b}{2}\right) \right\}$

***Question#6***

$3x + 4y = 25 \quad ; \quad \frac{3}{x} + \frac{4}{y} = 2$

***Solution:***

$3x + 4y = 25 \quad ...(1)$

$\frac{3}{x} + \frac{4}{y} = 2 \qquad ...(2)$

***From*** $(1)$

$4y = 25 - 3x$

$\Rightarrow y = \frac{25-3x}{4} \quad ....(3)$

*Putting value of y in eq.* $(2)$

$3\left(\frac{25-3x}{4}\right) + 4x = 2x\left(\frac{25-3x}{4}\right)$

*Multiplying by 4 we get*

$3(25 - 3x) + 16x = 2x(25 - 3x\,)$

$75 - 9x + 16x = 50x - 6x^2$

$75 + 7x = 50x - 6x^2$

$6x^2 - 50x + 7x + 75 = 0$

$6x^2 - 43x + 75 = 0$

$6x^2 - 18x - 25x + 75 = 0$

$6x(x - 3) - 25(x - 3) = 0$

$(x - 3)(6x - 25) = 0$

$x - 3 = 0 \qquad ,6x - 25 = 0$

$x = 3\,, \quad x = \frac{25}{6}$

***If*** $x = 3$ ***then, from*** $(3)$

$y = \frac{25-3(3)}{4} = \frac{25-9}{4} = \frac{16}{4}$

$\Rightarrow y = 4$

***If*** $x = \frac{25}{6}$ ***then,***

$y = \frac{25-3\left(\frac{25}{6}\right)}{4} = \frac{1}{4}\left(25 - \frac{25}{2}\right) = \frac{1}{4}\left(\frac{50-25}{2}\right) = \frac{1}{4}.\frac{25}{2} = \frac{25}{8}$

$\Rightarrow y = \frac{25}{8}$

**S. S** $= \left\{ (3\,,4)\,, \left(\frac{25}{2}\,, \frac{25}{8}\right) \right\}$

***Question#7***

$(x - 3)^2 + y^2 = 5\,; \; 2x = y + 6$

***Solution:***

$x^2 - 6x + 9 + 4y^2 = 5$

$x^2 + y^2 - 6x + 4 = 0 \quad ...(1)$

$2x - y = 4 \quad ...(2)$

***From*** $(1) \quad y = 2x - 6 \quad ...(3)$

***Putting value of y in eq.*** $(2)$

$x^2 + (2x - 6\,)^2 - 6x + 4 = 0$

$x^2 + 4x^2 - 24x + 36 - 6x^2 + 4 = 0$

$5x^2 - 30x + 40 = 0$

***Dividing by 5 we get***

$x^2 - 6x + 8 = 0$

$x^2 - 2x - 4x + 8 = 0$

$x(x - 2) - 4(x - 2) = 0$

$(x - 2)(x - 4) = 0$

$x - 2 = 0 \qquad ,x - 4 = 0$

$x = 2\,, \quad x = 4$

***If*** $x = 2$ ***then, from*** $(3)$

$y = 2(2) - 6$

$y = 4 - 6$

$\Rightarrow y = -2$

***If*** $x = 4$ ***then,***

$y = 2(4) - 4$

$y = 8 - 6 = 2$

$\Rightarrow y = 2$

**S. S** $= \{\,(2\,,2)\,, (4\,,2)\,\}$

***Question#8***

$(x + 3)^2 + (y - 1)^2 = 5 \quad ; \quad x^2 + y^2 + 2x = 9$

***Solution:***

$x^2 + 6x + 9 + y^2 - 2y + 1 - 5 = 0$

$x^2 + y^2 + 6x - 2y + 5 = 0 \quad ...(1)$

$x^2+y^2+2x=9$ ...(2)
or $x^2+y^2+2x-9=0$
***Subtracting eq.*** (2) ***from eq***(1)

$$x^2+y^2+6x-2y+5=0$$
$$x^2+y^2+2x \quad -9=0$$
$$- \quad - \quad - \quad \quad +$$
$$4x-2y \ +14 \ =0$$

***Dividing by*** 2 ***we get***
$2x-y \ +7 \ =0$
$y=2x \ +7 \ =0$ ...(3)
***Putting value of*** $y$ ***in eq.*** (2)
$x^2+(2x+7\ )^2+2x-9=0$ ...(1)
$x^2+4x^2+28x+49+2x-9=0$
$5x^2+30x+40=0$
***Dividing by*** 2 ***we get***
$x^2+6x+8=0$
$x^2+2x+4x+8=0$
$x(x+2)+4(x+2)=0$
$(x+2)(x+4)=0$
$x+2=0 \quad ,x+4=0$
$x=-2\,, \quad x=-4$
***If*** $x=-2$ ***then, from*** (3)
$y=2(-2)+7$
$y=-4+7$
$\Rightarrow y=3$
***If*** $x=-4$ ***then,***
$y=2(-4)+7$
$y=-8+7=-1$
$\Rightarrow y=-1$
**S. S** $=\{(-2,3),(-4,-1)\}$
***Question#9***
$\boldsymbol{x^2+(y+1)^2=18 \quad ; \quad (x+2)^2+y^2=21}$
***Solution:***
$x^2+y^2+2y+1-18=0$
$x^2+y^2+2y-17=0$ ...(1)
$x^2+y^2+4x+4-21=0$
$x^2+y^2+4x-17=0$ ...(2)
***Subtracting eq.*** (2) ***from eq***(1)

$$x^2+y^2+2y-17=0$$
$$x^2+y^2+4x-17=0$$
$$- \quad - \quad - \quad +$$
$$2y-4x \ =0$$

$2y-4x \ =0$
$\Rightarrow 2y=4x$
$\Rightarrow \boldsymbol{y=2x}$
***Putting value of*** $y$ ***in eq.*** (2)
$x^2+(2x\ )^2+4x-17=0$ ...(1)
$x^2+4x^2+4x-17=0$
$5x^2+4x-17=0$
***Using*** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$x=\frac{-(4)\pm\sqrt{(4)^2-4(5)(-17)}}{2(4)}$
$x=\frac{-4\pm\sqrt{16+340}}{8}$
$x=\frac{-4\pm\sqrt{356}}{10}$
$x=\frac{-4\pm\sqrt{4\times 89}}{10}$
$x=\frac{-4\pm 2\sqrt{89}}{10}$
$x=\frac{2(-2\pm\sqrt{89})}{10}$
$x=\frac{(-2\pm\sqrt{89})}{5}$
$x=\frac{-2+\sqrt{89}}{5} \ ,x=\frac{-2-\sqrt{89}}{5}$
***If*** $x=\frac{-2+\sqrt{89}}{5}$ ***then, from*** (3)
$y=2\left(\frac{-2+\sqrt{89}}{5}\right)$
$\Rightarrow y=\frac{-4+2\sqrt{89}}{5}$
***If*** $x=\frac{-2-\sqrt{89}}{5}$ ***then,***
$y=2\left(\frac{-2-\sqrt{89}}{5}\right)$
$\Rightarrow y=\frac{-4-2\sqrt{89}}{5}$
**S. S** $=\left\{\left(\frac{-2+\sqrt{89}}{5},\frac{-4+\sqrt{89}}{5}\right),\left(\frac{-2-\sqrt{89}}{5},\frac{-2-2\sqrt{89}}{5}\right)\right\}$
***Question#10***
$\boldsymbol{x^2+y^2+6x=1 \quad ; \quad x^2+y^2+2(x+y)=3}$
***Solution:***
$x^2+y^2+6x=1$ ...(1)
$x^2+y^2+2(x+y)=3$
or $x^2+y^2+2x+2y=3$ ...(3)
***Subtracting eq.*** (2) ***from eq***(1)

$$x^2+y^2+6x \quad =1$$
$$x^2+y^2+2x+2y=3$$
$$- \quad - \quad - \quad -$$
$$4x-2y \ =-2$$

***Dividing by*** 2 ***we get***
$4x-2y \ =-2$
$y=2x \ +1$ ...(3)
***Putting value of*** $y$ ***in eq.*** (2)
$x^2+(2x+1\ )^2+6x=0$ ...(1)
$x^2+4x^2+4x+1+6x=-1$
$5x^2+10x=0$
$5x(x+2)=0$
$5x=0 \quad ,x+2=0$
$x=0\,, \quad x=-2$
***If*** $x=0$ ***then, from*** (3)
$y=2(0)+1$
$\Rightarrow y=1$
***If*** $x=-2$ ***then,***
$y=2(-2)+1$
$y=-4+1=-3$
$\Rightarrow y=-3$
**S. S** $=\{(0,1),(-2,-3)\}$

**Case 2nd when both equations are Quadratic:**

a) **When both equations contain** $x^2$ *and* $y^2$ *terms.*

**See example:**

b) **When one of the equations is homogenous in** $x$ *and* $y$

**Homogenous Equations:**
The equation whose degree is 2 of every term is called homogenous quadratic equation:

$$e.g\ x^2-3xy+y^2=0$$

$$ax^2 + 2xy + by^2 = 0$$

See example 2, $Q4,5,6$

c) **When both equations are non-homogenous.**

$$x^2 - y^2 = 5$$

# Exercise 4.9

***Solve the following systems of Equations:***

***Question#1***

$\mathbf{2x^2 = 6 + 3y^2 \ ; \ 3x^2 - 5y^2 = 7}$

***Solution:***

$2x^2 = 6 + 3y^2$

$or \ \ 2x^2 - 3y^2 = 6$

$and \ \ 3x^2 - 5y^2 = 7$

***put*** $x^2 = u$ , $y^2 = v$ ***thus we get***

$2u - 3v = 6 \quad \ldots(1)$

$3u - 10v = 14 \quad \ldots(2)$

***To balance the coefficient of u multiplying eq. (1) by 3 and eq. (2) by 2 we get ,***

$6u - 10v = 14 \quad \ldots(3)$

$6u - 9v = 18 \quad \ldots(4)$

***Subtracting eq.*** $(3)$ ***from eq*** $(4)$

$$\begin{array}{r} 6u - 10v = 14 \\ 6u - 9v = 18 \\ - \quad + \qquad - \\ \hline -v \quad = -4 \end{array}$$

$\Rightarrow v = 4$

***Putting value of v in eq.*** $(1)$***, we get***

$2u - 3(4) = 6$

$2u - 12 = 6$

$2u = 6 + 12$

$2u = 18$

$\Rightarrow u = 9$

***If*** $u = 9$ ***then,***

$x^2 = 9$

$\Rightarrow x = \pm 3$

***If*** $v = 4$ ***then,***

$y^2 = 4$

$\Rightarrow y = \pm 2$

**S. S** $= \{ \pm 3 , \pm 2 \}$

***Question#2***

$\mathbf{8x^2 = y^2 \ ; \ x^2 + 2y^2 = 19}$

***Solution:***

$8x^2 = y^2$

$or \ \ 8x^2 - y^2 = 0$

$and \ \ x^2 + 2y^2 = 19$

***put*** $x^2 = u$ , $y^2 = v$ ***thus we get***

$8u - v = 0 \quad \ldots(1)$

$u + 2v = 19 \quad \ldots(2)$

***To balance the coefficient of v multiplying eq. (1) by 2 we get,***

$16u - 2v = 0 \quad \ldots(4)$

***Adding eq.*** $(2)$ ***and eq*** $(3)$

$$\begin{array}{r} u + 2v = 19 \\ 16u - 2v = 0 \\ \hline 17u \qquad = 19 \end{array}$$

$\Rightarrow 17u = 19$

$\Rightarrow u = \frac{19}{17}$

***Putting value of u in eq. (1), we get***

$8(\frac{19}{17}) - v = 0$

$\Rightarrow v = \frac{152}{17}$

***If*** $u = \frac{19}{17}$ ***then,***

$x^2 = \frac{19}{17}$

$\Rightarrow x = \pm\sqrt{\frac{19}{17}}$

***If*** $v = \frac{152}{17}$ ***then,***

$y^2 = \frac{152}{17}$

$y = \pm\sqrt{\frac{152}{17}}$

$\Rightarrow y = \pm\sqrt{\frac{4\times 38}{17}}$

$\Rightarrow y = \pm 2\sqrt{\frac{38}{17}}$

**S. S** $= \left\{ \pm\sqrt{\frac{19}{17}} , \pm 2\sqrt{\frac{38}{17}} \right\}$

***Question#3***

$\mathbf{2x^2 - 8 = 5y^2 \ ; \ x^2 - 13 = 2y^2}$

***Solution:***

$2x^2 - 8 = 5y^2$

$or \ \ 2x^2 - 5y^2 = 8$

$x^2 - 13 = -2y^2$

$or \ \ x^2 - 2y^2 = 13$

***put*** $x^2 = u$ , $y^2 = v$ ***thus we get***

$2u - 5v = 8 \quad \ldots(1)$

$u + 2v = 13 \quad \ldots(2)$

***To balance the coefficient of u multiplying eq. (2) by 2 we get ,***

$2u + 4v = 26 \quad \ldots(3)$

***Subtracting eq.*** $(1)$ ***from eq*** $(3)$

$$\begin{array}{r} 2u + 4v = 26 \\ 2u - 5v = \ \ 8 \\ - \quad + \qquad - \\ \hline +9v \ = 18 \end{array}$$

$\Rightarrow v = 2$

***Putting value of v in eq.*** $(2)$***, we get***

$u + 2(2) = 13$

$u + 4 = 13$

$u = 13 - 4$

$\Rightarrow u = 9$

***If*** $u = 9$ ***then,***

$x^2 = 9$

$\Rightarrow x = \pm 3$

***If*** $v = 2$ ***then,***

$y^2 = 2$

$\Rightarrow y = \pm\sqrt{2}$

**S. S** $= \{\ \pm 3, \pm\sqrt{2}\}$

***Question#4***

$x^2 - 5xy + 6y^2 = 0\ ;\ x^2 + y^2 = 45$

***Solution:***

$x^2 - 5xy + 6y^2 = 0 \quad \dots(1)$

$x^2 + y^2 = 45 \quad \dots(2)$

***Factorizing homogeneous equation we get***

$x^2 - 2xy - 3xy + 6y^2 = 0$

$x(x - 2y) - 3y(x - 2y) = 0$

$(x - 2y)(x - 3y) = 0$

$x - 2y = 0\ ,\ x - 3y = 0$

$x = 2y\ ,\ x = 3y$

***If*** $x = 2y$ ***then, from*** (2) ***If*** $x = 3y$ ***then, from*** (2)

$(2y)^2 + y^2 = 45 \qquad (3y)^2 + y^2 = 45$

$4y^2 + y^2 = 45 \qquad 9y^2 + y^2 = 45$

$5y^2 = 45 \qquad 10y^2 = 45$

$y^2 = 9 \qquad y^2 = \frac{45}{10}$

$\Rightarrow y = \pm 3 \qquad y^2 = \frac{9}{2} \Rightarrow y = \pm\frac{3}{\sqrt{2}}$

***If*** $y = 3$ ***then,*** $\qquad$ ***If*** $y = \frac{3}{\sqrt{2}}$ ***then,***

$x = 2(3) = 6 \qquad x = 3\left(\frac{3}{\sqrt{2}}\right) = \frac{9}{\sqrt{2}}$

***If*** $y = -3$ ***then,*** $\qquad$ ***If*** $y = -\frac{3}{\sqrt{2}}$ ***then,***

$x = 2(-3) = -6 \qquad x = 3\left(-\frac{3}{\sqrt{2}}\right) = -\frac{9}{\sqrt{2}}$

**S. S** $= \left\{\ (6,3), (6,-3), \left(\frac{9}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right), \left(\frac{-9}{\sqrt{2}}, \frac{-3}{\sqrt{2}}\right)\right\}$

***Question#5***

$12x^2 - 25xy + 12y^2 = 0\ ;\ 4x^2 + 7y^2 = 148$

***Solution:***

$12x^2 - 25xy + 12y^2 = 0 \quad \dots(1)$

$4x^2 + 7y^2 = 148 \quad \dots(2)$

***Factorizing homogeneous equation we get***

$12x^2 - 16xy - 9xy + 12y^2 = 0$

$4x(3x - 4y) - 3y(3x - 4y) = 0$

$(3x - 4y)(4x - 3y) = 0$

$3x - 4y = 0\ ,\ 4x - 3y = 0$

$x = \frac{4y}{3}\ ,\ x = \frac{3y}{4}$

***If*** $x = \frac{4y}{3}$ ***then, from(2) If*** $x = \frac{3y}{4}$ ***then, from*** (2)

$4\left(\frac{4y}{3}\right)^2 + 7y^2 = 148 \qquad 4\left(\frac{3y}{4}\right)^2 + 7y^2 = 148$

$\frac{64}{9}y^2 + 7y^2 = 148 \qquad \frac{9}{4}y^2 + 7y^2 = 148$

$64y^2 + 63y^2 = 1332 \qquad 9y^2 + 28y^2 = 592$

$127y^2 = 1332 \qquad 37y^2 = 592$

$y^2 = \frac{1332}{127} \qquad y^2 = \frac{592}{37} = 16$

$y = \pm\sqrt{\frac{1332}{127}} \qquad y = \pm 4$

$y = \pm\sqrt{\frac{4\times 333}{127}} \qquad$ ***If*** $y = 4$ ***then***

$\Rightarrow y = \pm 2\sqrt{\frac{333}{127}} \qquad x = \frac{3}{4}(4)$

***If*** $y = 2\sqrt{\frac{333}{127}}$ ***then,*** $\qquad x = 3$

$x = \frac{4}{3}\left(2\sqrt{\frac{333}{127}}\right) \qquad$ ***If*** $y = -4$ ***then***

$= \frac{8}{3}\sqrt{\frac{333}{127}} \qquad x = \frac{3}{4}(4)$

***If*** $y = -2\sqrt{\frac{333}{127}}$ ***then,*** $\qquad x = -3$

$x = \frac{4}{3}\left(-2\sqrt{\frac{333}{127}}\right)$

$= -\frac{8}{3}\sqrt{\frac{333}{127}}$

**S. S** $= \left\{\left(\frac{8}{3}\sqrt{\frac{333}{127}}, 3\right), (6,-3), \left(\frac{9}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right), \left(\frac{-9}{\sqrt{2}}, \frac{-3}{\sqrt{2}}\right)\right\}$

***Question#6***

$12x^2 - 11xy + 2y^2 = 0\ ;\ 2x^2 + 7xy = 60$

***Solution:***

$12x^2 - 11xy + 2y^2 = 0 \quad \dots(1)$

$2x^2 + 7xy = 60 \quad \dots(2)$

***Factorizing homogeneous equation, we get***

$12x^2 - 8xy - 3xy + 2y^2 = 0$

$4x(3x - 2y) - y(3x - 2y) = 0$

$(3x - 2y)(4x - y) = 0$

$3x - 2y = 0\ ,\ 4x - y = 0$

$3x = 2y\ ,\ 4x = y$

$x = \frac{2y}{3}\ ,\ x = \frac{y}{4}$

***If*** $x = \frac{2y}{3}$ ***then, from*** (2) ***If*** $x = \frac{y}{4}$ ***then, from*** (2)

$2\left(\frac{2y}{3}\right)^2 + 7\left(\frac{2}{3}y\right)y = 60 \qquad 2\left(\frac{y}{4}\right)^2 + 7\left(\frac{1}{4}y\right)y = 60$

$\frac{8}{9}y^2 + \frac{14}{3}y^2 = 60 \qquad \frac{1}{8}y^2 + \frac{7}{4}y^2 = 480$

$8y^2 + 42y^2 = 540 \qquad 15y^2 = 480$

$50y^2 = 540 \qquad y^2 = \frac{480}{15} = 32$

$y^2 = \frac{540}{50} = \frac{54}{5} \qquad y^2 = \pm\sqrt{32}$

$y = \pm\sqrt{\frac{9\times 6}{5}} \qquad y = \pm\sqrt{16\times 2} = \pm 4\sqrt{2}$

$y = \pm 3\sqrt{\frac{6}{5}} \qquad$ ***If*** $y = 4\sqrt{2}$ ***then***

***If*** $y = 3\sqrt{\frac{6}{5}}$ ***then,*** $\qquad x = \frac{1}{4}(4\sqrt{2}) = \sqrt{2}$

$x = \frac{2}{3}\left(3\sqrt{\frac{6}{5}}\right) \qquad$ ***If*** $y = -4\sqrt{2}$ ***then***

$= 2\sqrt{\frac{6}{5}} \qquad x = \frac{1}{4}(-4\sqrt{2})$

***If*** $y = -3\sqrt{\frac{6}{5}}$ ***then,*** $\qquad x = -\sqrt{2}$

$x = \frac{2}{3}\left(-3\sqrt{\frac{6}{5}}\right)$

$= -2\sqrt{\frac{6}{5}}$

**S. S** $=$

$\left\{ \left(2\sqrt{\frac{6}{5}}, 3\sqrt{\frac{6}{5}}\right), \left(-2\sqrt{\frac{6}{5}}, -3\sqrt{\frac{6}{5}}\right), (\sqrt{2}, 4\sqrt{2}), (-\sqrt{2}, -4\sqrt{2}) \right\}$

***Question#7***

$x^2 - y^2 = 16$ ; $xy = 15$

***Solution:***

$x^2 - y^2 = 16$ ... (1)

$xy = 15$ ... (2)

***To balance constant terms multiply eq. (1) by 15 and eq. (2) by 16 we get,***

$15x^2 - 15y^2 = 240$ ... (3)

$xy = 240$ ... (4)

***Subtracting eq.*** (4) ***from eq***(3)

$15x^2 - 15y^2 - 16xy = 0$

$15x^2 - 16xy - 15y^2 = 0$

***Factorizing***

$15x^2 - 25xy + 9xy - 15y^2 = 0$

$5x(3x - 5y) + 3y(3x - 5y) = 0$

$(3x - 5y)(5x + 3y) = 0$

$3x - 5y = 0$ , $5x + 3y = 0$

$3x = 5y$ , $5x = -3y$

$x = \frac{5y}{3}$ , $x = \frac{-3y}{5}$

***If*** $x = \frac{5y}{3}$ ***then, from*** (2)

$\left(\frac{5y}{3}\right)^2 = 15$

$y^2 = 15 \times \frac{3}{5}$

$y^2 = 9$

$y = \pm 3$

***If*** $y = 3$ ***then,***

$x = \frac{5}{3}(3)$

$x = 5$

***If*** $y = -3$ ***then,***

$x = \frac{5}{3}(-3) = -5$

***If*** $x = \frac{-3y}{5}$ ***then, from*** (2)

$\left(\frac{-3}{5}y\right)y = 15$

$\frac{-3}{5}y^2 = 15$

$y^2 = 15\left(\frac{-5}{3}\right) = \pm\sqrt{25}$

$y = \pm 5i$

***If*** $y = 5i$ ***then***

$x = \frac{-3(5i)}{5}$

$x = \frac{-15i}{3} = -3i$

***If*** $y = 5i$ ***then***

$x = \frac{-3(-5i)}{5} = \frac{15i}{3} = 3i$

**S. S** $= \{ (5, 3), (-5, -3), (-3i, 5i), (3i, -5i) \}$

***Question#8***

$x^2 + xy = 9$ ; $x^2 - y^2 = 2$

***Solution:***

$x^2 + xy = 9$ ... (1)

$x^2 - y^2 = 2$ ... (2)

***To balance constant terms multiply eq. (1) by 15 and eq. (2) by 16 we get,***

$2x^2 + 2xy = 18$ ... (3)

$9x^2 - 9y^2 = 18$ ... (4)

***Subtracting eq.*** (1) ***from eq*** (3)

$9x^2 - 9y^2 = 18$

$2x^2 + 2xy = 18$

$- \quad - \quad -$

$7x^2 - 9y^2 - 2xy = 0$

$7x^2 - 2xy - 9y^2 = 0$

$7x^2 + 7xy - 9xy - 9y^2 = 0$

$7x(x + y) - 9y(x + y) = 0$

$(x + y)(7x - 9y) = 0$

$x + y = 0$ , $7x - 9y = 0$

$x = -y$ , $7x = 9y$

$x = -y$ , $x = \frac{9y}{7}$

***If*** $x = -y$ ***then, from*** (2)

$(-y)^2 - y^2 = 2$

$y^2 - y^2 = 2$

$0 = 2$

*Which is impossible*

***If*** $x = \frac{9y}{7}$ ***then, from*** (2)

$\left(\frac{9y}{7}\right)^2 - y^2 = 2$

$\frac{81}{49}y^2 - y^2 = 2$

$81y^2 - 49y^2 = 98$

$32y^2 = 98$

$y^2 = \frac{98}{32} = \frac{49}{16}$

$y = \pm\frac{7}{4}$

***If*** $y = \frac{7}{4}$ ***then***

$x = \frac{9}{7}\left(\frac{7}{4}\right) = \frac{9}{4}$

***If*** $y = -\frac{7}{4}$ ***then***

$x = \frac{9}{7}\left(-\frac{7}{4}\right) = -\frac{9}{4}$



**S. S** $= \left\{ \left(-\frac{9}{4}, -\frac{7}{4}\right), \left(\frac{9}{4}, \frac{7}{4}\right) \right\}$

***Question#9***

$y^2 - 7 = 2xy$ ; $2x^2 + 3 = xy$

***Solution:***

$y^2 - 7 = 2xy$ or $y^2 - 2xy = 7$ ... (1)

$2x^2 + 3 = xy$ or $2x^2 - xy = -3$ ... (2)

***To balance constant terms, multiply eq. (1) by 3 and eq. (2) by 7 we get,***

$3y^2 - 6xy = 21$ ... (3)

$14x^2 - 7xy = -21$ ... (4)

***Adding eq.*** (3) ***and eq*** (4)

$3y^2 - 6xy = 21$

$14x^2 - 7xy = -21$

$14x^2 + 3y^2 - 13xy = 0$

$14x^2 - 13xy + 3y^2 = 0$

$14x^2 - 7xy - 6xy + 3y^2 = 0$

$7x(2x - y) - 3y(2x - y) = 0$

$(2x - y)(7x - 3y) = 0$

$2x - y = 0$ , $7x - 3y = 0$

$x = \frac{y}{2}$ , $x = \frac{3y}{7}$

***If*** $x = \frac{y}{2}$ ***then, from*** (2)

$y^2 - 2\left(\frac{1y}{2}\right)y = 7$

$y^2 - y^2 = 7$

$0 = 7$

***If*** $x = \frac{3y}{7}$ ***then, from*** (2)

$y^2 - 2\left(\frac{3y}{7}\right)y = 7$

$y^2 - \frac{6}{7}y^2 = 7$

$7y^2 - 6y^2 = 49$

***Which is impossible***

$y^2 = 49$

$y = \pm 7$

***If*** $y = 7$

***then,***

$x = \frac{3}{7}(7) = 3$

***If*** $y = -7$ ***then***

$x = \frac{3}{7}(-7) = -3$

**S. S** $= \{ (3,7), (-3,-7)\}$

***Question#10***

$x^2 + y^2 = 5 \; ; \; xy = 2$

***Solution:***

$x^2 + y^2 = 5 \quad \dots(1)$

$xy = 2 \quad \dots(2)$

***To balance constant terms multiply eq.*** $(1)$ ***by 2 and eq.*** $(2)$ ***by 5 we get,***

$2x^2 + 2y^2 = 10 \quad \dots(3)$

$5xy = 10 \quad \dots(4)$

***Subtracting eq.*** $(1)$ ***from eq*** $(3)$

$2x^2 + 2y^2 = 10$

$5xy = 10$

$- \quad -$

$2x^2 + 2y^2 - 5xy = 0$

$2x^2 - 5xy + 2y^2 = 0$

$2x^2 - xy - 4xy + 2y^2 = 0$

$x(x - 2y) - 2y(x - 2y) = 0$

$(x - 2y)(2x - y) = 0$

$x - 2y = 0 \;,\; 2x - y = 0$

$x = 2y \;,\; x = \frac{1}{2}y$

***If*** $x = 2y$ ***then, from*** $(2)$

$(2y)y = 2$

$2y^2 = 2$

$y^2 = 1$

$y = \pm 1$

***then,***

***If*** $y = 1$ ***then,***

$x = 2(1)$

$x = 2$

***then,***

***If*** $y = 2$ ***then,***

$x = 2(-1) = -2$

***If*** $x = \frac{1}{2}y$ ***then, from*** $(2)$

$\left(\frac{1}{2}y\right)y = 2$

$y^2 = 4$

$y = \pm 2$

***If*** $y = 2$

$x = \frac{1}{2}(2)$

$x = 1$

***If*** $y = -2$

$x = \frac{1}{2}(-2)$

$x = -1$

**S. S** $= \{ (2, \mathbf{1}), (-2, -\mathbf{1}), (\mathbf{1}, 2), (-\mathbf{1}, -2)\}$

**Problems on Quadratic Equation:**

Remember following steps to solve problems expressed symbolically, lead to quadratic equations in one or two variables.

1. Suppose the unknown quantities $x$ *or* $y$ *etc.*
2. Translate the problems into symbolically
   $5$ *is greater then* $3$ *by* $2 = 5 - 3$
3. $x$ *is greater than* $3$ *by* $x - 3$
4. $5$ *is greater ten by* $5 - y$
5. *Xxis greater than* $y$ *by* $x - y$

# Exercise 4.10

***Question#1***

***The product of one less than a certain positive number and two less than three times the number is 14. Find the number.***

***Solution:***

*Let* $x$ *be a certain positive number , then one less than* $x$ *means* $x - 1$

*two less than* $3x$ *means* $3x - 2.$

***Now according to the given condition***

$(\textit{one less than } x\,)(\textit{two less than three times } x) = 14$

*i.e.*

$(x - 1)(3x - 2) = 14$

$3x^2 - 2x - 3x + 2 - 14 = 0$

$3x^2 - 5x - 12 = 0$

$3x^2 - 9x + 4x - 12 = 0$

$3x(x - 3) + 4(x - 3) = 0$

$(x - 3)(3x + 4) = 0$

$x - 3 = 0 \Rightarrow x = 3 \;,$

$3x + 4 = 0 \Rightarrow x = \frac{-4}{3}$ ***(impossible being negative)***

***Hence*** $x = 3$ ***is required positive number.***

***Question#2***

***The sum of a positive number and its square is 380. Find the number.***

***Solution:***

*Let* $x$ *be a certain positive number , then its square will be* $x^2$

***Now according to the given condition***

$x^2 + x = 380$

$x^2 + x - 380 = 0$

$x^2 + 20x - 19x - 380 = 0$

$x(x + 20) - 19(x + 20) = 0$

$(x - 19)(x + 20) = 0$

$x - 19 = 0 \Rightarrow x = 19 \;,$

$x + 20 = 0 \Rightarrow x = -20$ ***(impossible being negative)***

***Hence*** $x = 19$ ***is required positive number.***

***Question#3***

***Divide 40 into two parts such that the sum of their squares is greater than 2 times their product by 100.***

***Solution:***

*Let* $x$ *be one part then other part will be* $40 - x$

*Sum of squares of parts* $= x^2 + (40 - x)^2$

*Product of the parts* $= x\,(40 - x)$

***Now according to the given condition***

$\left[x^2+(40-x)^2\right]-2[x(40-x)]=100$
$x^2+1600x-80x+x^2-80x+2x^2=100$
$x^2+1600x-80x+x^2-80x+2x^2-100=0$
$4x^2-160x+1500=0$
*Dividing by 4*
$x^2-40x+375=0$
$x^2-25x-15x+375=0$
$x(x-25)-15(x-25)=0$
$(x-25)(x-15)=0$
$x-15=0 \Rightarrow x=15$
$x-25=0 \Rightarrow x=25$
*If one part is 15 then the other part* $=40-15=25$
*If one part is 25 then the other part* $=40-25=15$

**Question#4**

**The sum of a positive number and its reciprocal is $\frac{26}{5}$ .Find the number.**

**Solution:**

*Let* $x$ *be a certain positive number ,*
**Now according to the given condition**
$x+\frac{1}{x}=\frac{26}{5}$
**Multiply by $5x$ we get,**
$5x^2+5=26x$
$5x^2-26x+5=0$
$5x^2-25x-x+5=0$
$5x(x-5)-1(x-5)=0$
$(x-5)(5x-1)=0$
$x-5=0 \Rightarrow x=5$ ,
$5x-1=0 \Rightarrow x=\frac{1}{5}$
**Hence $x=5$ and $x=\frac{1}{5}$ are required positive numbers.**

**Question#5**

**A number exceeds its square root by 56. Find the number.**

**Solution:**

*Let* $x$ *be the number , then its square root* $=\sqrt{x}$
$x=\sqrt{x}+56$
$x-56=\sqrt{x}$
*Squaring both sides*
$(x-56)^2=(\sqrt{x})^2$
$x^2-112x+3136=x$
$x^2-112x-x+3136=0$
$x^2-113x+3136=0$
$x^2-64x-49x+3136=0$
$x(x-64)-49(x-64)=0$
$(x-64)(x-49)=0$
$x-64=0 \Rightarrow x=64$
$x-49=0 \Rightarrow x=49$
$x=49$ *does not satisfy the given condition*
**Hence $x=64$ is required number**

**Question#6**

**Find two consecutive numbers, whose product is 132.**
**(Hint: Suppose the numbers are $x$ and $x+1$).**

**Solution:**

*Let* $x$ *and* $x+1$ *be two consecutive numbers.*
**Then according to the given condition**
$x(x+1)=132$
$x^2+x-132=0$
$x^2+12x-11x-132=0$
$x(x+12)-11(x+12)=0$
$(x-11)(x+12)=0$
$x-11=0 \Rightarrow x=11$ ,
$x+12=0 \Rightarrow x=-12$
**If $x=11$ then,**
$x+1=11+1=12$
**If $x=-12$ then,**
$x+1=-12+1=11$
**Hence two consecutive numbers are 11 , 12 , or − 11 , −12**

**Question#7**

**The difference between the cubes of two consecutive even numbers is 296. Find them.**
**(Hint: Let two consecutive even numbers be x and x + 2)**

**Solution:**

*Let* $x$ *and* $x+2$ *be two consecutive numbers.*
**Then according to the given condition**
$(x+2)^3-x^3=296$
$x^3+8+3(x^2)(2)+3(x)(2)^2-x^3-296=0$
$6x^2+12x-288=0$
*Dividing by 6 we get*
$x^2+2x-48=0$
$x^2+8x-6x-48=0$
$x(x+8)-6(x+8)=0$
$(x-6)(x+8)=0$
$x-6=0 \Rightarrow x=6$ ,
$x+8=0 \Rightarrow x=-8$
**If $x=6$ then,**
$x+2=6+2=8$
**If $x=-8$ then,**
$x+2=-8+2=-6$
**Hence two consecutive numbers are 6 , 8 , or − 6 , −8**

**Question#8**

**A farmer bought some sheep for Rs. 9000. If he had paid Rs. 100 less for each, he would have got 3 sheep more for the same money. How many sheep did he buy, when the rate in each case is uniform?**

**Solution:**

*Let* $x$ *be the number of sheep*
*Amount for* $x$ *sheep* $=9000$
*Amount for 1 sheep* $=\frac{9000}{x}$

*Amount for* $x+3$ *sheep* $= \frac{9000}{x+3}$

***according to the given condition***

$\frac{9000}{x} - 100 = \frac{9000}{x+3}$

***Multiply by*** $\boldsymbol{x(x+3)}$ ***we get,***

$x(x+3)\,.\frac{9000}{x} - 100.x(x+3) = x(x+3)\,.\frac{9000}{x+3}$

$9000(x+3) - 100x(x+3) = 9000x$

*Dividing by 100 we get*

$90(x+3) - x(x+3) = 90x$

$90x + 270 - x^2 - 3x = 90x$

$0 = x^2 + 3x + 90x - 90x - 270$

$x^2 + 3x - 270 = 0$

$x^2 + 18x - 15x - 270 = 0$

$x(x+18) - 15(x+18) = 0$

$(x-15)(x+18) = 0$

$x - 15 = 0 \Rightarrow x = 15$ ,

$x + 18 = 0 \Rightarrow x = -18$ *(impossible)*

***Hence*** $\boldsymbol{x = 15}$ ***is required number of sheep.***

***<u>Question#9</u>***

***A man sold his stock of eggs for Rs. 240. If he had 2 dozen more, he would have got the same money by selling the whole for Rs. 0.50 per dozen cheaper. How many dozen eggs did he sell?***

***<u>Solution:</u>***

*Let total dozen eggs to be sold* $= x$

*Amount for* $x$ *dozen eggs* $= 240$

*Amount for 1 dozen egg* $= \frac{240}{x}$

*Amount for* $x+2$ *dozen eggs* $= \frac{240}{x+2}$

***according to the given condition***

$\frac{240}{x} - 0.50 = \frac{240}{x+2}$

***Multiply by*** $\boldsymbol{x(x+2)}$ ***we get,***

$\boldsymbol{x(x+2)}\,.\frac{240}{x} - \boldsymbol{x(x+2)}\,.0.50 = \boldsymbol{x(x+2)}\,.\frac{240}{x+2}$

$\boldsymbol{(x+2)}\frac{240}{x} - 0.50\boldsymbol{x(x+2)} = 240x$

$240x + 480 - 0.50x^2 - x = 240x$

$-0.50x^2 - x + 480 = 0$

$-0.50x^2 - x - 480 = 0$

***Multiply by*** $2$ ***we get,***

$x^2 + 2x - 960 = 0$

$x^2 + 32x - 30x - 960 = 0$

$x(x+32) - 30(x+32) = 0$

$(x-30)(x+32) = 0$

$x - 30 = 0 \Rightarrow x = 30$ ,

$x + 32 = 0 \Rightarrow x = -32$ *(impossible)*

***Hence*** $\boldsymbol{x = 30}$ *dozen eggs were sold by the stockist.*

***<u>Question#10</u>***

***A cyclist travelled 48 km at a uniform speed. Had he travelled 2 km/hour slower, he would have taken 2 hours more to perform the journey. How long did he take to cover 48 km?***

***<u>Solution:</u>***

*Let speed to cover 48km* $= x$

*Time to cover 48km* $= t$

*As Distance* $=$ *Speed* $\times$ *time*

*So,*

$48 = xt$ *or* $xt = 48 \ldots (1)$

*Now speed to cover 48km for travelling 2 km/hr. slower* $= x - 2$

*Distance* $=$ *Speed* $\times$ *time*

$48 = (x-2)(t+2)$

$48 = xt + 2x - 2t - 4$

$48 = 48 + 2x - 2t - 4$

$2x - 2t - 4 = 0$

$x - t - 2 = 0$

$x = t + 2$

***Putting value of*** $\boldsymbol{x}$ ***in eq.(1), we get***

$(t+2)t = 48$

$t^2 + 2t = 48$

$t^2 + 2t - 48 = 0$

$t^2 - 6t + 8t - 48 = 0$

$t(t-6) + 8(t-6) = 0$

$(t-6)(t+8) = 0$

$t - 6 = 0 \Rightarrow t = 6$ ,

$t + 8 = 0 \Rightarrow t = -8$ ***(impossible)***

***So,*** $\boldsymbol{t = 6}$ ***is the required time.***

***<u>Question#11</u>***

***The area of a rectangular field is 297 square meters. Had it been 3 meters longer and one meter shorter, the area would have been 3 square meters more. Find its length and breadth.***

***<u>Solution:</u>***

*Let length of original rectangle* $= x$

*Width of original rectangle* $= y$

$\because$ *Area* $=$ *Length* $\times$ *Width*

*So,*

$297 = xy \quad \ldots (1)$

*After changing the length and width*

*Now, length of new rectangle* $= (x+3)(y-1)$

*But given that area* $= 297 + 3 = 300$

*So,*

$300 = (x+3)(y-1)$

$300 = xy - x + 3y - 3$

$300 = 297 - x + 3y - 3$

$300 - 294 + x - 3y = 0$

$x = 3y - 6 \quad \ldots .(2)$

***Putting value of*** $\boldsymbol{x}$ ***in eq.(1), we get***

$297 = (3y-6)y$

$3y^2 - 6y = 297$

$y^2 - 2y - 99 = 0$

$y^2 + 9y - 11y - 99 = 0$

$y(y+9) - 11(y+9) = 0$

$(y+9)(y-11) = 0$

$y + 9 = 0 \Rightarrow y = -9$ ***(impossible)***

$y - 11 = 0 \Rightarrow y = 11$

***If*** $\boldsymbol{y = 11}$ ***then, from eq.*** $(2)$

$x = 3(11) - 6 = 33 - 6 = 27$

**So,**

*length of original rectangle* $= x = 27m$

*Width of original rectangle* $= y = 11m$

**Question#12**

**The length of a rectangular piece of paper exceeds its breadth by 5 cm. If a strip 0.5 cm wide be cut all around the piece of paper, the area of the remaining part would be 500 square cms. Find its original dimensions.**

**Solution:**

*Let breadth(width ) of original rectangle* $= x$

*length of original rectangle* $= x + 5$

*After cutting a strip of 0.5 cm from all around*

*Change in breadth* $= x - 2(0.5) = x - 1$

*Change in length* $= x + 5 - 2(0.5) = x + 4$

$\because$ *Area = Length × Width*

*Now, breadth of new rectangle* $= x - 1$

*length of new rectangle* $= x + 4$

$\because$ *Area = Length × Width*

*So,*

$\because Area = (x + 4)(x - 1)$

*But Area* $= 500cm^2$

$500 = (x + 3)(x - 1)$

$x^2 - x + 4x - 4 = 500$

$x^2 + 3x - 504 = 0$

$x^2 + 24x - 21x - 504 = 0$

$x(x + 24) - 21(x + 24) = 0$

$(x - 21)(x + 24) = 0$

$x - 21 = 0 \Rightarrow x = 21$ ,

$x + 24 = 0 \Rightarrow x = -24$ *(impossible)*

**If $x = 21$ then,**

$x + 5 = 21 + 5 = 26$

**So,**

*length of original rectangle* $= x = 26cm$

*Width of original rectangle* $= y = 21cm$

**Question#13**

**A number consists of two digits whose product is 18. If the digits are interchanged, the new number becomes 27 less than the original number. Find the number.**

**Solution:**

*Let unit digit* $= x$

*Tens digit* $= y$

**According to the given condition**

$xy = 18 \quad \dots(1)$

$x + 10y - 27 = y + 10x$

$x + 10y - 27 - y - 10x = 0$

$9y - 9x - 27 = 0$

$y = x + 3 \qquad \dots(2)$

**Putting value of $y$ in eq.(1), we get**

$x(x + 3) = 18$

$x^2 + 3x - 18 = 0$

$x^2 - 3x + 6x - 18 = 0$

$x(x - 3) + 6(x - 3) = 0$

$(x - 3)(x + 6) = 0$

$x - 3 = 0 \Rightarrow x = 3$ ,

$x + 6 = 0 \Rightarrow x = -6$

**If $x = 3$ then, from (2)**

$y = 3 + 3 = 6$

*Then number* $= x + 10y = 3 + 10(6)$

$= 3 + 60 = 63$

**If $x = -6$ then, from (2)**

$y = -6 + 3 = -3$

*Then number* $= x + 10y = -6 + 10(-3)$

$= -6 - 30 = -36$

**Hence required number is 63 or −36**

**Question#14**

**A number consists of two digits whose product is 14. If the digits are interchanged, the resulting number will exceed the original number by 45. Find the number.**

**Solution:**

*Let unit digit* $= x$

*Tens digit* $= y$

**According to the given condition**

$xy = 14 \quad \dots(1)$

$x + 10y + 45 = y + 10x$

$x + 10y + 45 - y - 10x = 0$

$9y - 9x + 45 = 0$

$y = x - 3 \qquad \dots(2)$

**Putting value of $y$ in eq.(1), we get**

$x(x - 5) = 14$

$x^2 - 5x - 14 = 0$

$x^2 + 2x - 7x - 14 = 0$

$x(x + 2) - 7(x + 2) = 0$

$(x + 2)(x - 7) = 0$

$x + 2 = 0 \Rightarrow x = -2$ ,

$x - 7 = 0 \Rightarrow x = 7$

**If $x = 7$ then, from (2)**

$y = 7 - 5 = 2$

*Then number* $= x + 10y = 7 + 10(2)$

$= 7 + 20 = 27$

**If $x = -2$ then, from (2)**

$y = -2 - 5 = -7$

*Then number* $= x + 10y =$

$-2 + 10(-7) = -2 - 70 = -72$

**Hence required number is 27 or −72**

**Question#15**

**The area of a right triangle is 210 square meters. If its hypotenuse is 37 meters long. Find the length of the base and the altitude.**

**Solution:**

*Given that in right triangle*

*Area* $= 210m^2$

*Hypotenuse* $= 37$

*Let Base* $= x$ *, Perpendicular* $= y$

**We know that**

*Area of triangle* $= \frac{1}{2}(Base)(Altitude)$

$210 = \frac{1}{2}(x)(y)$

$xy = 420 \Rightarrow 2xy = 840 \quad \ldots(1)$

**By Pythagoras theorem**

$(\boldsymbol{Hyp})^2 = (\boldsymbol{Base})^2 + (\boldsymbol{Prep})^2$

$\boldsymbol{Hyp} = \sqrt{(\boldsymbol{Base})^2 + (\boldsymbol{Prep})^2}$

**Putting values, we get ,**

$37 = \sqrt{x^2 + y^2}$

$x^2 + y^2 = (37)^2$

$x^2 + y^2 = 1369 \ \ldots(2)$

**Subtracting eq. (1) from eq. (2)**

$x^2 + y^2 - 2xy = 1369 - 840$

$x^2 + y^2 - 2xy = 529$

$(x - y)^2 = (23)^2$

$x - y = 23$

$x = y + 23$

**Putting value of $x$ in eq.(1), we get**

$2(y + 23)y = 840$

$y^2 + 23y = 420$

$y^2 + 23y - 420 = 0$

$y^2 - 12y + 35y - 420 = 0$

$y(y - 12) + 35(y - 12) = 0$

$(y + 35)(y - 12) = 0$

$y + 35 = 0 \Rightarrow y = -35$ ***(impossible)***

$y - 12 = 0 \Rightarrow y = 12$

**If $y = 12$ then, from (3)**

$x = 12 + 23 = 35$

**so,**

**Base** $= 35m$

**Perpendicular** $= 12m$

***Question#16***

***The area of a rectangle is 1680 square meters. If its diagonal is 58 meters long, find the length and the breadth of the rectangle. 17.***

***Solution:***

*Let length of rectangle* $= x$

*Width of rectangle* $= y$

*diagonal of rectangle* $= z$

*So,*

$1680 = xy \quad \ldots(1)$

*Given that*

$z = 58$

**By Pythagoras theorem**

$(Hyp)^2 = (Base)^2 + (Prep)^2$

**By the figure**

$z^2 = x^2 + y^2$

$(58)^2 = x^2 + y^2$

$x^2 + y^2 = 3364 \quad \ldots(2)$

**From (1)**

$xy = 1680 \Rightarrow 2xy = 3360 \quad \ldots(3)$

*Subtracting eq. (3) from eq. (2)*

$x^2 + y^2 - 2xy = 3364 - 3360$

$x^2 + y^2 - 2xy = 4$

$(x - y)^2 = (2)^2$

$x - y = 22$

$x = y + 2$

**Putting value of $x$ in eq.(1), we get**

$(y + 2)y = 1680$

$y^2 + 2y - 1680 = 0$

$y^2 + 42y - 40y - 1680 = 0$

$y(y + 42) - 40(y + 42) = 0$

$(y + 42)(y - 40) = 0$

$y + 42 = 0 \Rightarrow y = -42$ ***(impossible)***

$y - 40 = 0 \Rightarrow y = 40$

**If $y = 40$ then, from (4)**

$x = 40 + 2 = 42$

*so,*

*Let length of rectangle* $= 42m$

*Width of rectangle* $= 40m$

***Question#17***

***To do a piece of work, A takes 10 days more than B. Together they finish the work in 12 days. How long would B take to finish it alone?***

***Hint: If someone takes x days to finish a work. The one day's work will be $\frac{1}{x}$ .***

***Solution:***

*Let B can do work in days* $= x$

*Work done by B in one day* $= \frac{1}{x}$

*A can do work in days* $= x + 10$

*Work done by A in one day* $= \frac{1}{x+10}$

*Work done by both A and B in one day* $= \frac{1}{x} + \frac{1}{x+10}$

**Given that,**

*Work done by both A and B in one day* $= \frac{1}{12}$

**So,**

$\frac{1}{x} + \frac{1}{x+10} = \frac{1}{12}$

**Multiplying by $12(x + 10)$**

$12(x + 10) . \frac{1}{x} + 12(x + 10) . \frac{1}{x+10} = \frac{1}{12} . 12(x + 10)$

$12(x + 10) + 12x = x(x + 10)$

$12x + 120 + 12x = x(x + 10)$

$12x + 120 + 12x = x^2 + 10x$

$x^2 + 10x - 24x - 120 = 0$

$x^2 - 14x - 120 = 0$

$x^2 - 20x + 6x - 120 = 0$

$x(x - 20) + 6(x - 20) = 0$

$(x - 20)(x + 6) = 0$

$x - 20 = 0 \Rightarrow x = 20$ ,

$x + 6 = 0 \Rightarrow x = -6$ *(impossible)*

***Hence B can finish his work alone in 20 days.***

***Question#18***

***To complete a job, A and B take 4 days working together. An alone takes twice as long as B alone to finish the same job. How***

***long would each one alone takes to do the job?***

***Solution:***

*Let B can do the job in days* $= x$

*Work done by B in one day* $= \frac{1}{x}$

*A can do the job in days* $= 2x$

*Work done by A in one day* $= \frac{1}{2x}$

*Work done by both A and B in one day* $= \frac{1}{x} + \frac{1}{2x}$

***Given that,***

*both A and B can do the job in days* $= 4$

*both A and B can do the job in one days* $= \frac{1}{4}$

***So,***

$\frac{1}{x} + \frac{1}{2x} = \frac{1}{4}$

***Multiplying by*** $4x$

$4x \cdot \frac{1}{x} + 4x \cdot \frac{1}{2x} = \frac{1}{4} \cdot 2x$

$4 + 2 = x \Rightarrow x = 6$

***If*** $x = 6$ ***then, from*** $(4)$

$2x = 2(6) = 12$

***Hence B can do job in 6 days while A can do job in 12 days.***

***Question#19***

***An open box is to be made from a square piece of tin by cutting a piece 2 dm square from each corner and then folding the sides of the remaining piece. If the capacity of the box is to be finish 128 c.dm, find the length of the side of the piece***

***Solution:***

*Let length of piece of square tin= xdm*

*After cutting* $2dm^2$ *from each corner*

*Length of box* $= x - 4$ *dm*

*Width of box* $= x - 4$ *dm*

*Height of box* $= 2$ *dm*

*We know that*

*Volume of box = Length × Width × Height*

*So,*

$128 = (x - 4)(x - 4).2$

$(x - 4)^2 = 64$

$(x - 4)^2 = (8)^2$

$x - 4 = 8 \Rightarrow x = 8 + 4 = 12$

*So,*

$x = 12dm$ *is the length of the square tin piece*

***Question#20***

***A man invests Rs. 100,000 in two companies. His total profit is Rs.3080. If he receives Rs. 1980 from one company and at the rate 1% more from the other, find the amount of each investment.***

***Solution:***

*Let I and II be the two companies. Now let*

*Investment in company A* $= x$ *Rs.*

*Investment in company B* $= 100000 - x$ *Rs.*

*Profit role in company I* $= yz$

*Profit role in company II* $= (y + 1)z$

*As we know that,*

$Profit = \frac{Amount \times Rate \times Period}{100}$

So,

$1980 = \frac{x \times y \times 1}{100} \Rightarrow xy = 198000 \quad ....(1)$

***Also,***

$3080 = \frac{(100000 - x) \times (y+1) \times 1}{100}$

$(100000 - x)(y + 1) = 3080$

$100000y + 100000 - xy - x = 308000$

$100000y - xy - x = 308000 - 100000$

$100000y - 198000 - x = 208000$

$100000y - x = 208000 + 198000$

$100000y - x = 406000 \quad ...(2)$

From $(1)$ $x = \frac{198000}{y} \quad ...(3)$

***Putting value of*** $x$ ***in eq.***$(2)$***, we get***

$100000y - \frac{198000}{y} = 406000$

$100000y^2 - 198000 = 406000y$

$50y^2 - 99 = 203y$

$50y^2 - 203y - 99 = 0$

***Using*** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \frac{-(-203) \pm \sqrt{(-203)^2 - 4(50)(-99)}}{2(50)}$

$y = \frac{203 \pm \sqrt{61009}}{100}$

$y = \frac{203 \pm 247}{100}$

$y = \frac{450}{100}, y = \frac{-44}{100}$

$y = 4.5, y = -0.44$ *(Impossible)*

Putting value of y in eq. (3)

$x = \frac{198000}{4.5} \Rightarrow x = 44,000$

*Investment in company I* $= 44,000$ *Rs.*

*Thus amount invested in II=*

*100,000-44000=56000*